

UVOD V **KVANTNO RAČUNALNIŠTVO**

Matija Pretnar

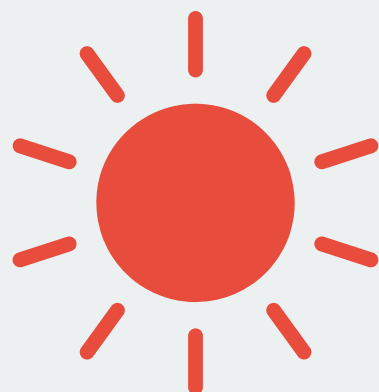


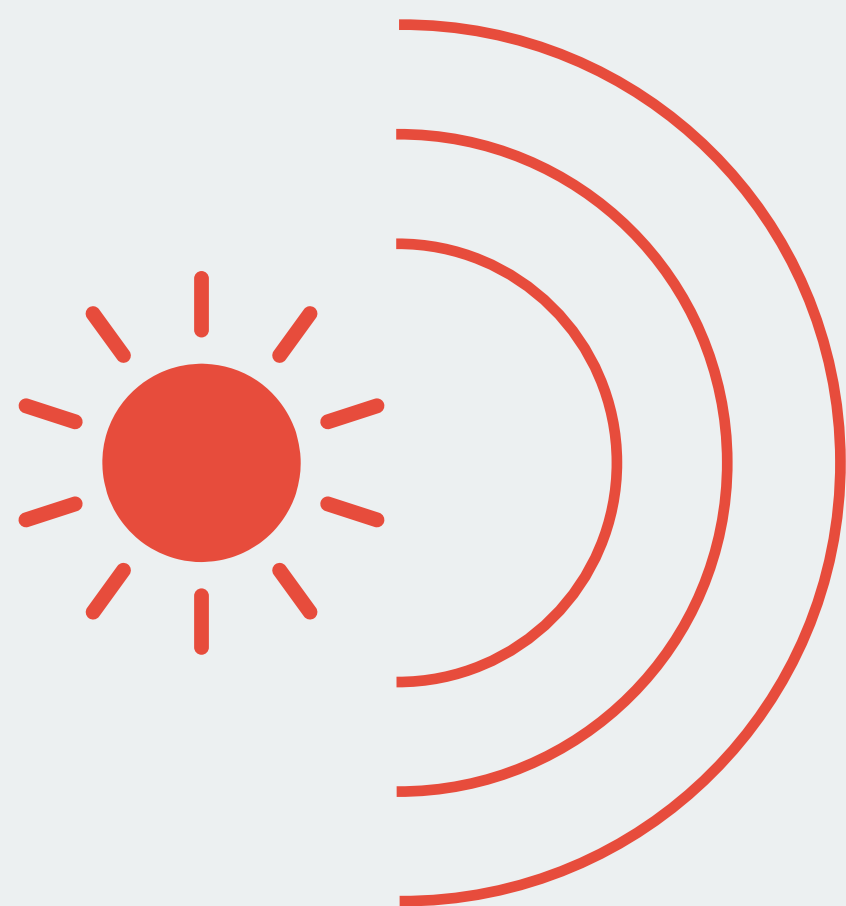
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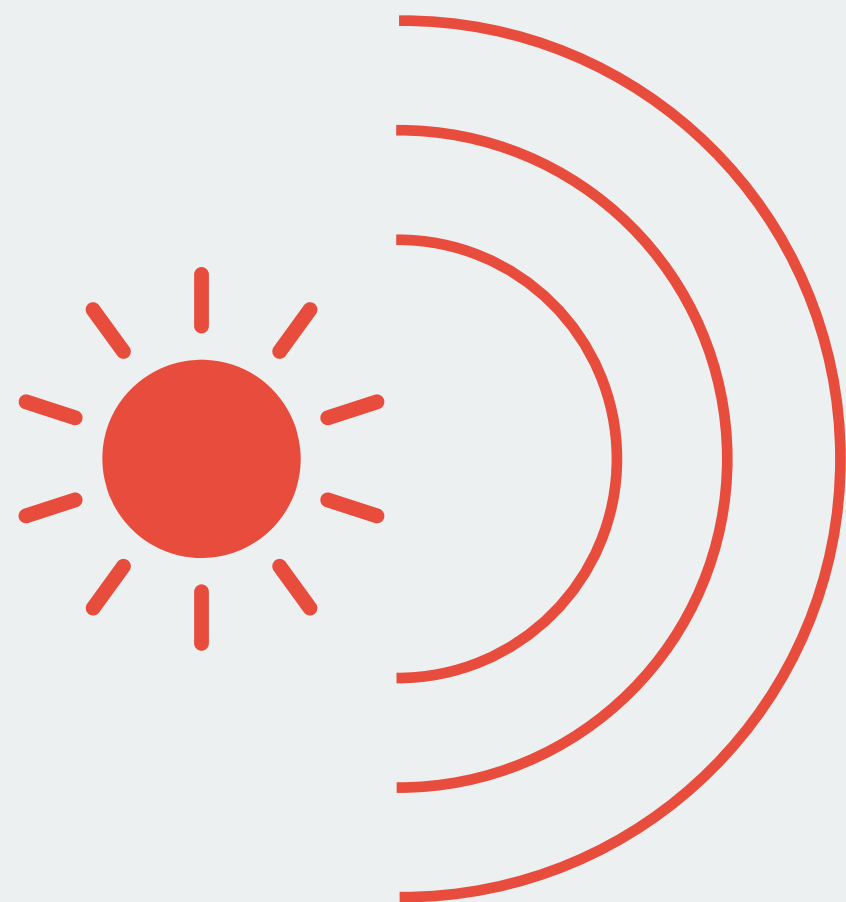
UNIVERZA V LJUBLJANI
Fakulteta za matematiko in fiziko

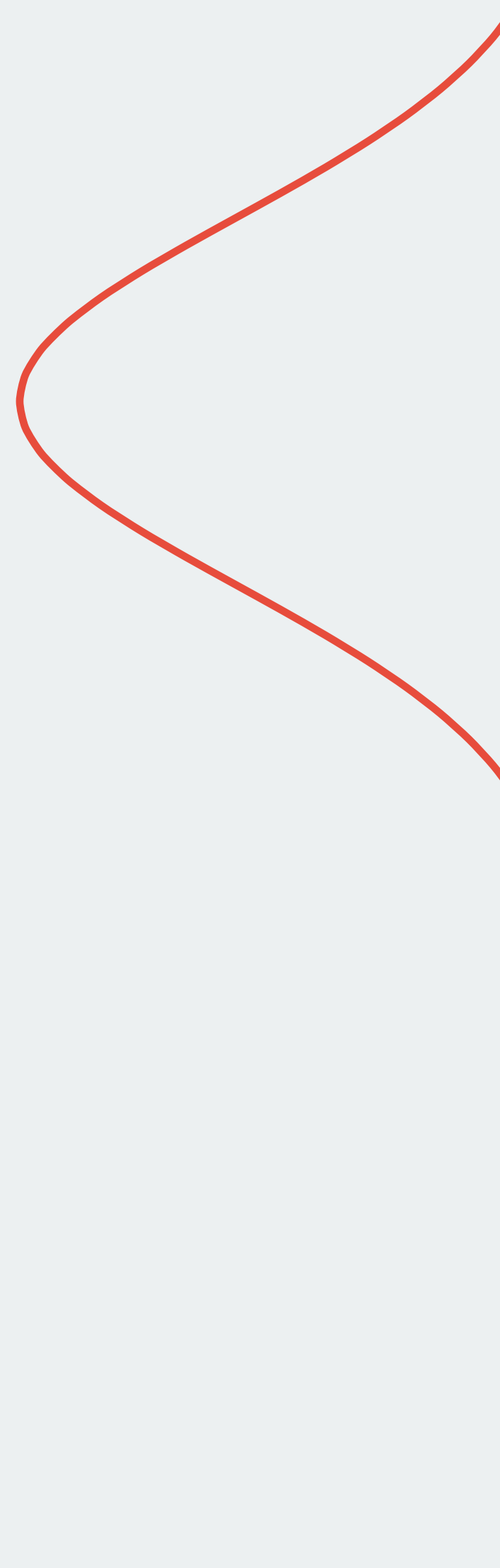
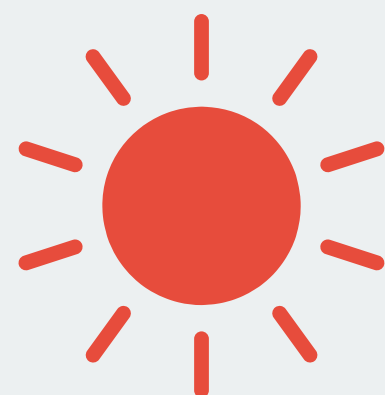
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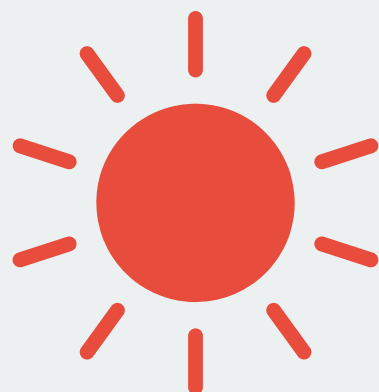
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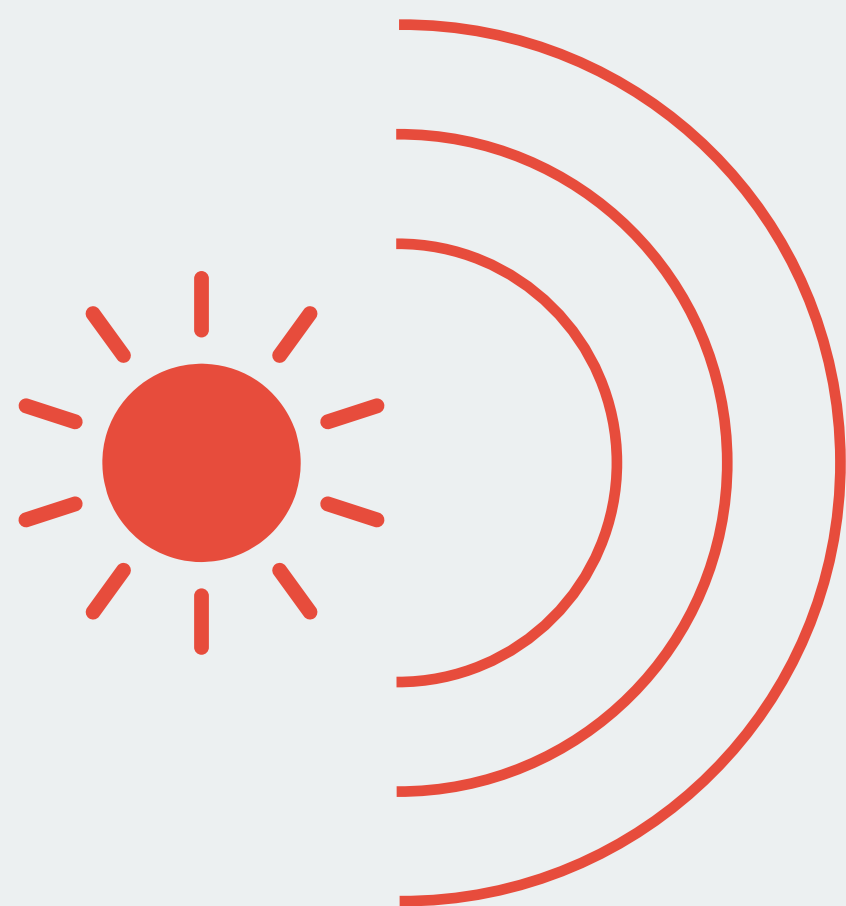




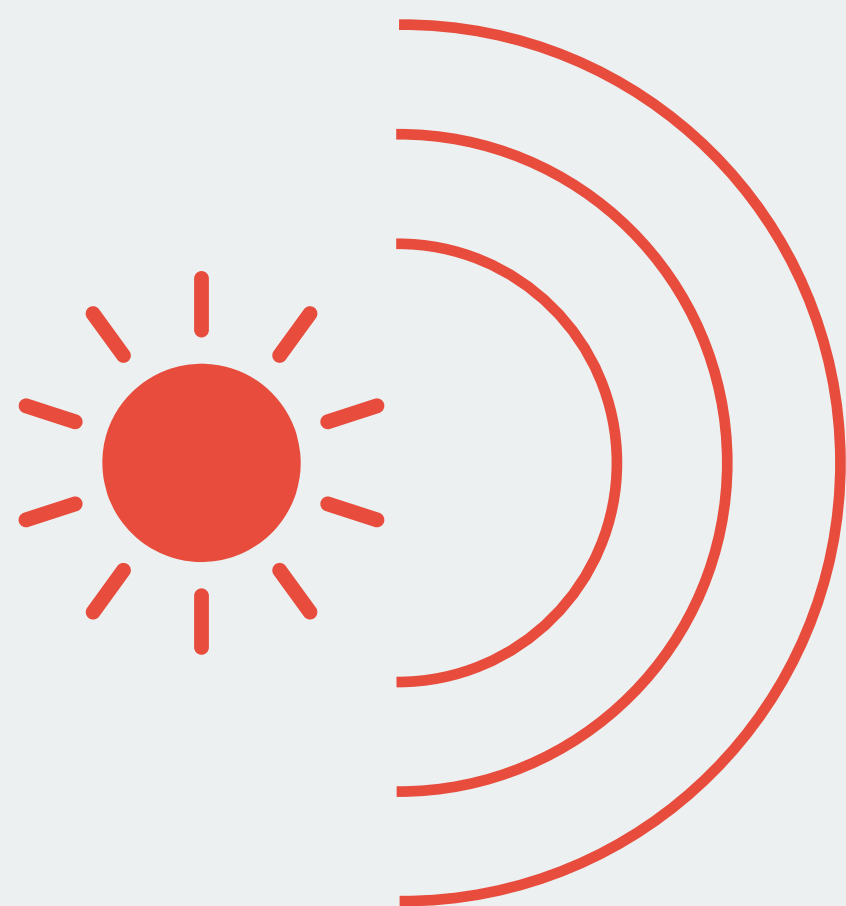


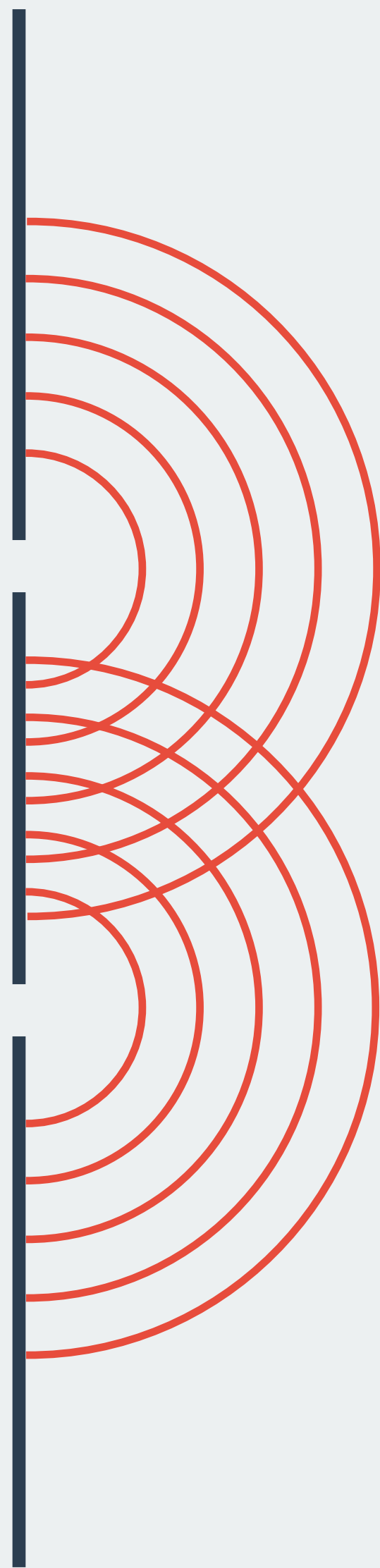
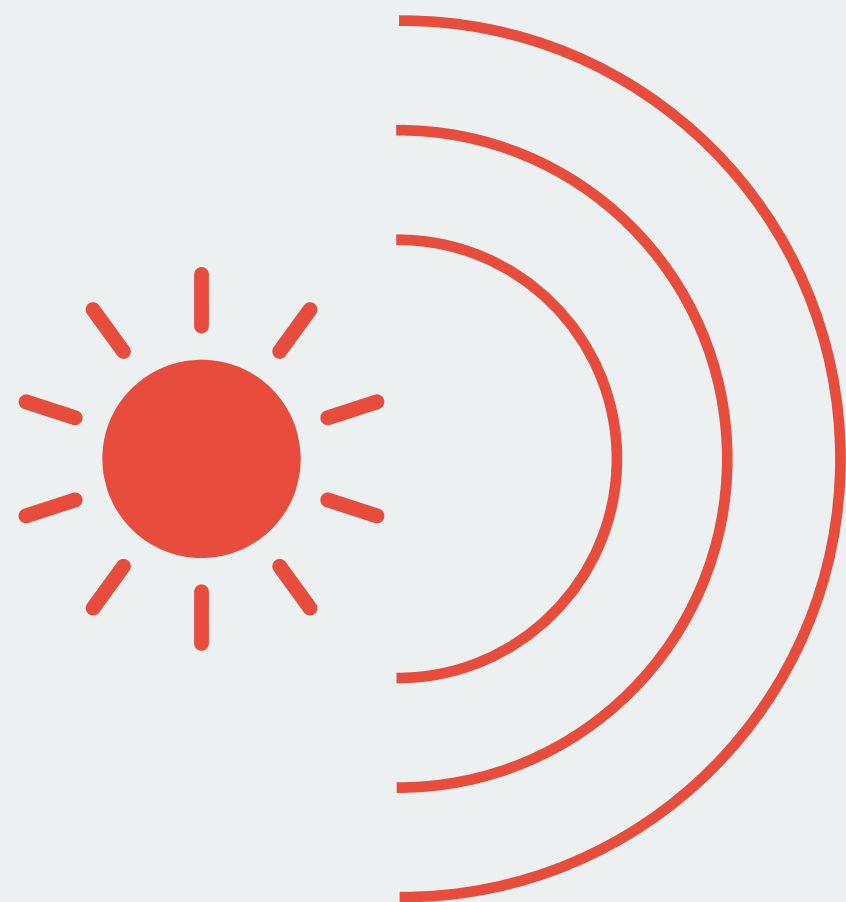




