$$v ::= x \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{fun}\ x \mapsto c \mid v \triangleright \gamma_v$$

$$\begin{aligned} c ::= {} & \mathsf{if}\ v\ \mathsf{then}\ c_1\ \mathsf{else}\ c_2 \mid v_1\, v_2 \mid \\ & \mathsf{do}\ x \leftarrow c_1\ \mathsf{in}\ c_2 \mid \mathsf{return}\ v \mid \dots \mid c \triangleright \gamma_c \end{aligned}$$

$$A ::= \mathsf{bool} \mid A \to \underline{C}$$

$$\underline{C} ::= A\,!\,\Delta$$

$$\frac{c : \underline{C} \qquad \gamma : \underline{C} \leq \underline{C}'}{c \triangleright \gamma : \underline{C}'}$$

$$\gamma_v ::= \langle \mathsf{bool} \rangle \mid \gamma_v \to \gamma_c$$

$$\gamma_c ::= \gamma_v\,!\,\gamma_D$$

$$(\mathsf{do}\ x \leftarrow c_1\ \mathsf{in}\ c_2)^* = \begin{cases} \mathsf{let}\ x = c_1^*\ \mathsf{in}\ c_2^* & \text{if } c_1 \text{ pure } / \ c_1 : A\,!\,\emptyset \\ c_1^* \gg\!= (\mathsf{fun}\ x \to c_2^*) \end{cases}$$

$$(A\,!\,\Delta)^* = \begin{cases} A^* & \Delta = \emptyset \\ A^*\ \mathsf{comp} \end{cases}$$

$$(c \triangleright \gamma)^* = c^* \mid\!> \gamma^*$$

$$(\gamma_v\,!\,\gamma_D)^* = \begin{cases} \vdots \end{cases}$$

ALGOL 2030

let applyzero f = f 0

applyzero cos

ExALGOL 2030

$$\text{let applyzero}_{\alpha\,\beta}\ \underset{(\omega : \text{int} \leq \alpha)}{}\ (f : \alpha \to \beta) = f\ (0 \triangleright \omega)$$

$$\forall \alpha, \beta.\ (\text{int} \leq \alpha) \Rightarrow (\alpha \to \beta) \to \beta$$

$$\text{applyzero}_{\text{float float int2float}}\ \text{cos}$$

OCAML

let apply_zero w f = f (w 0)

(int -> 'a) -> ('a -> 'b) -> 'b

apply_zero int_to_float cos

$$\text{let applyzero}'_{\beta}\ (f : \text{int} \to \beta) = f\ 0$$

$$\text{applyzero}'_{\text{float}}\ (\text{cos} \triangleright (\text{int2float} \to \langle \text{float} \rangle))$$

$$= \text{cos} \circ \text{int2float}$$

let applyIf p f x =

  if p x then f x else x

$$(\alpha_1^+ \to \mathrm{bool}) \to (\alpha_2^+ \to \alpha_3^-) \to \alpha_4^- \to \alpha_5^+$$

$$\gamma \;\; \vee\!/$$

$$(\alpha_4^\pm \to \mathrm{bool}) \to (\alpha_4^\pm \to \alpha_5^\pm) \to \alpha_4^\pm \to \alpha_5^\pm$$

$$\begin{tikzcd}
\alpha_1^+ & \alpha_2^+ & \alpha_5^+ \\
\alpha_4^- \arrow[u] \arrow[ur] \arrow[urr] & & \alpha_3^- \arrow[u]
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\alpha_5^+ \\
\alpha_4^- \arrow[u]
\end{tikzcd}$$

$$\ldots \; \mathrm{applyIf}_{A_1 A_2 A_3 A_4 A_5}\, \gamma_1\, \gamma_2\, \gamma_3\, \gamma_4 \; \ldots$$

$$\rightsquigarrow$$

$$\ldots \; (\mathrm{applyIf}'_{A_4' A_5'}\, \gamma_3') \triangleright \gamma \; \ldots$$

In general

$$\Gamma \vdash_{\Xi} v : A$$

$$\mathrm{Simpl}(\Xi, \Gamma, A) = (\Xi', \sigma : \Xi \to \Xi')$$

$$\sigma(\Gamma) \vdash_{\Xi'} \sigma(v) : \sigma(A)$$

$$\forall \eta \models \Xi.\ \exists \eta' \models \Xi'.$$

$$\begin{array}{lll} \eta \geq \eta' \circ \sigma \quad \ldots & \gamma_A : \eta'(\sigma(A)) & \leq \eta(A) \\ & \gamma_\Gamma & \geq \end{array}$$

$$[\![\Gamma \vdash_{\Xi} v : A]\!]_\eta =$$

$$[\![\gamma_A]\!] \circ [\![\sigma(\Gamma) \vdash_{\Xi'} \sigma(v) : \sigma(A)]\!]_{\eta'} \circ [\![\gamma_\Gamma]\!]$$

comp irrelevant