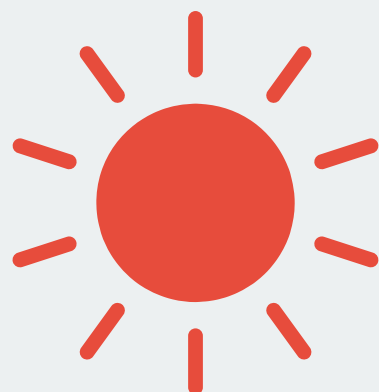




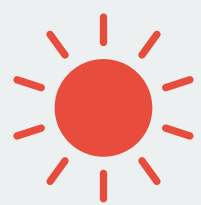




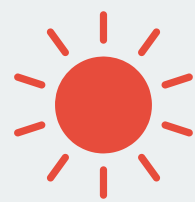


























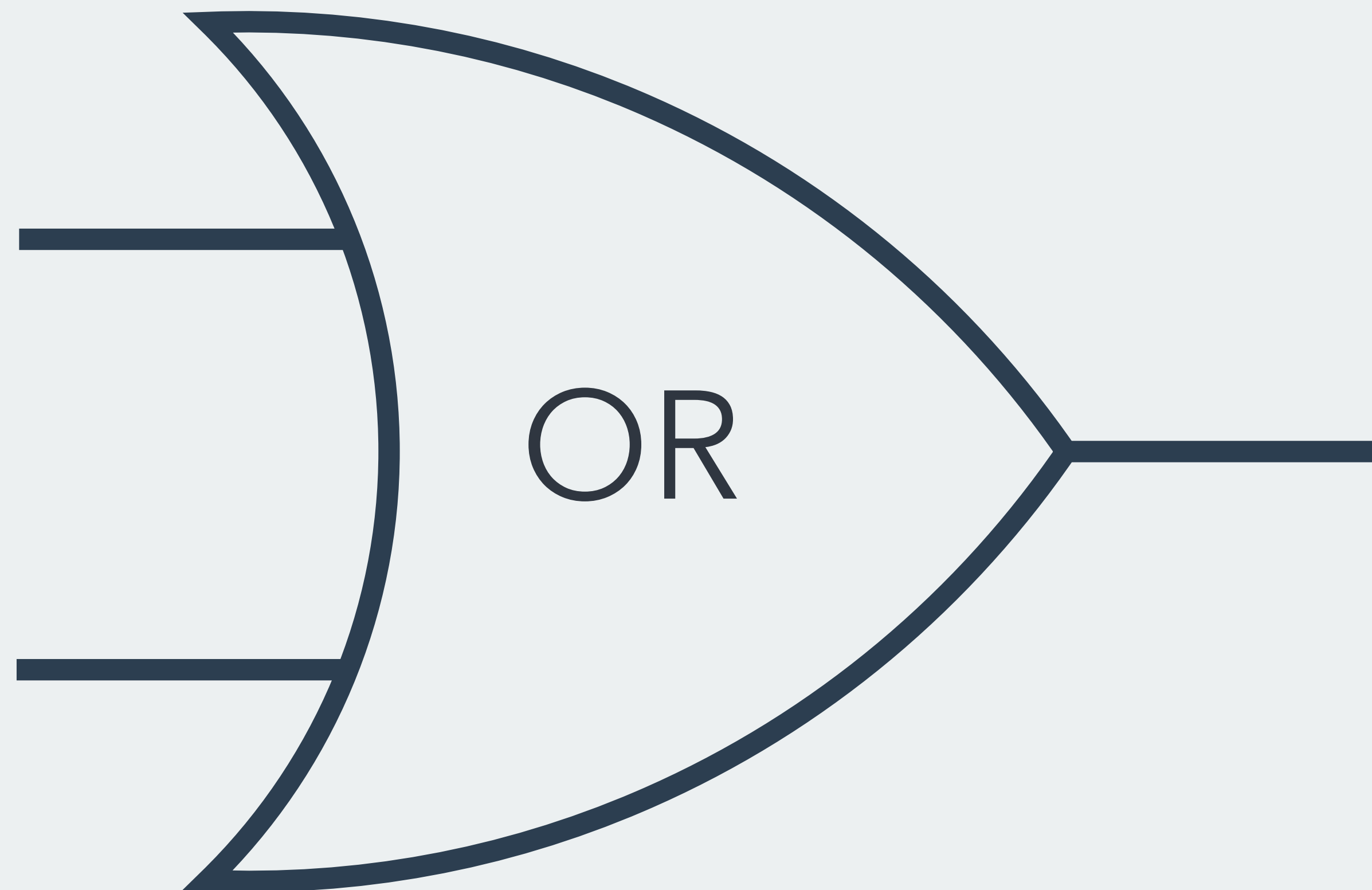
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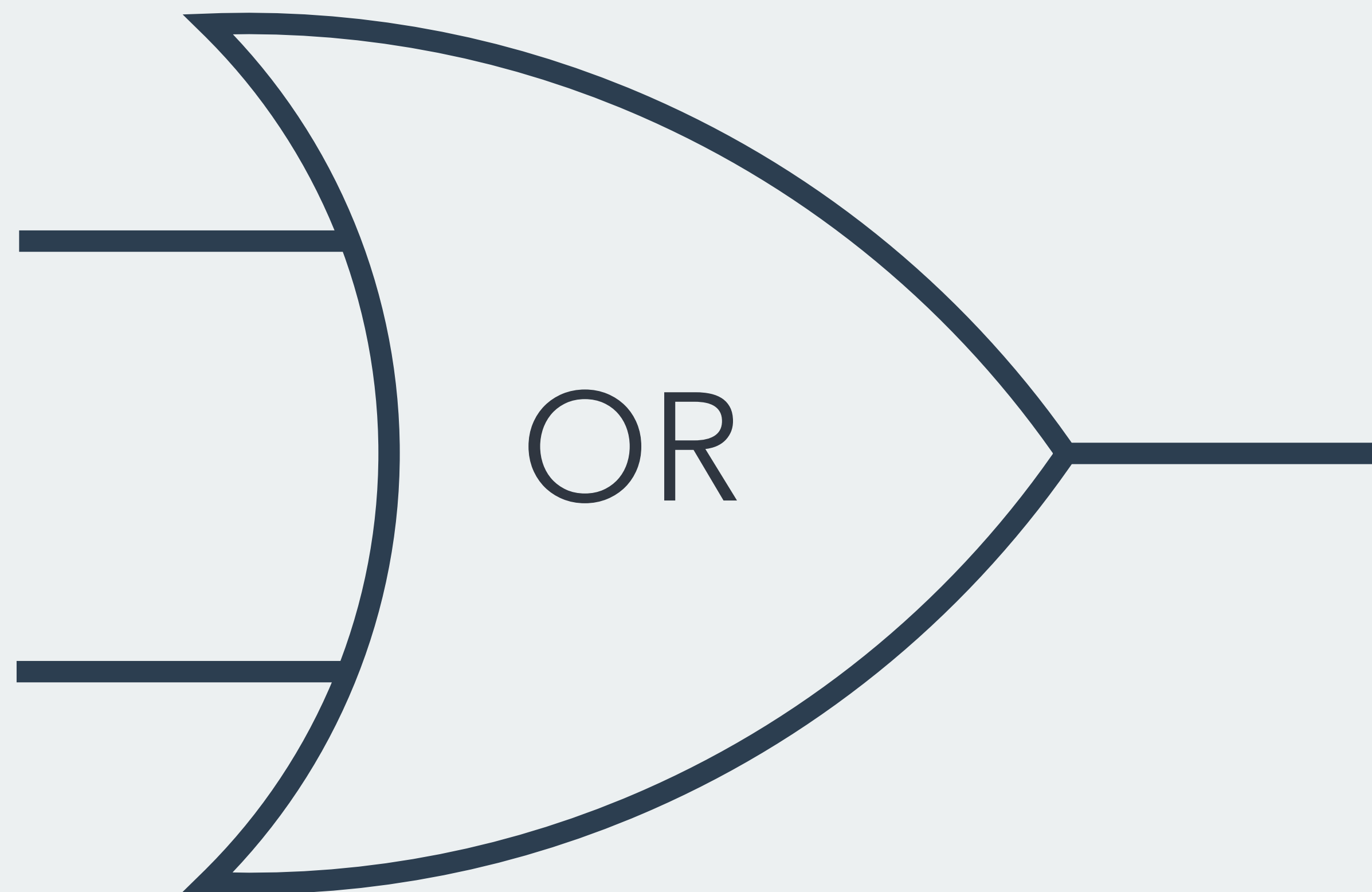
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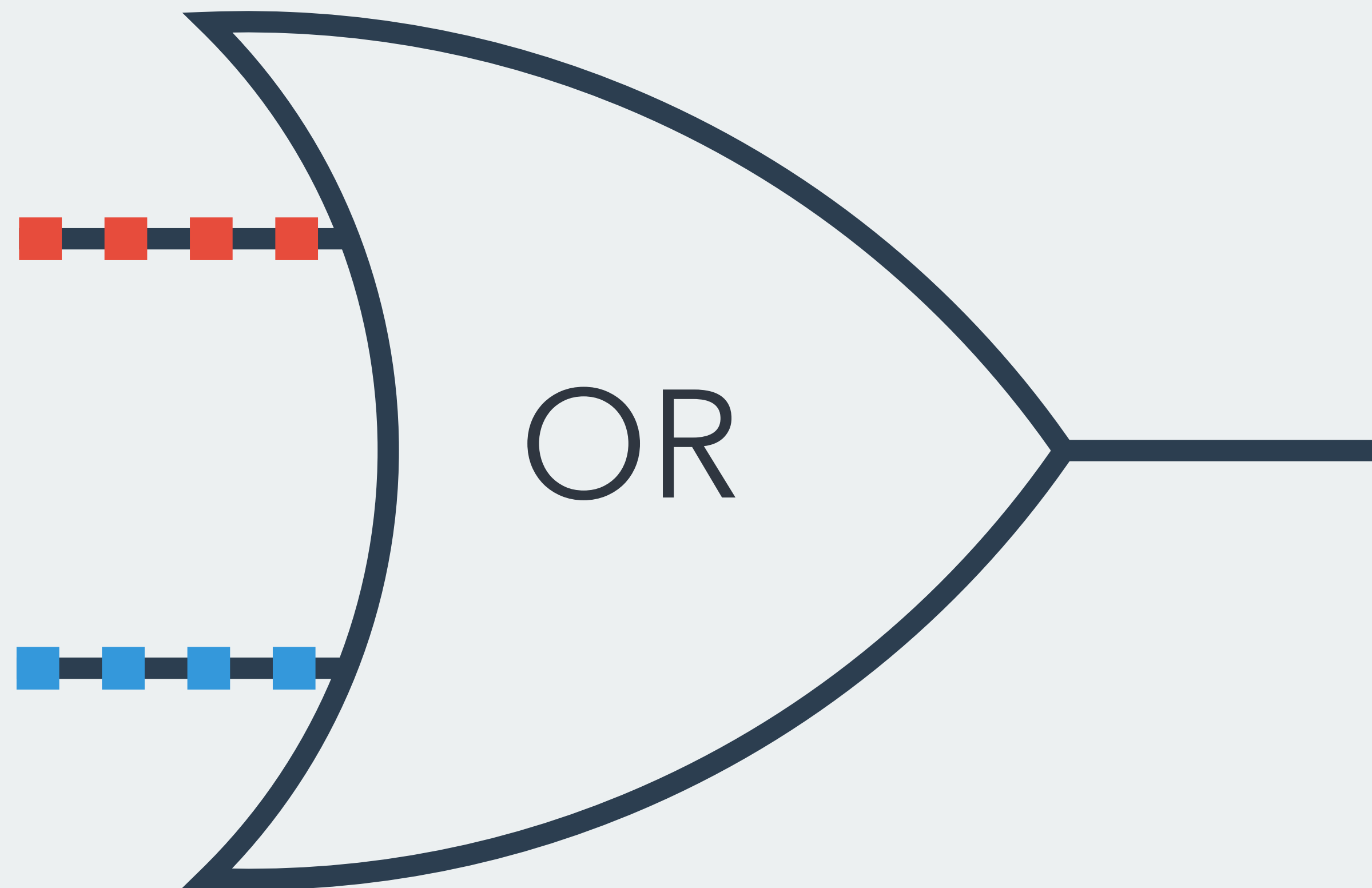


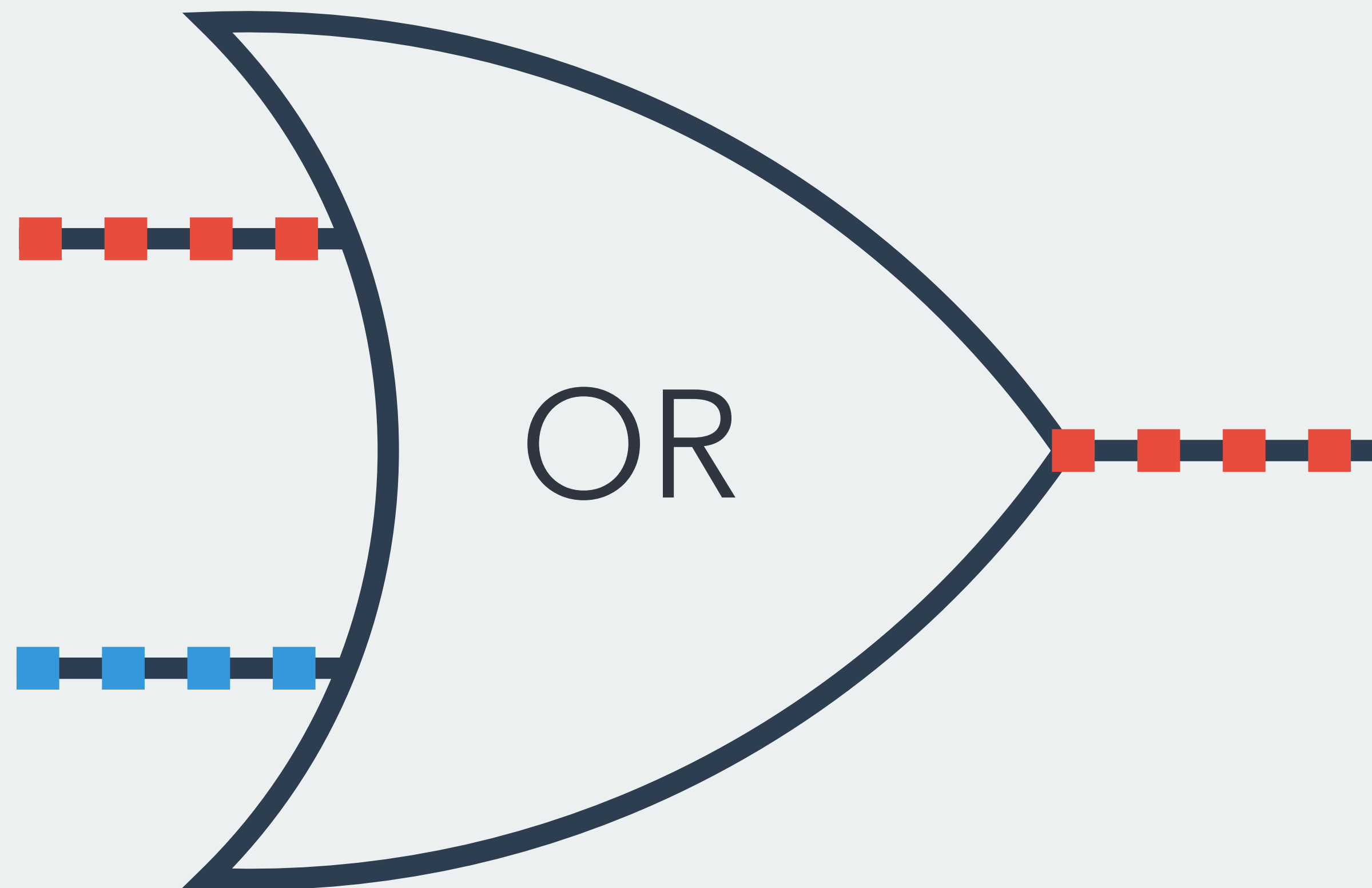
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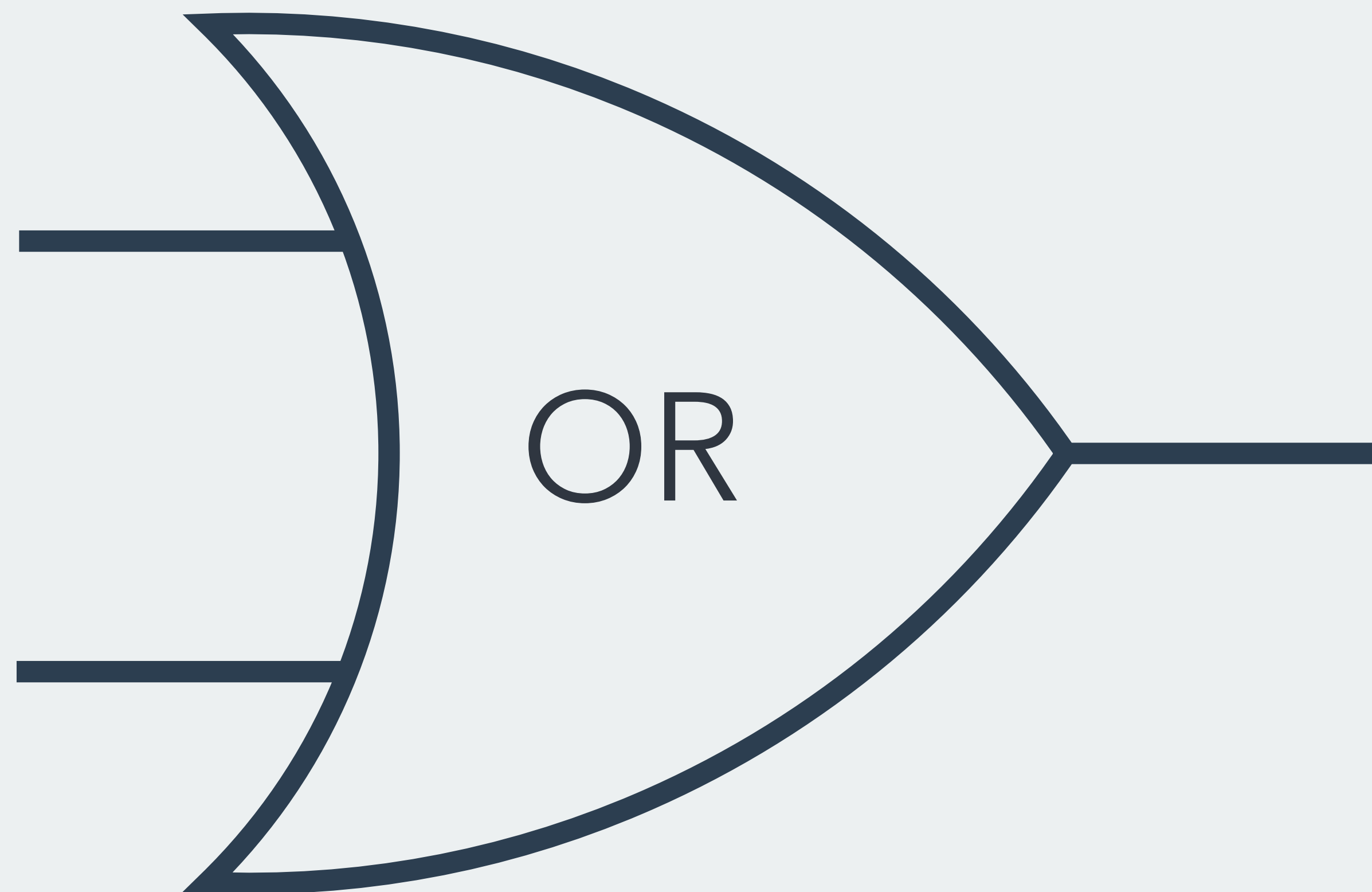
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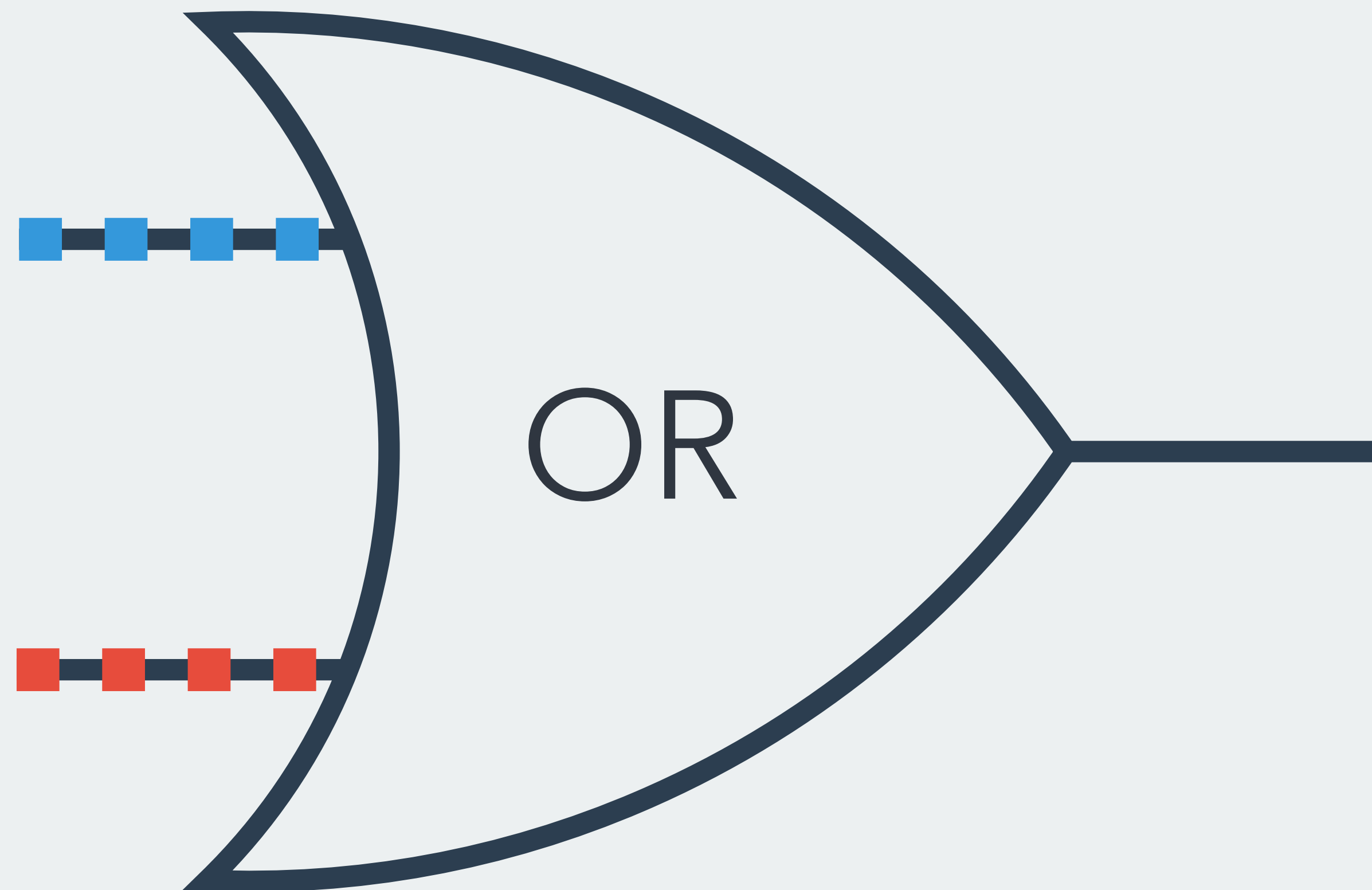


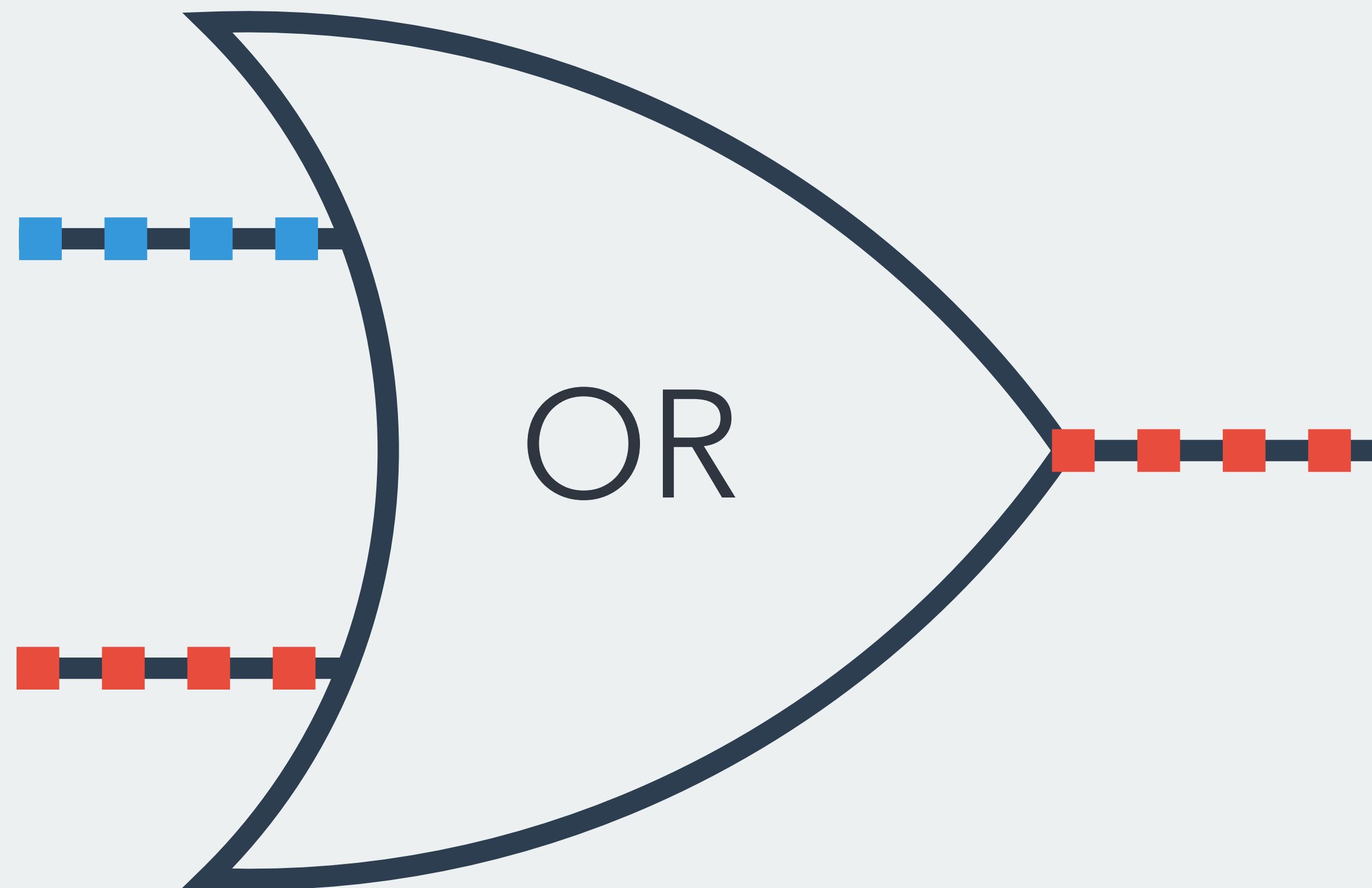


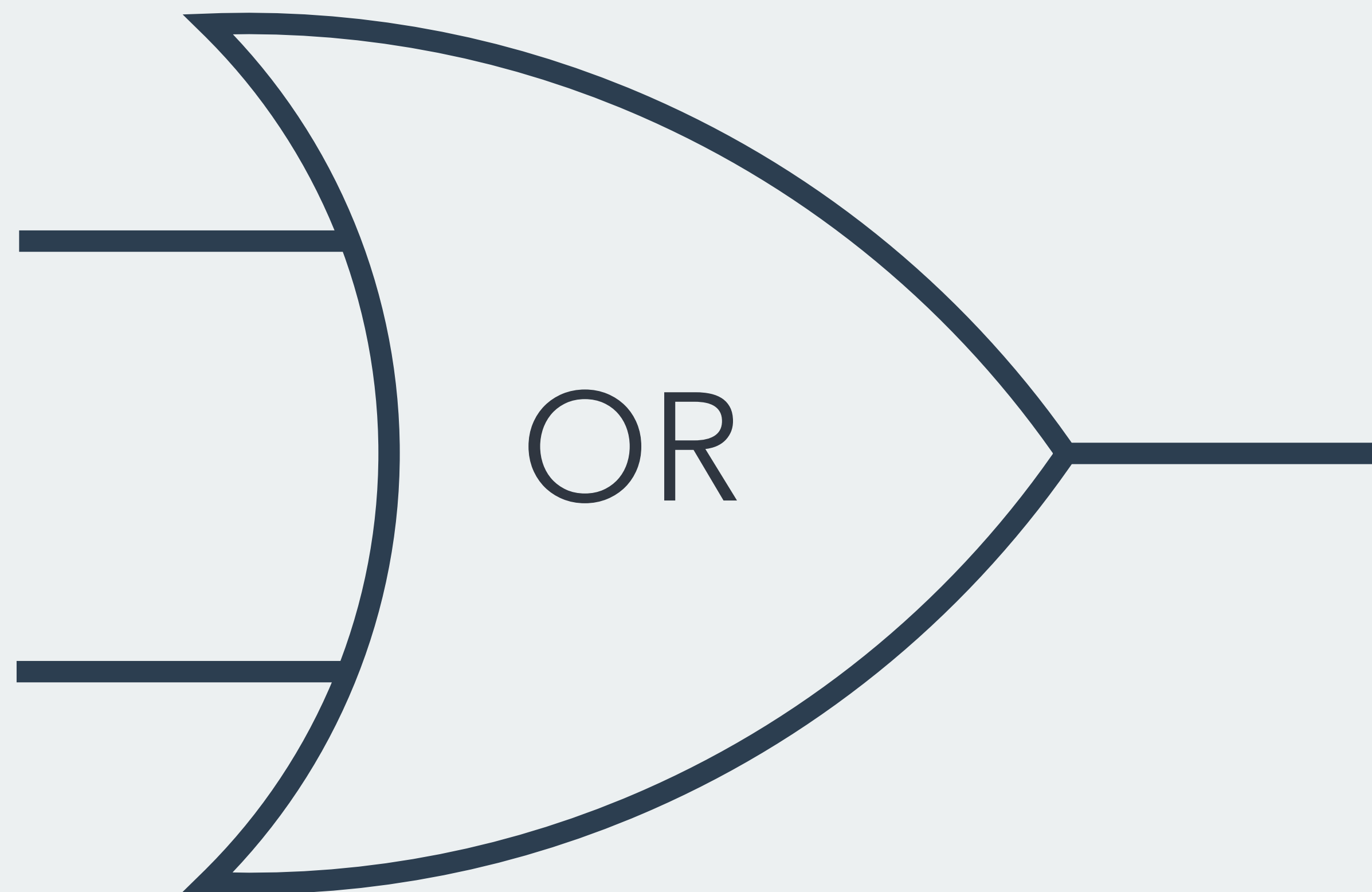


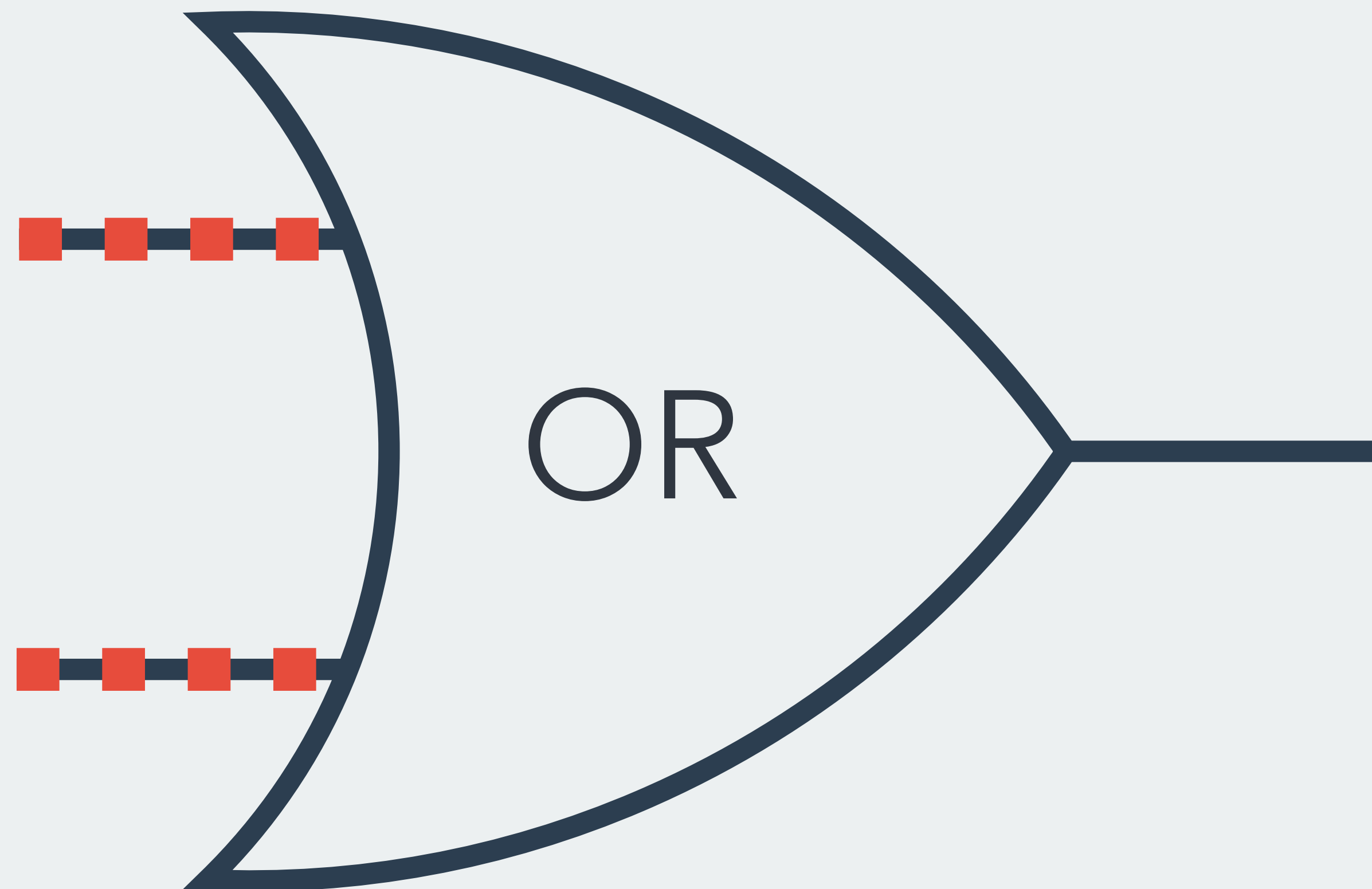


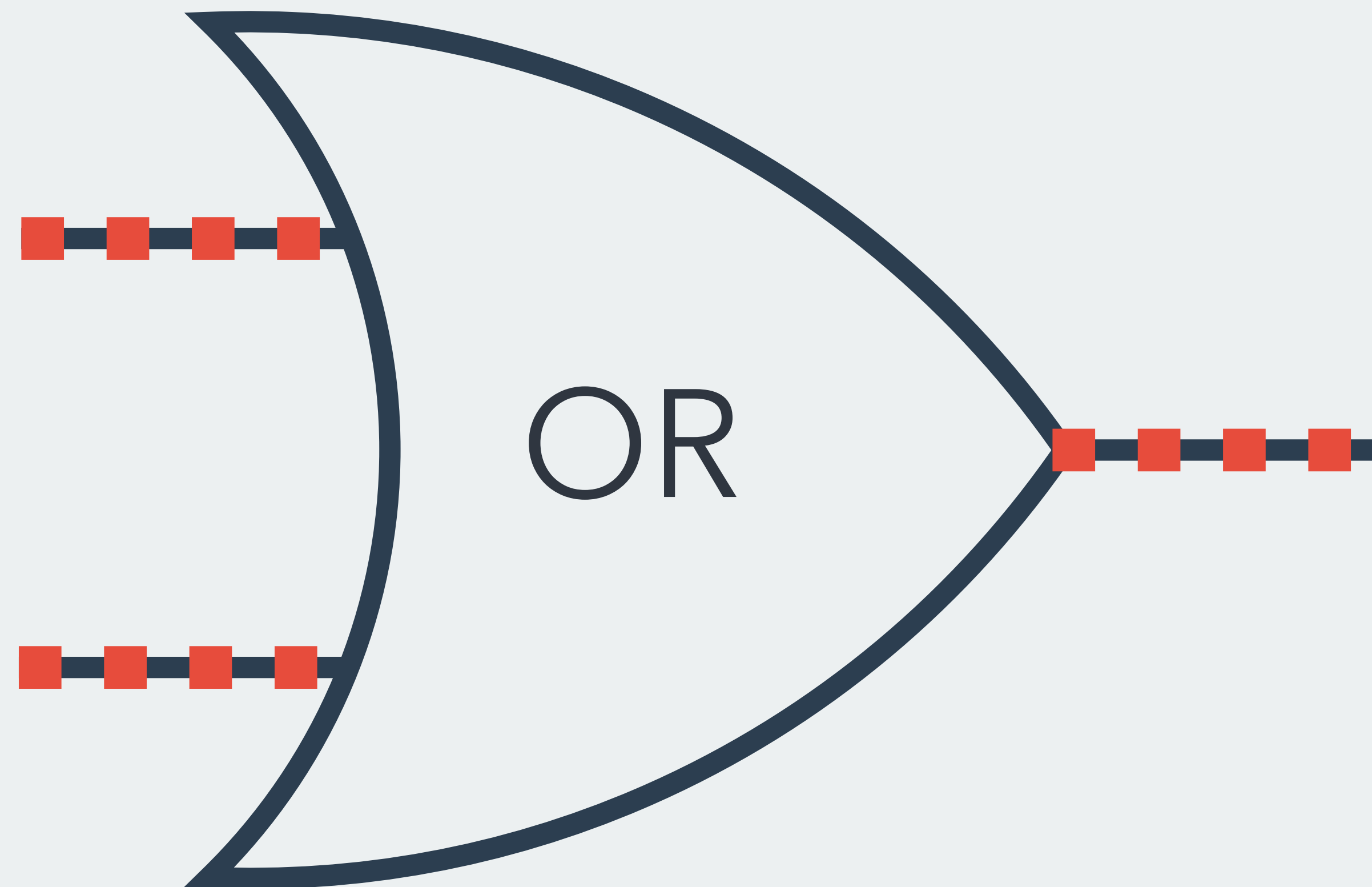


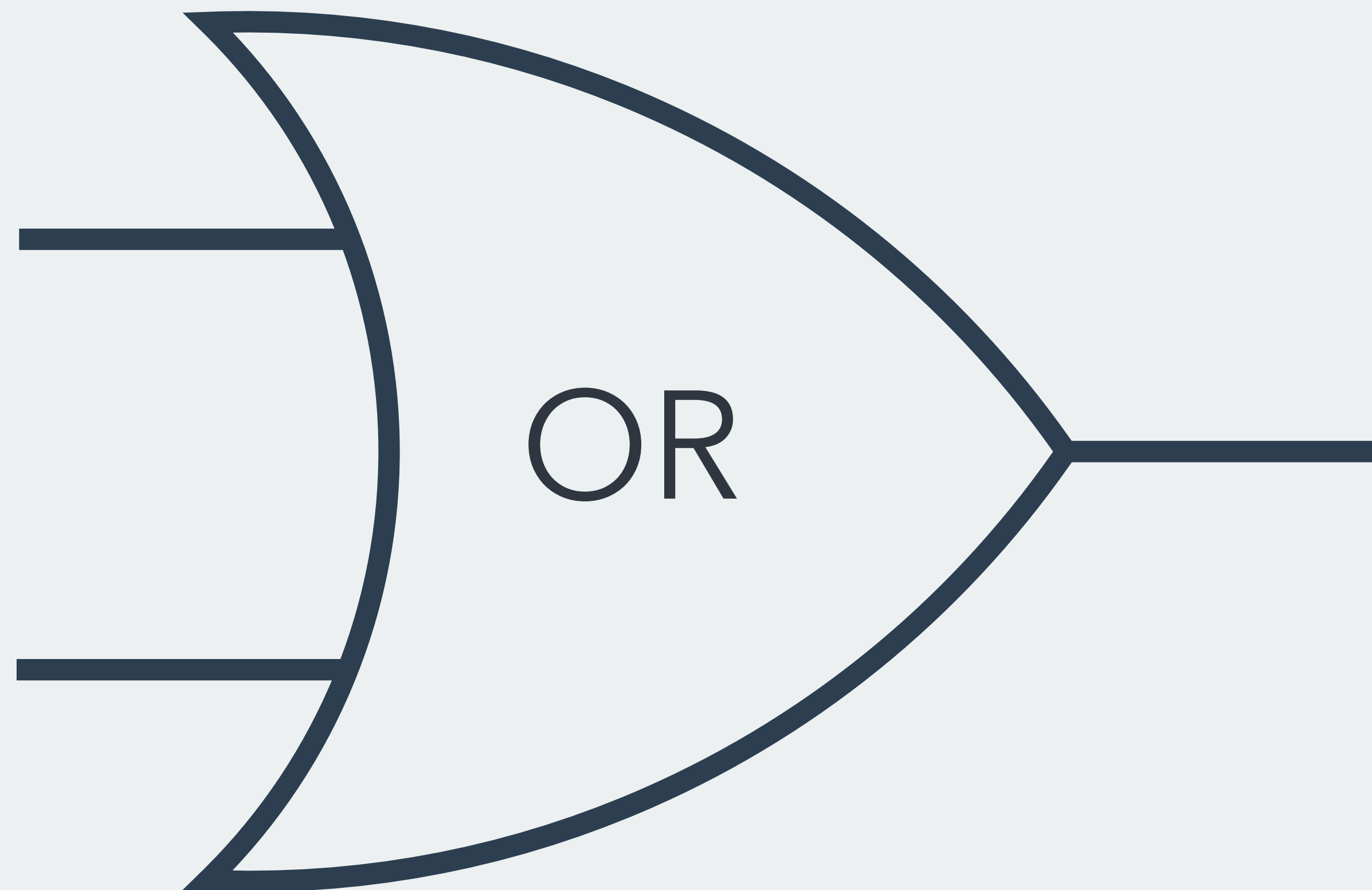


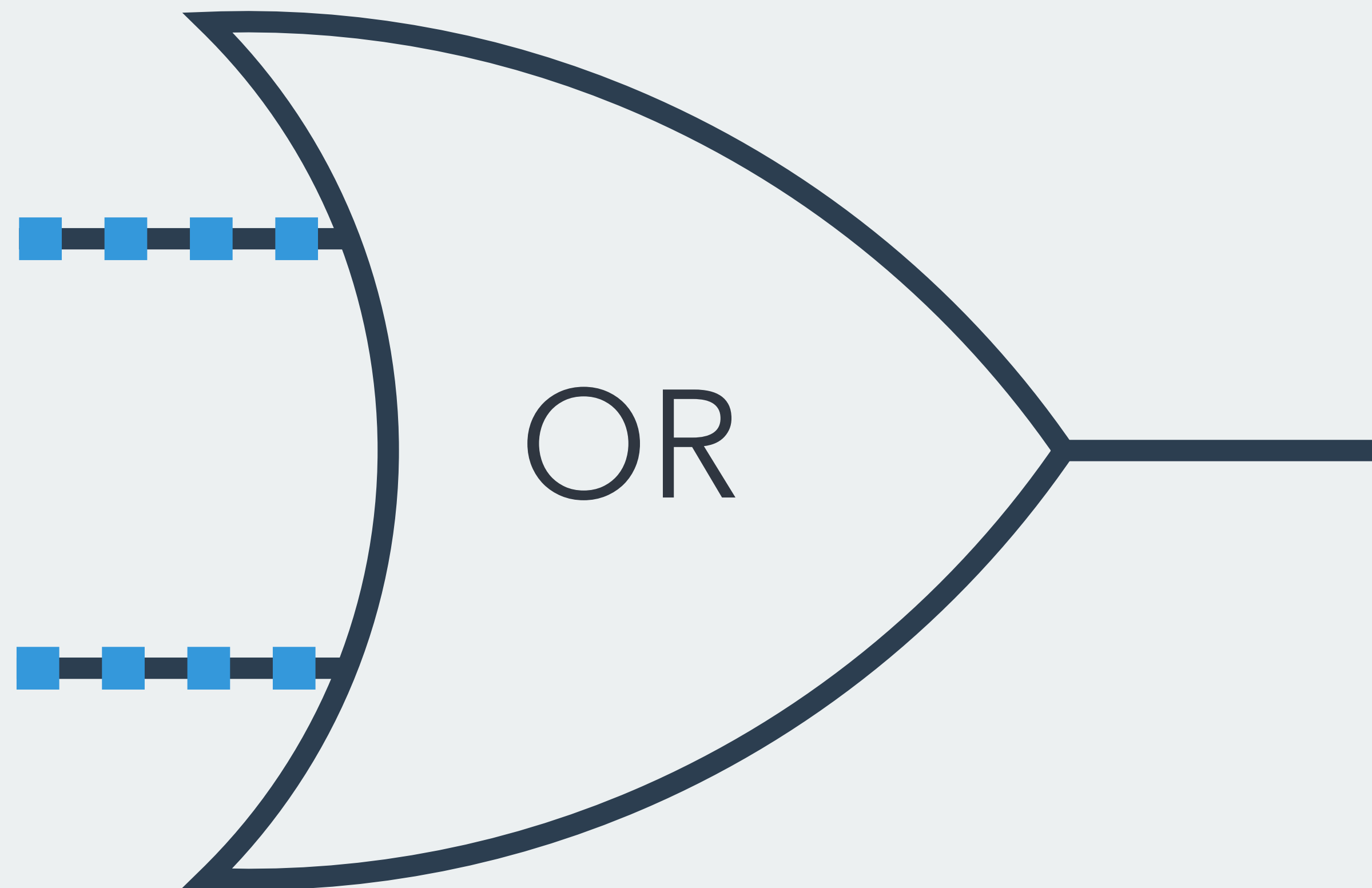


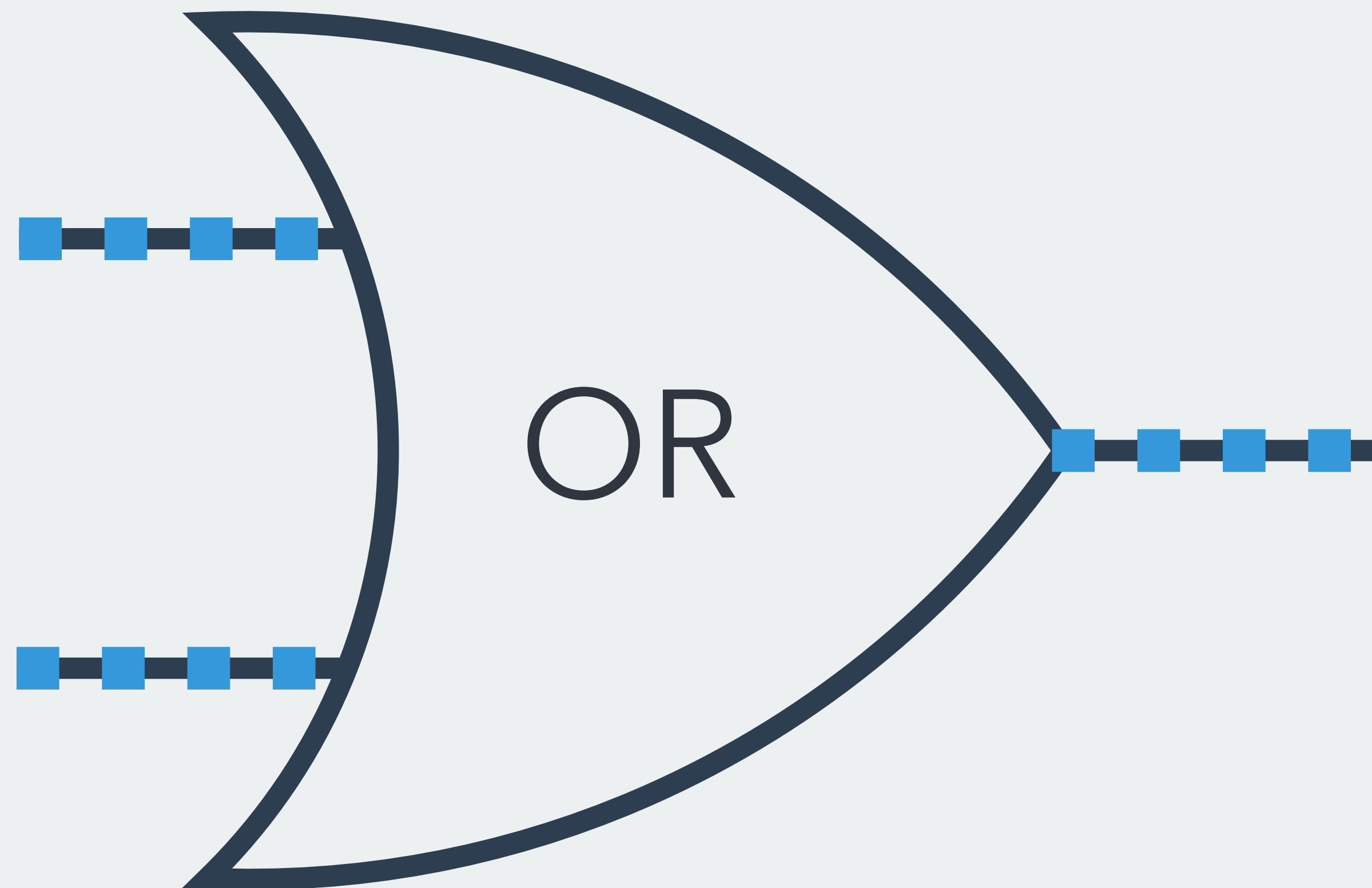


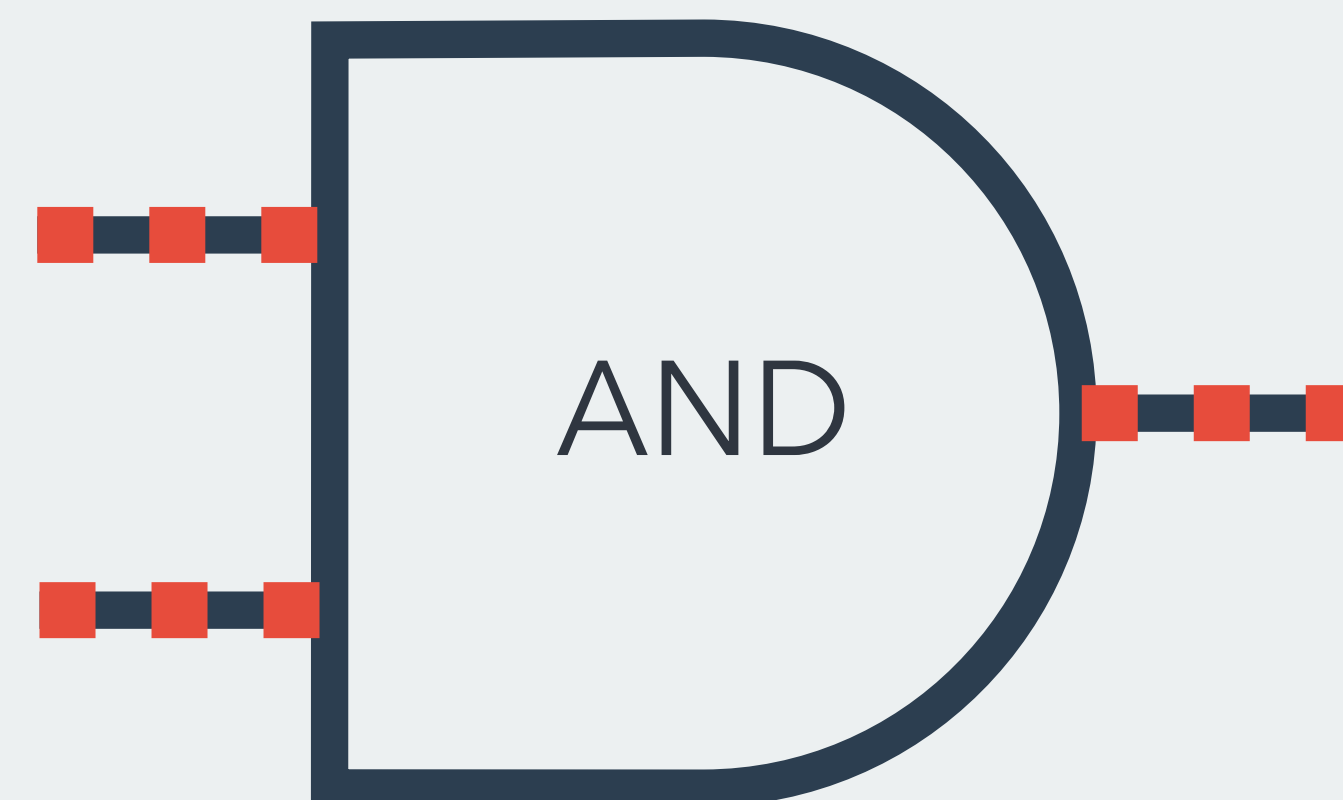
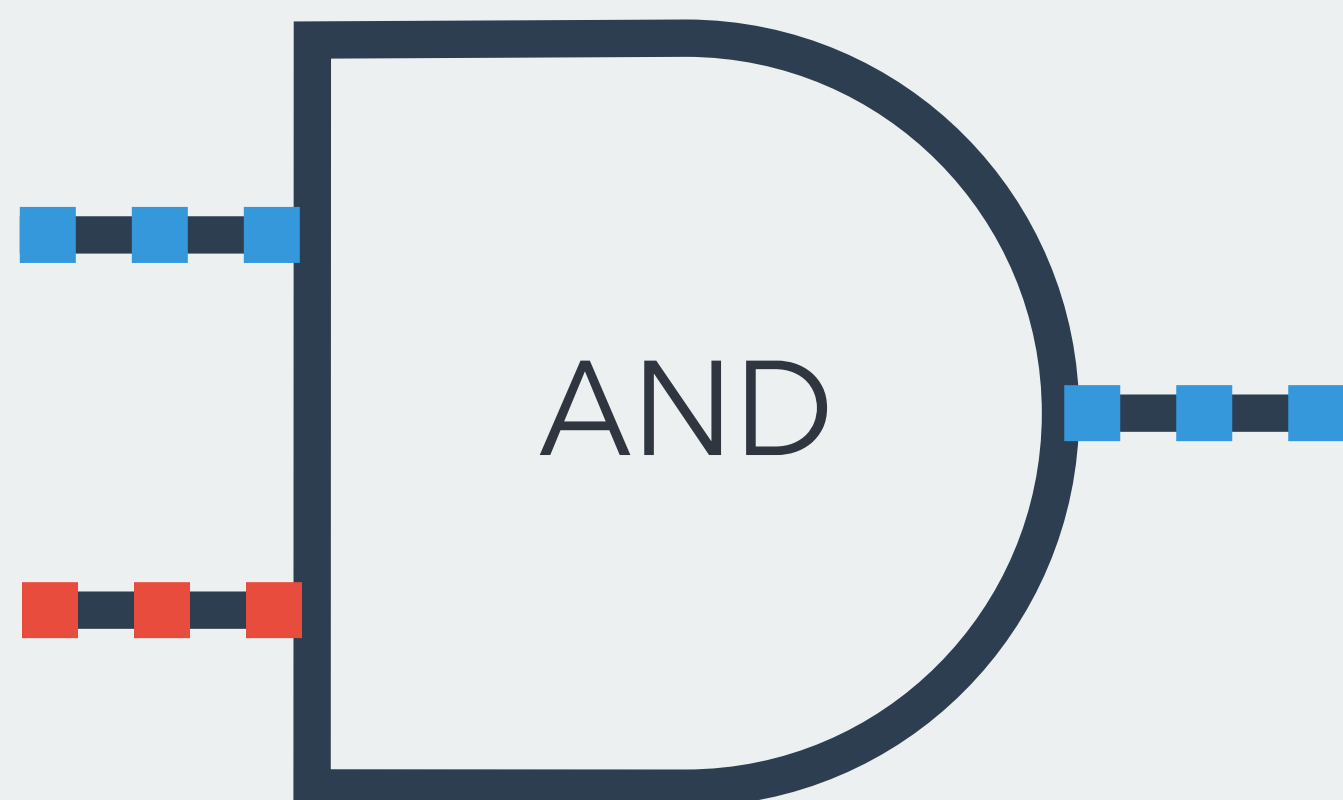
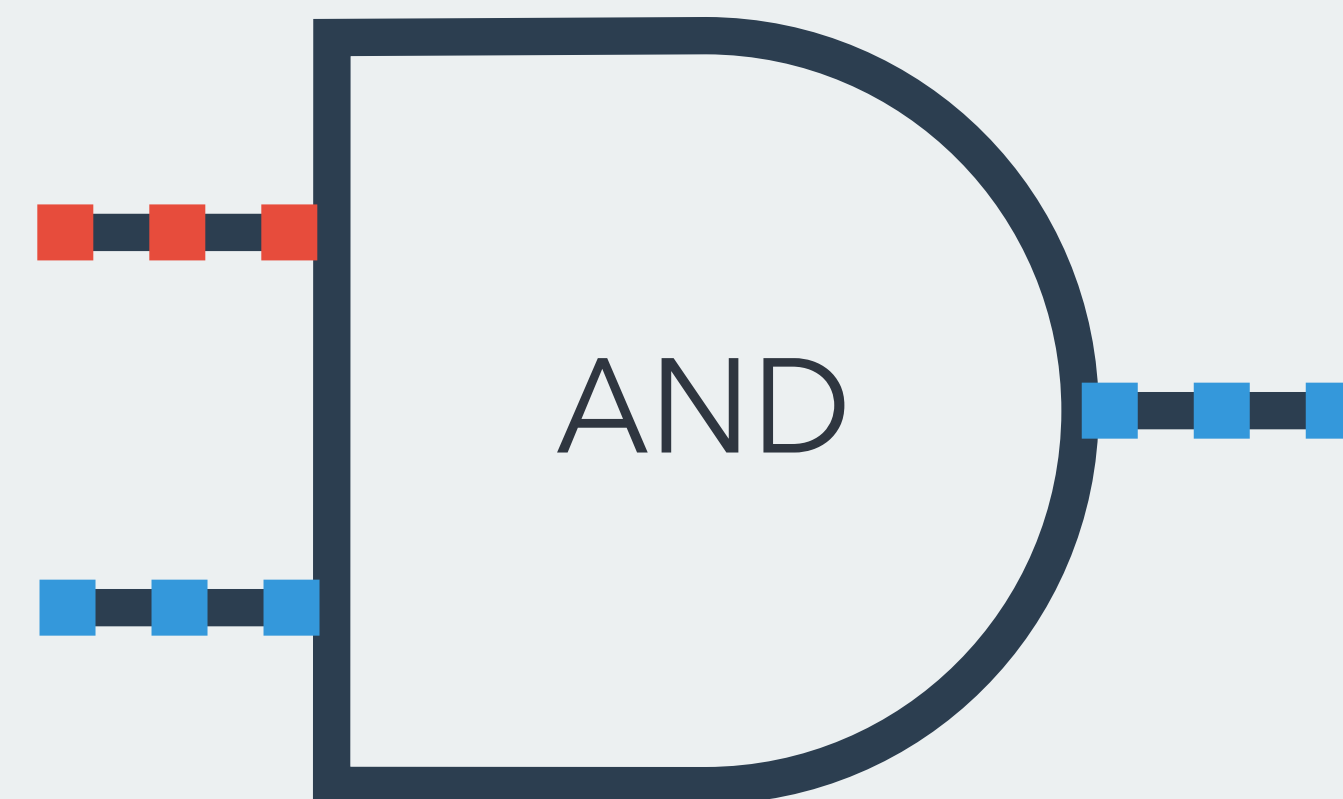
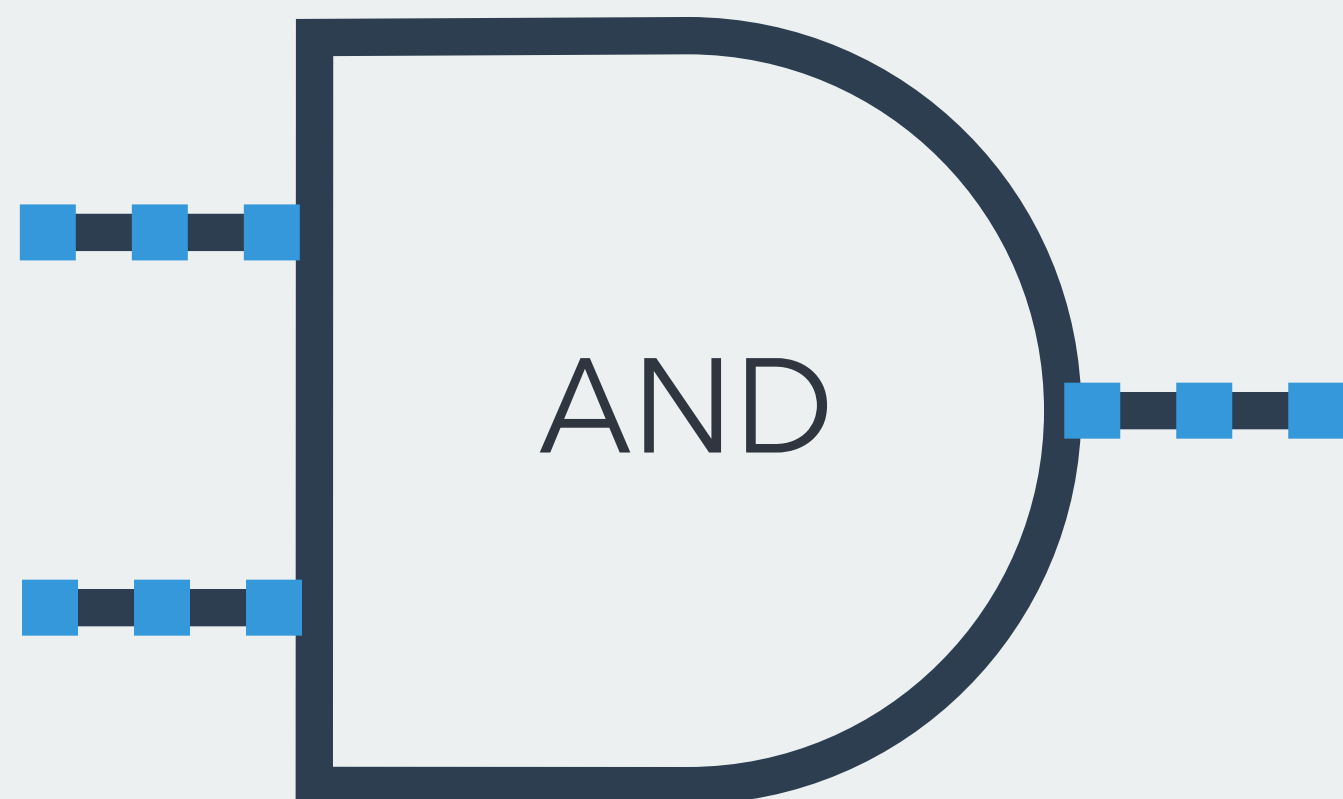


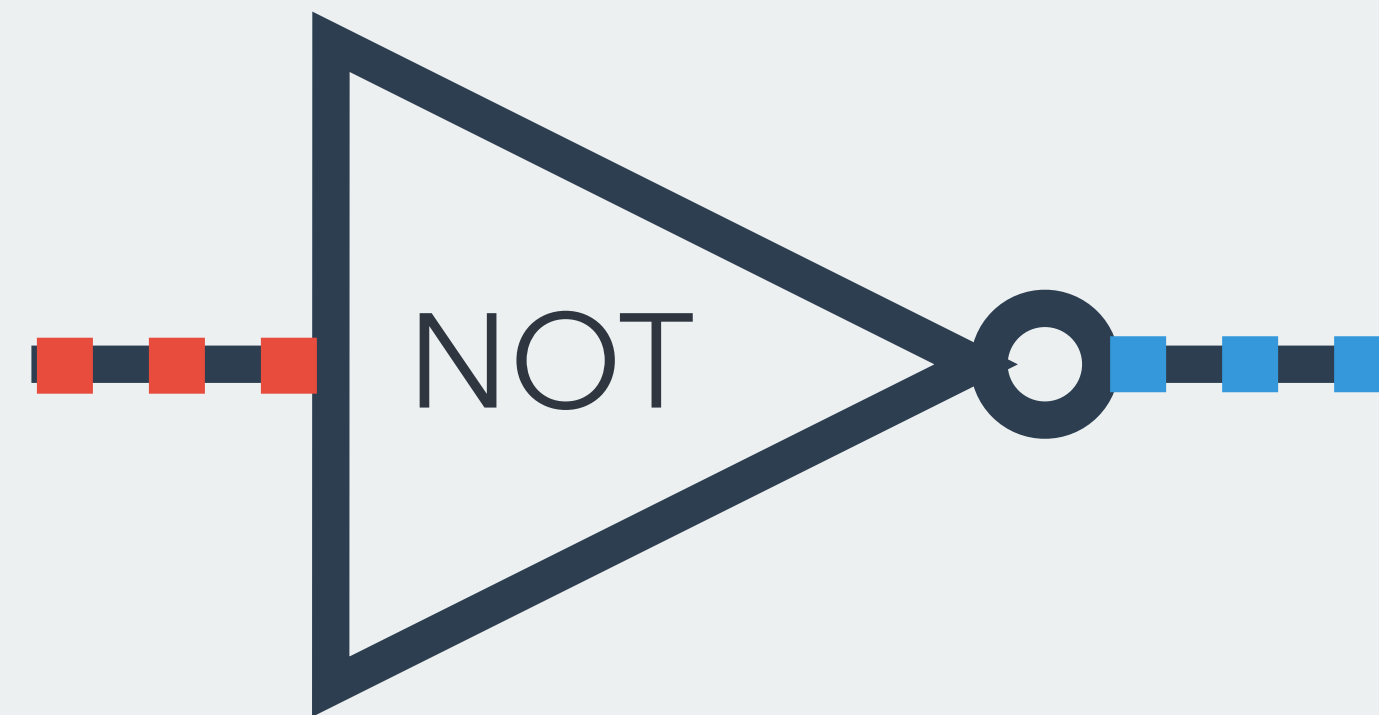
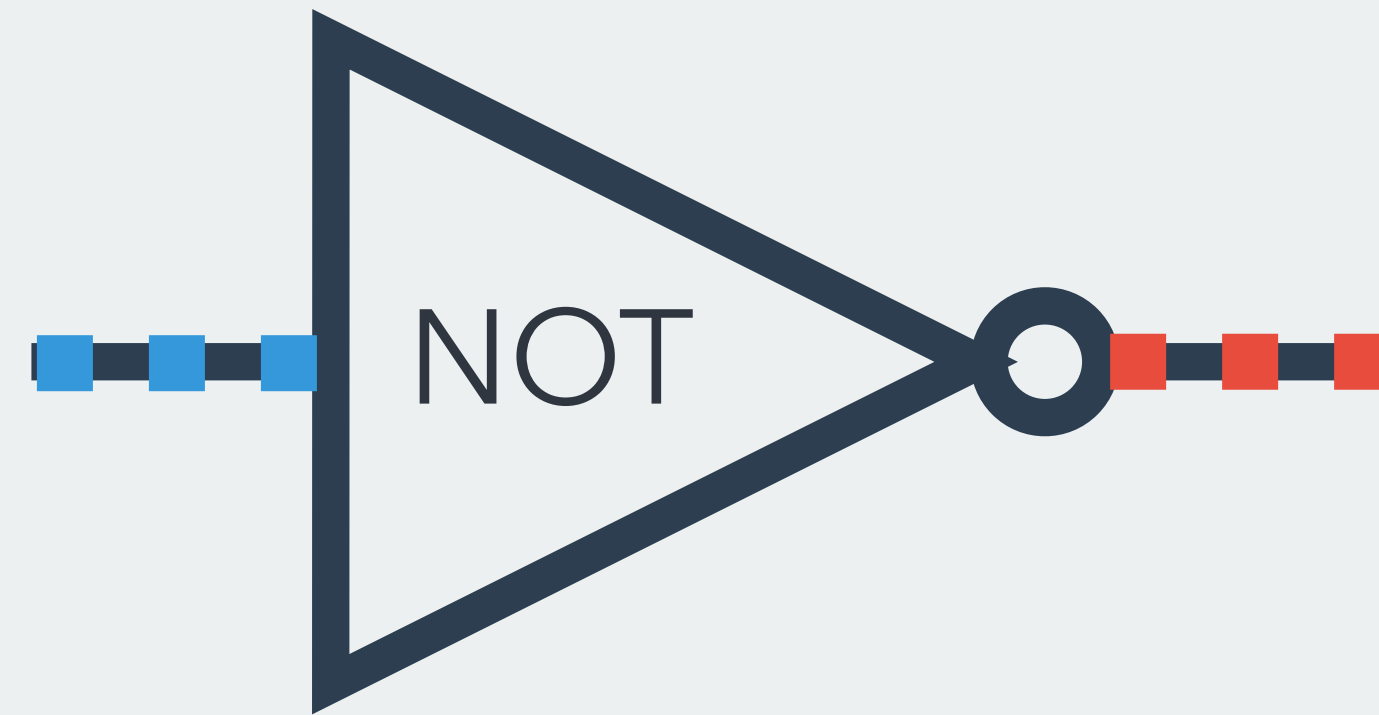


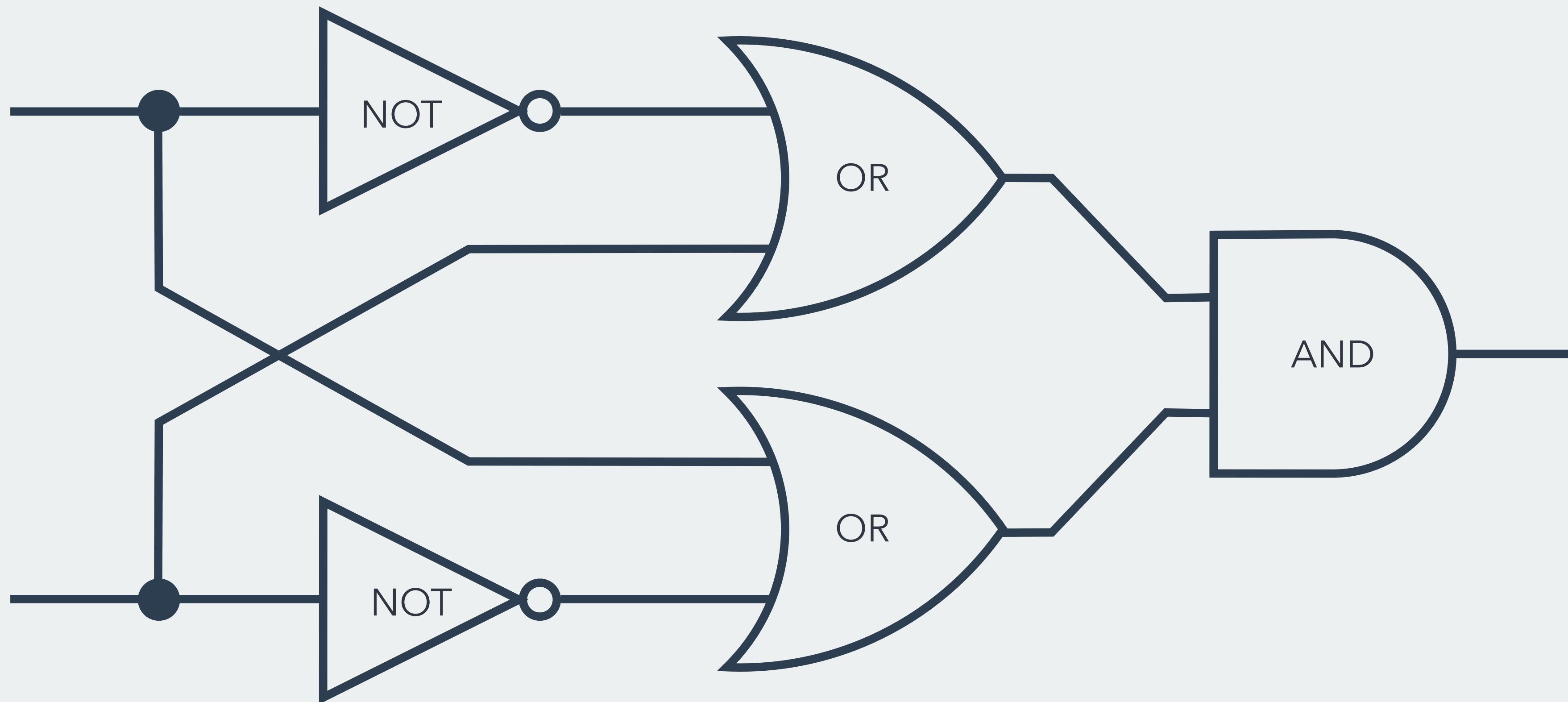


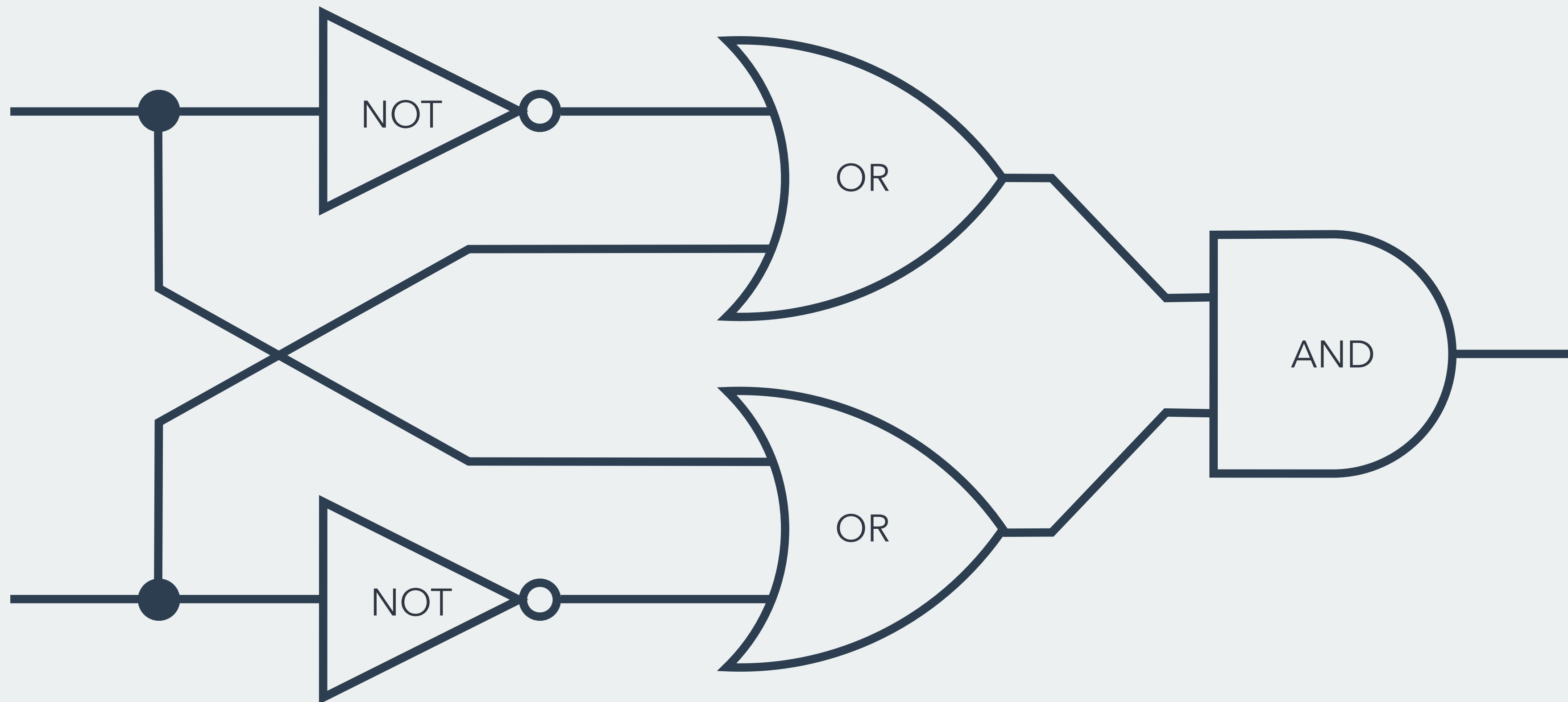


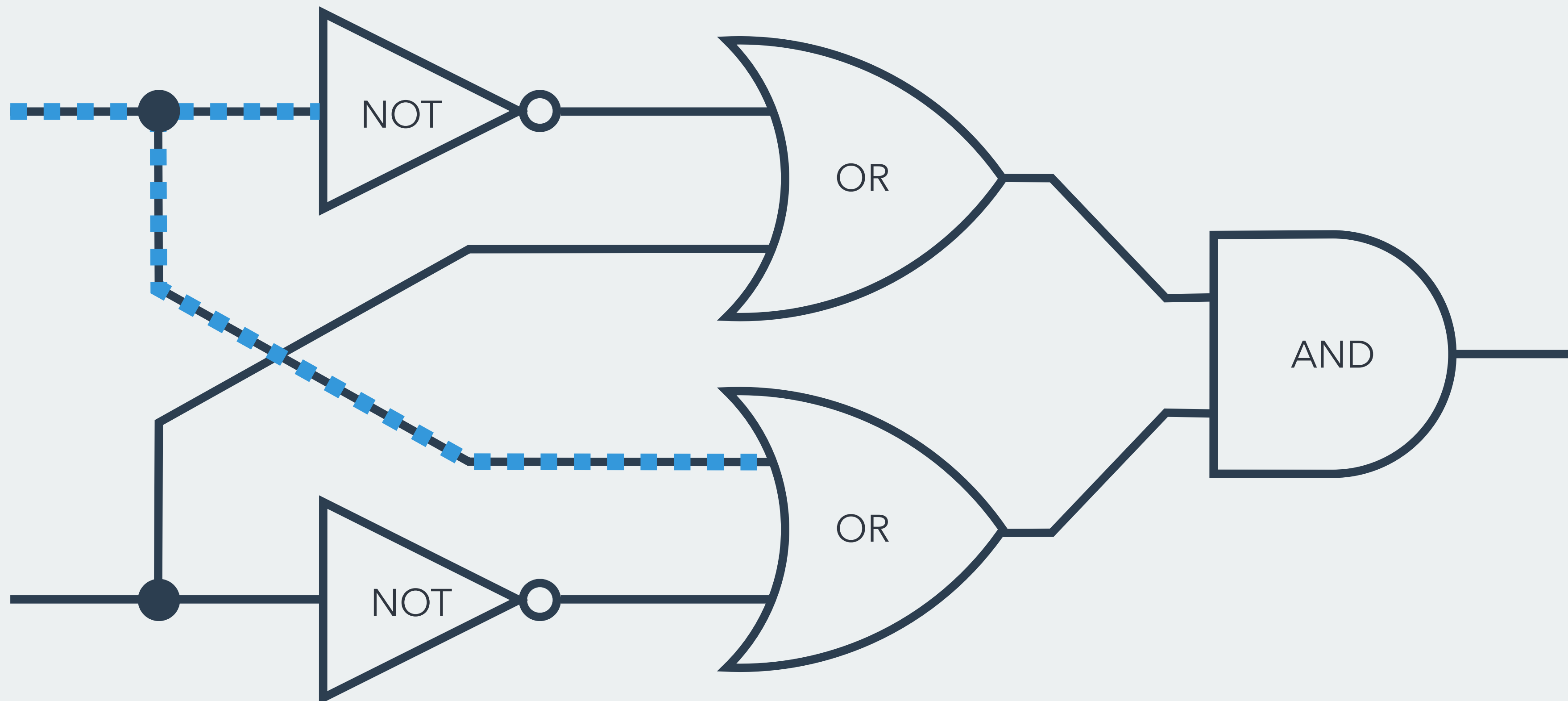


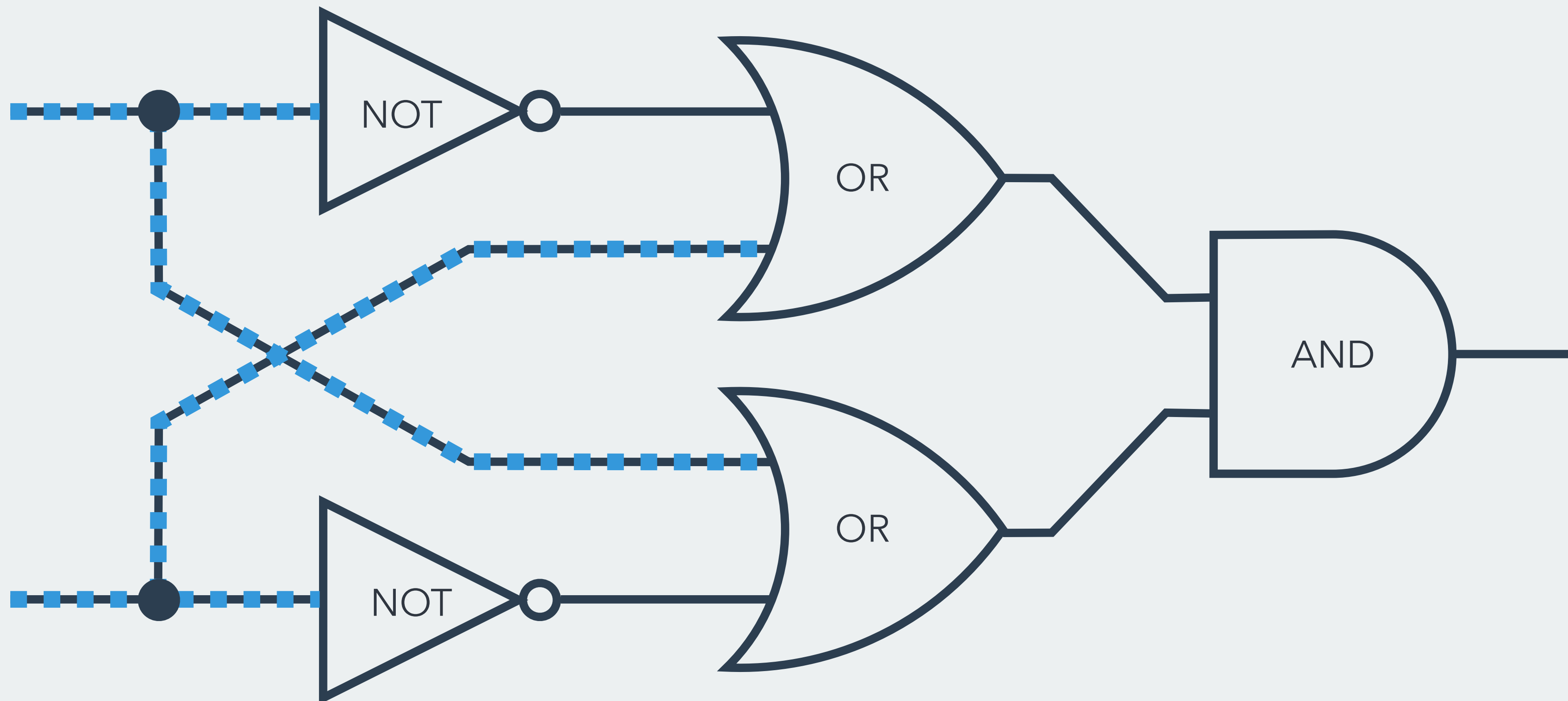


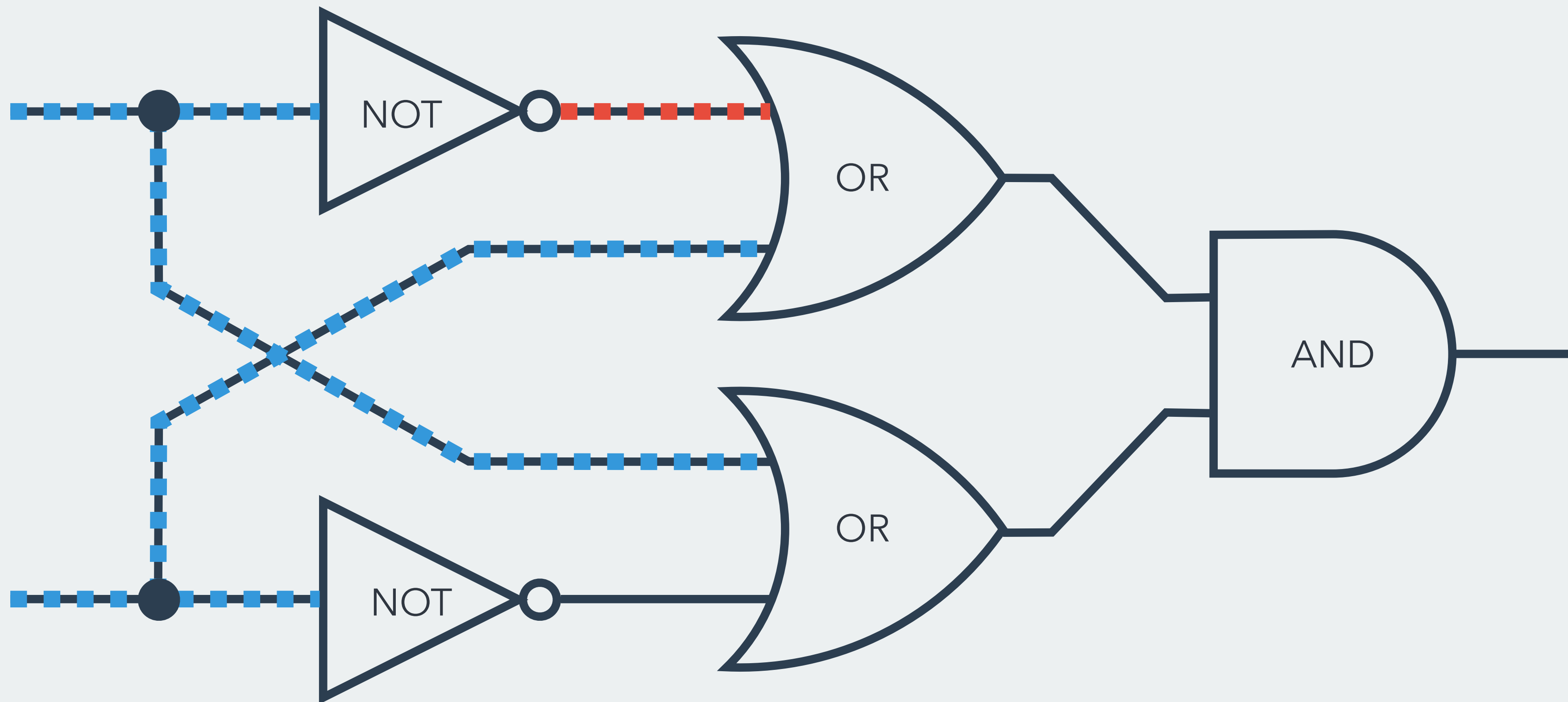


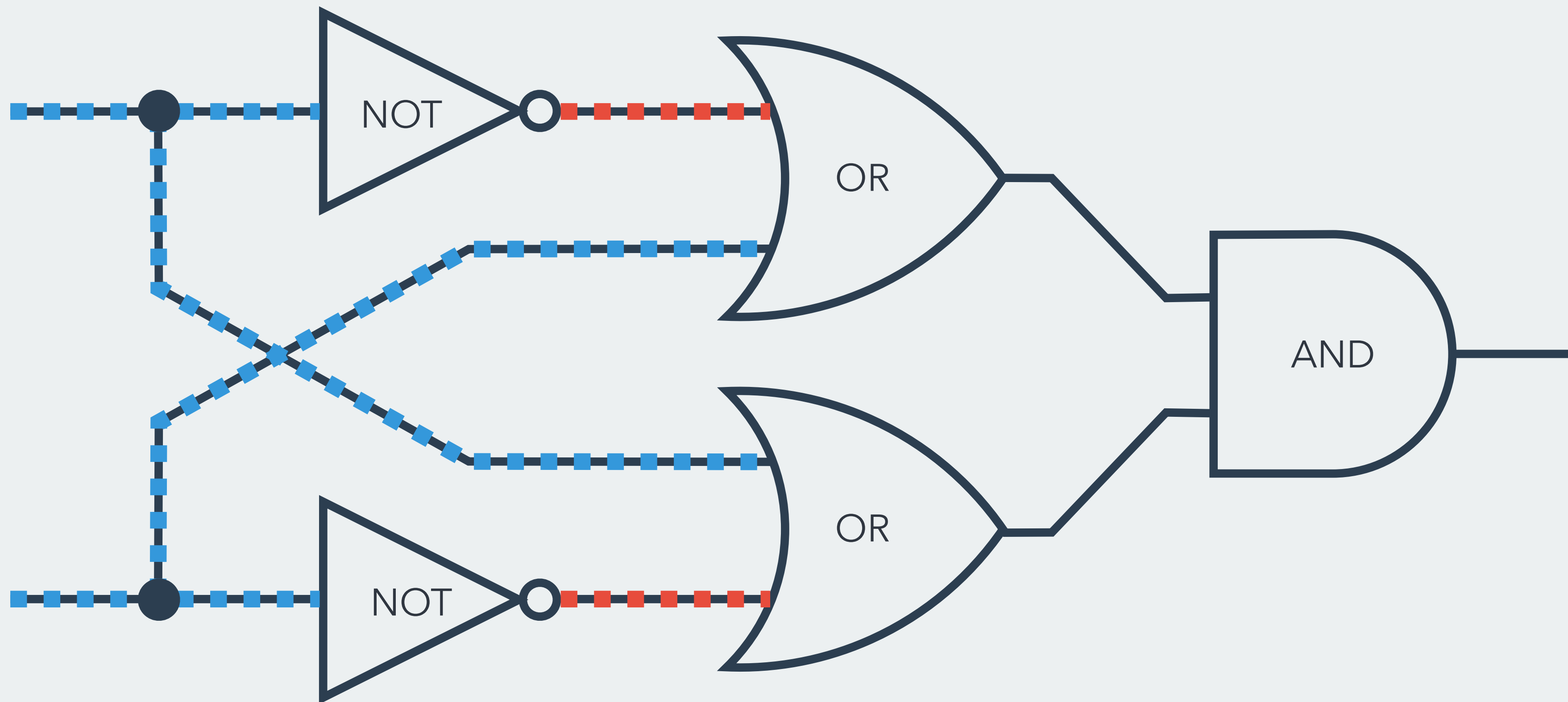


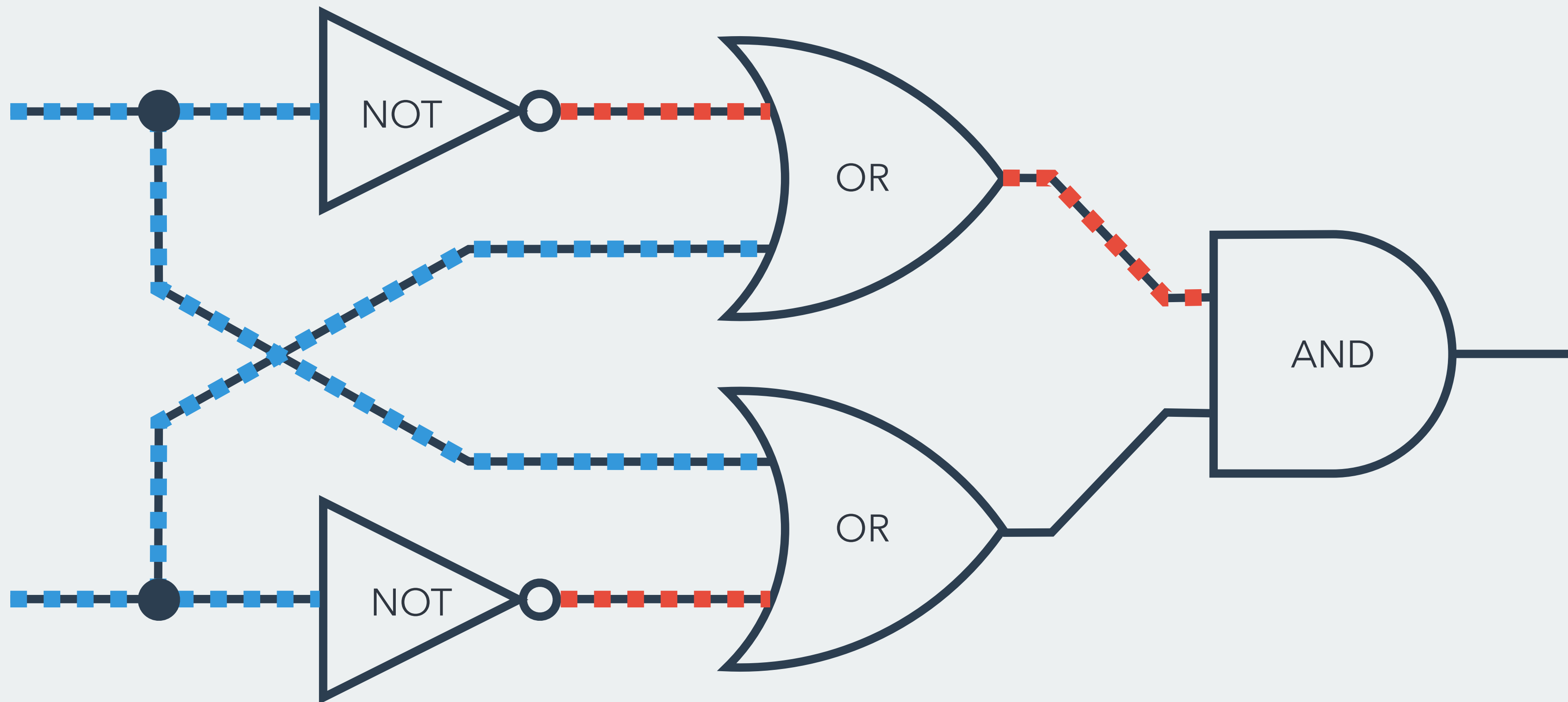


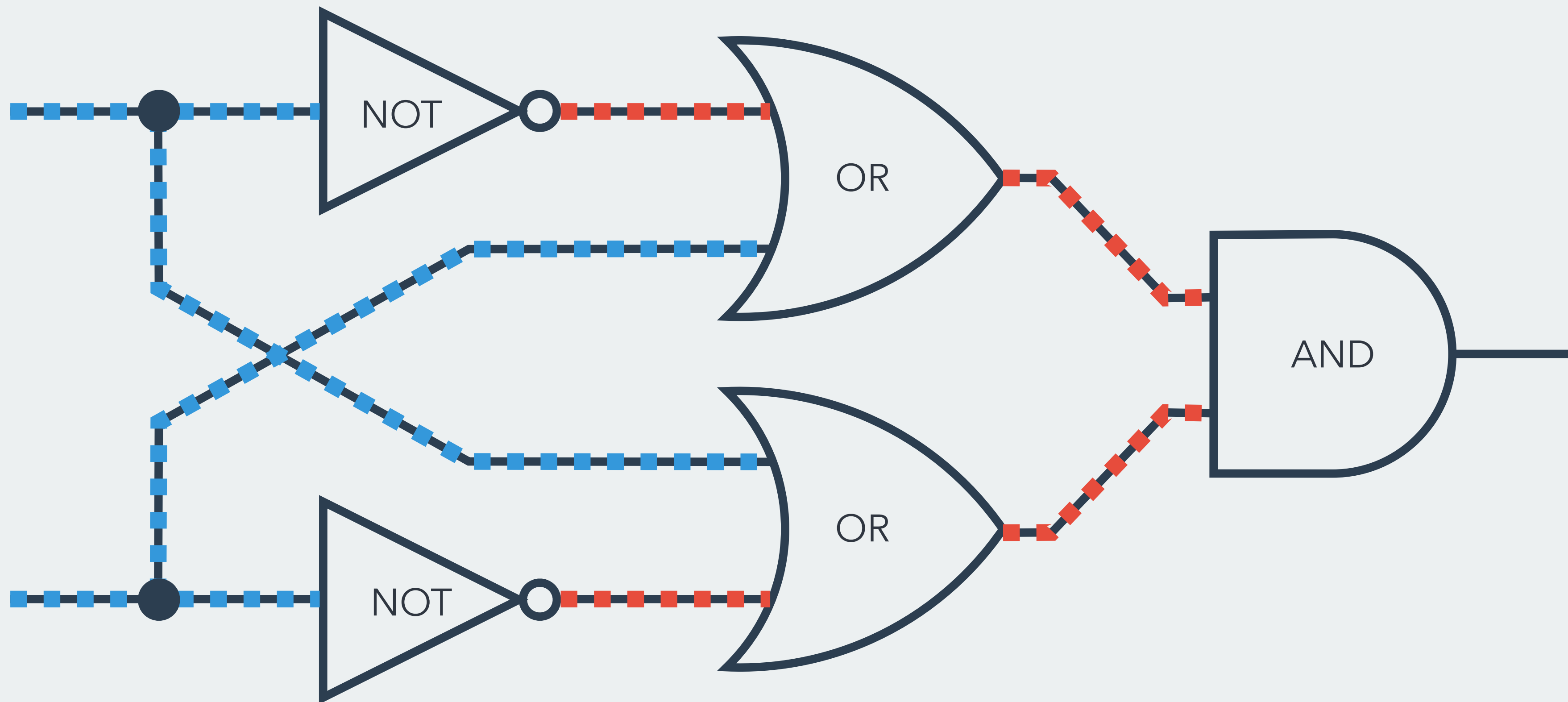


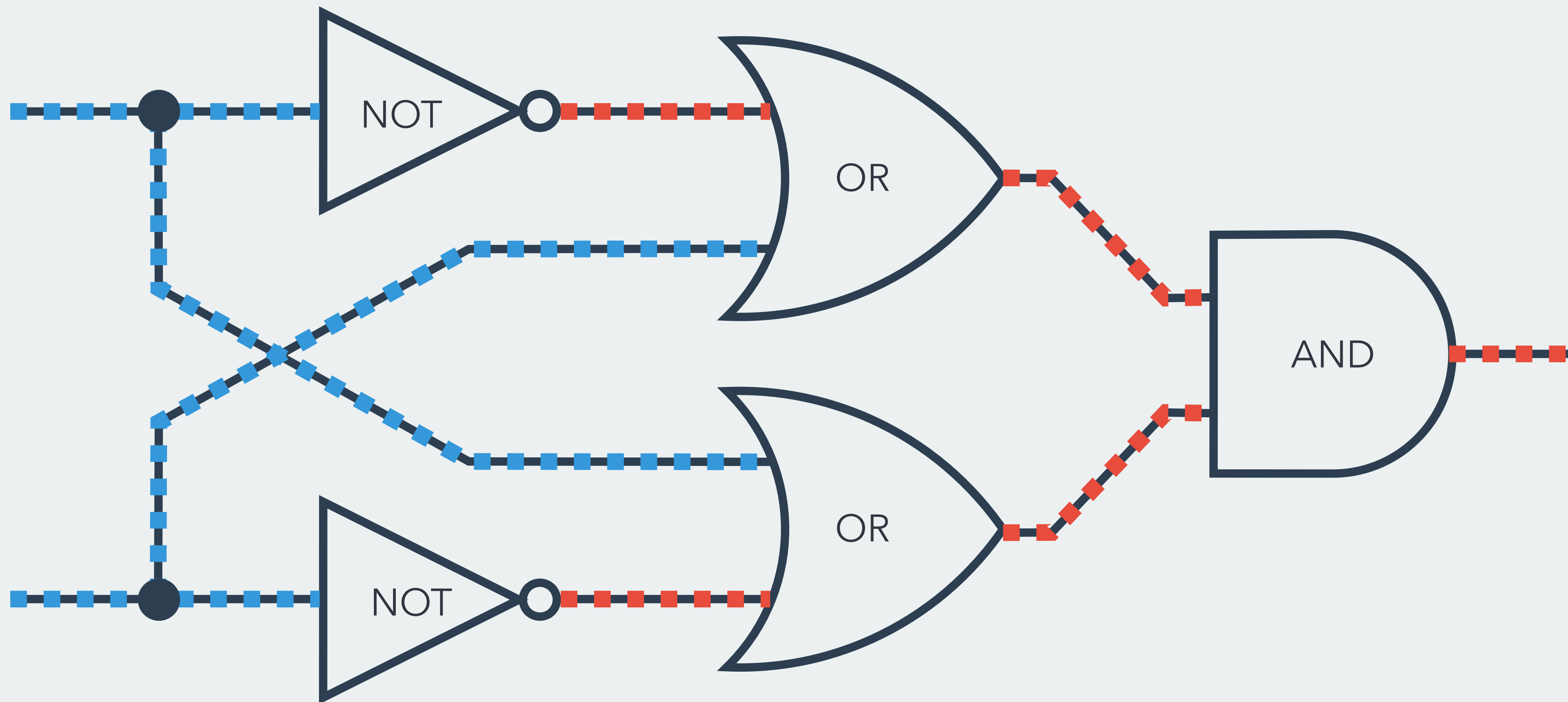


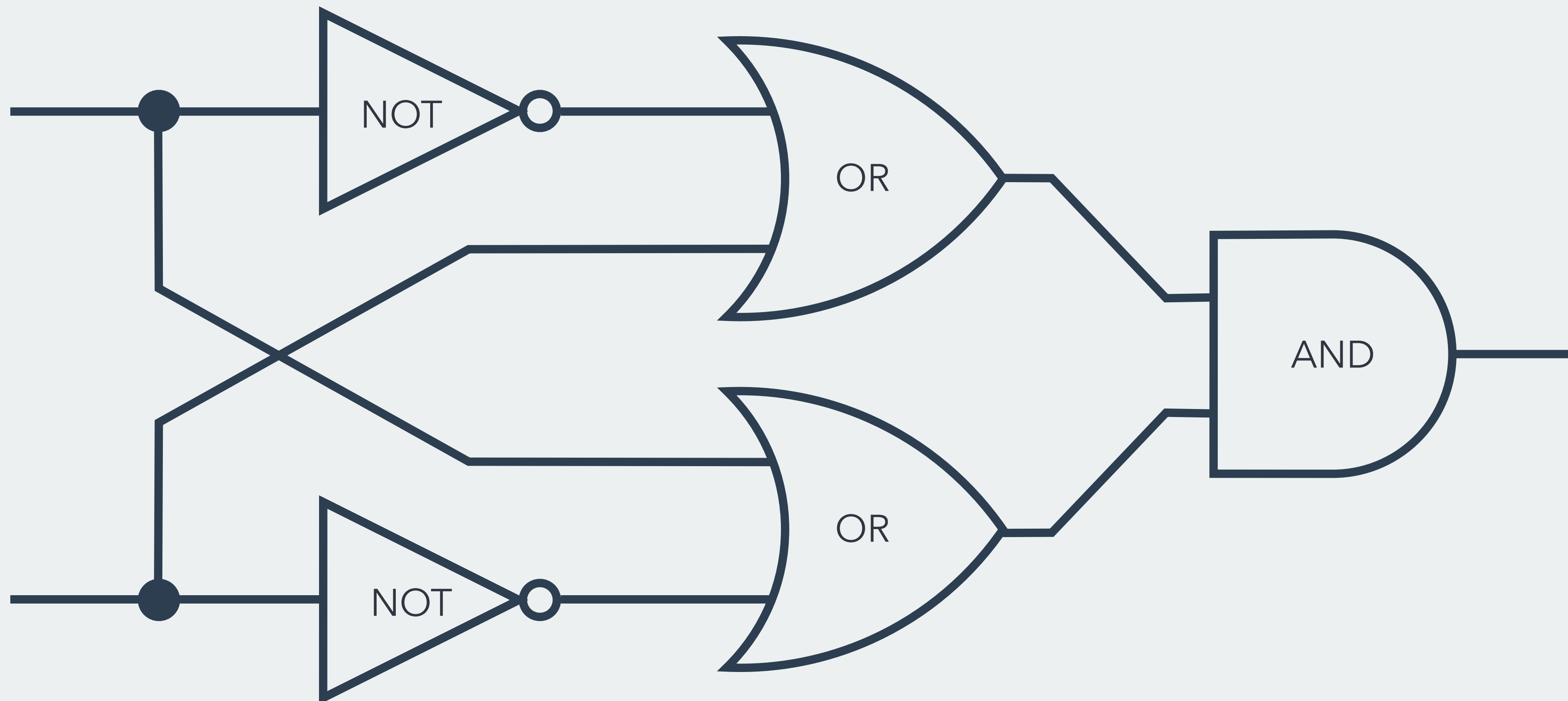


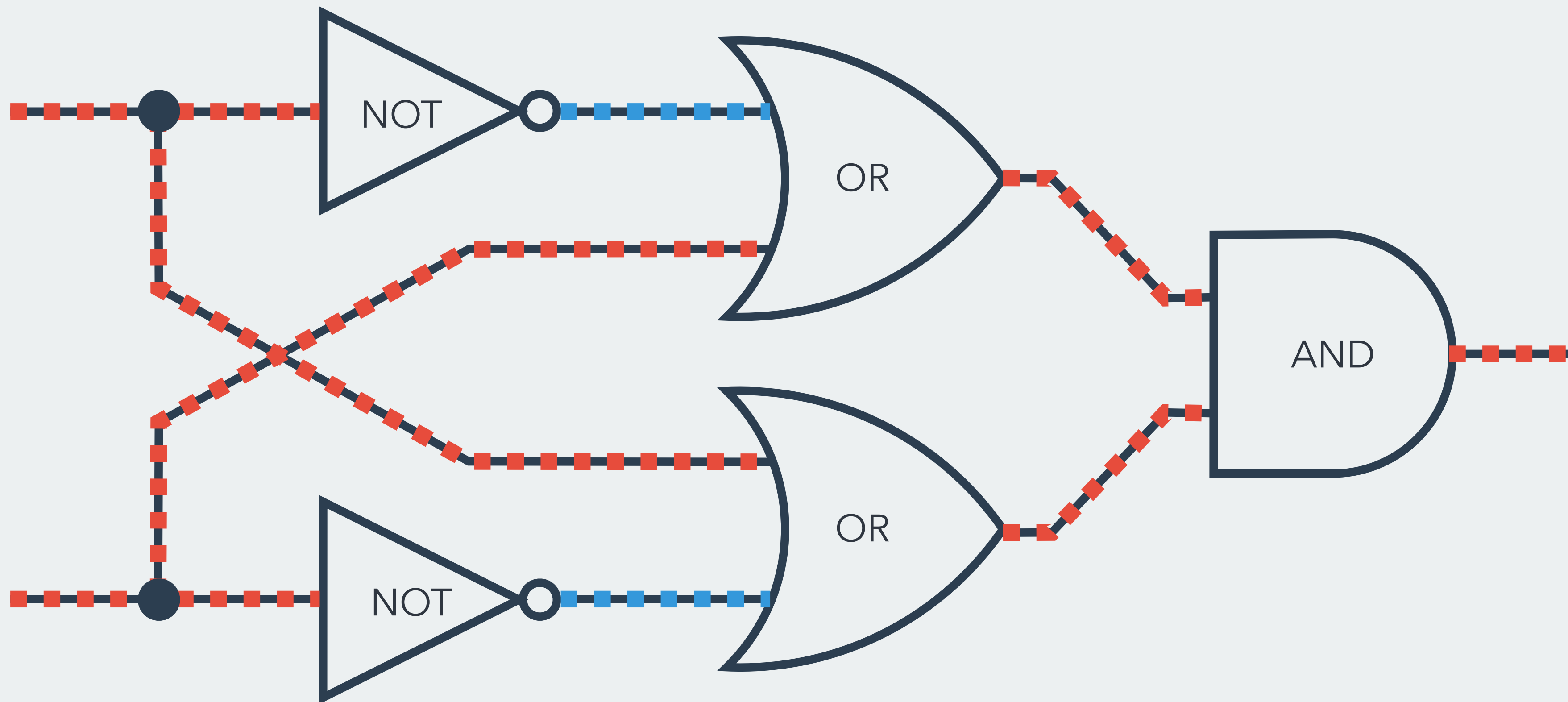


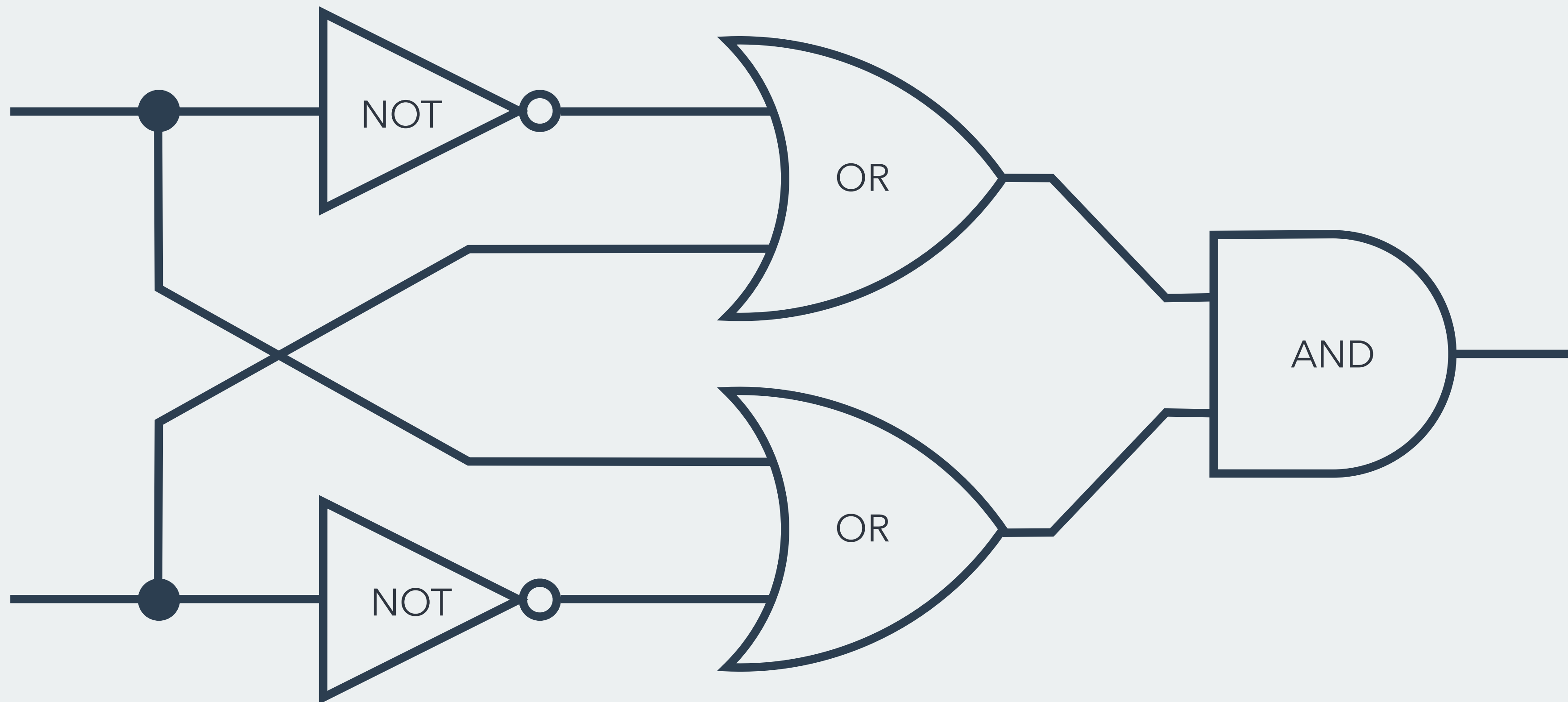


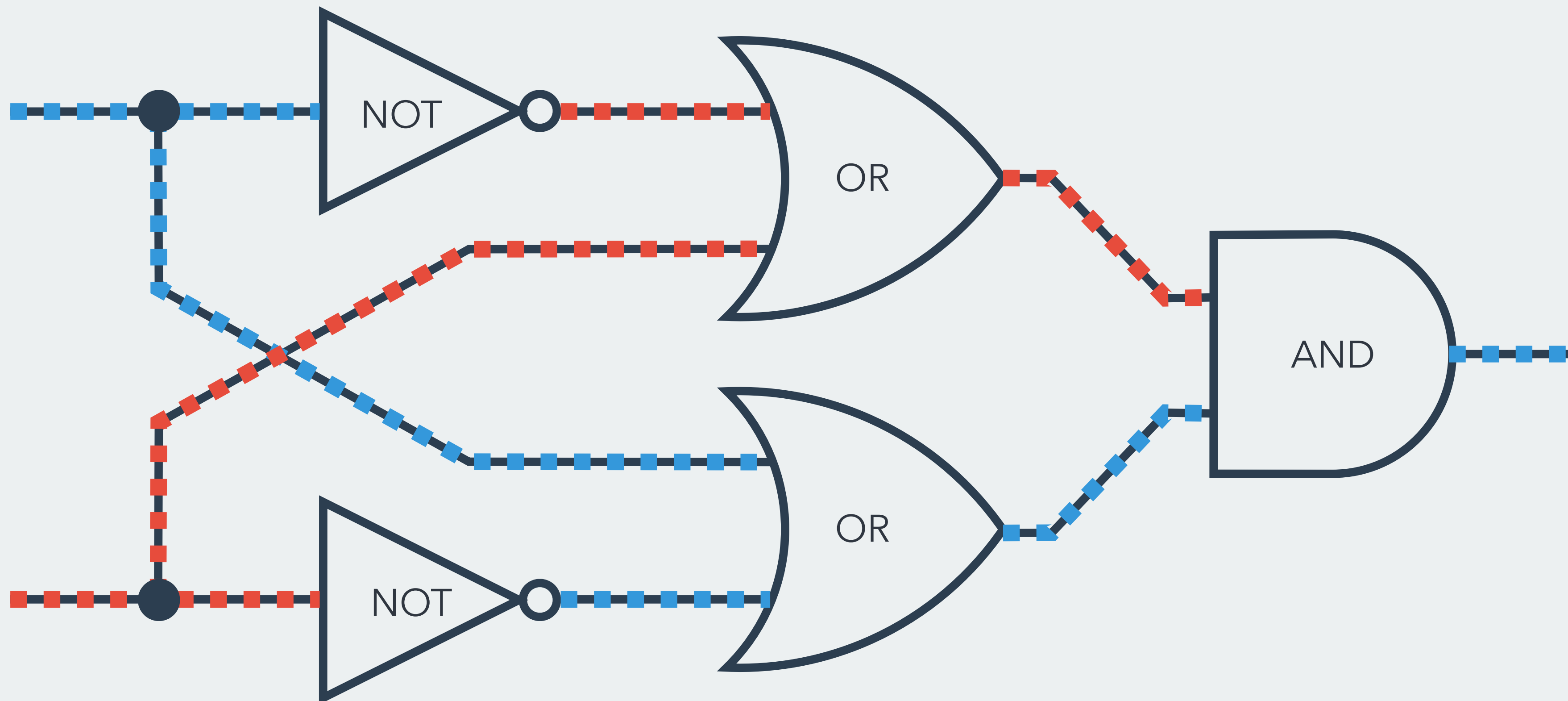


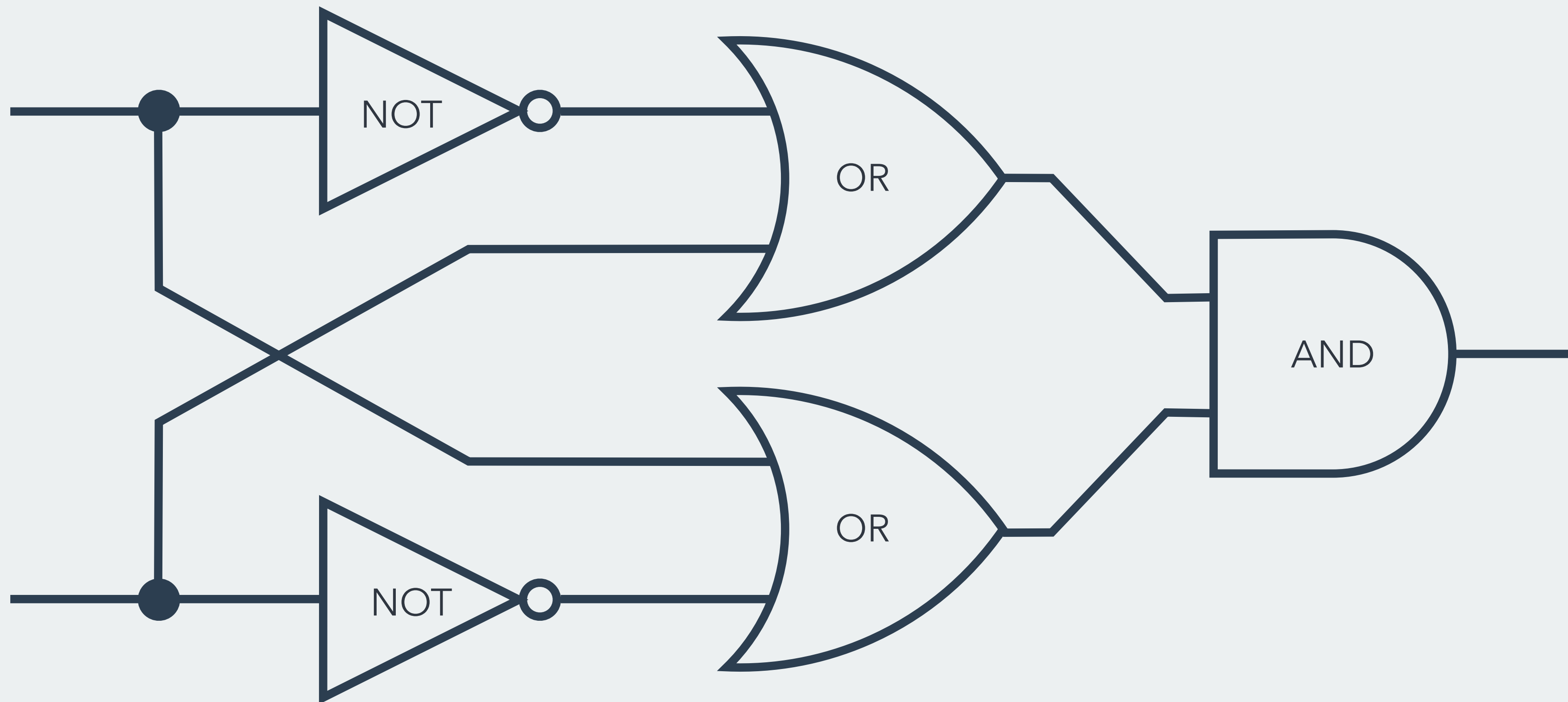


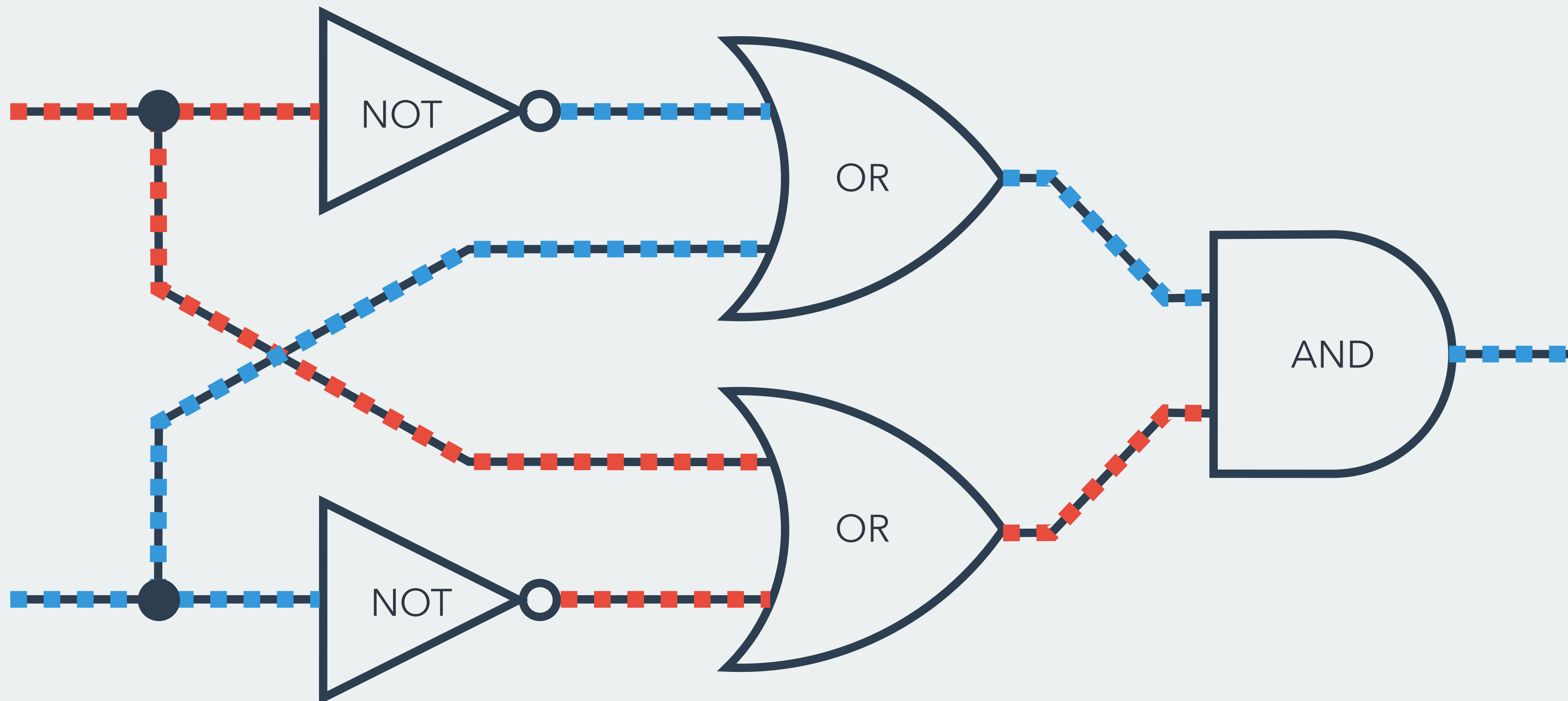


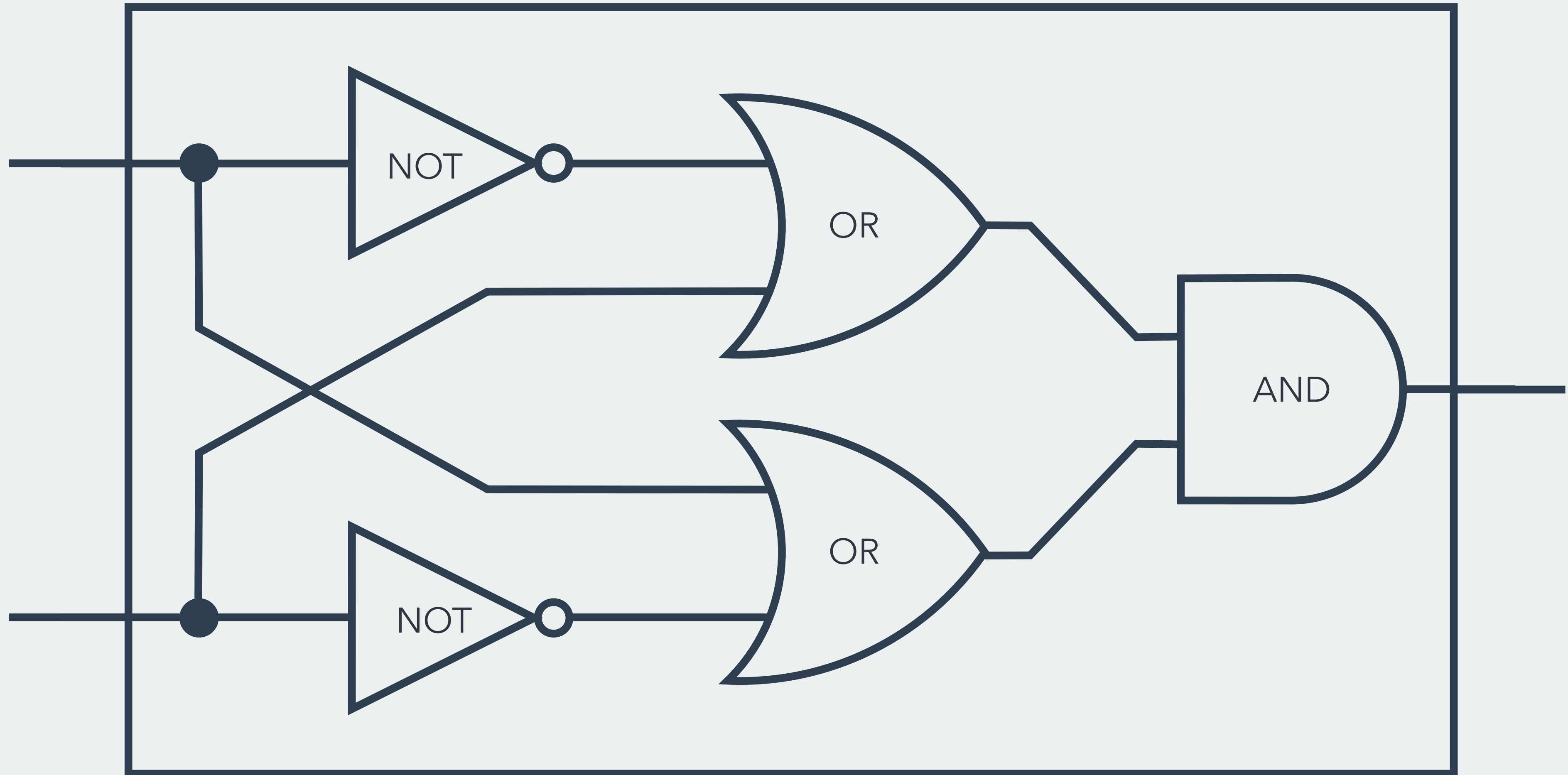




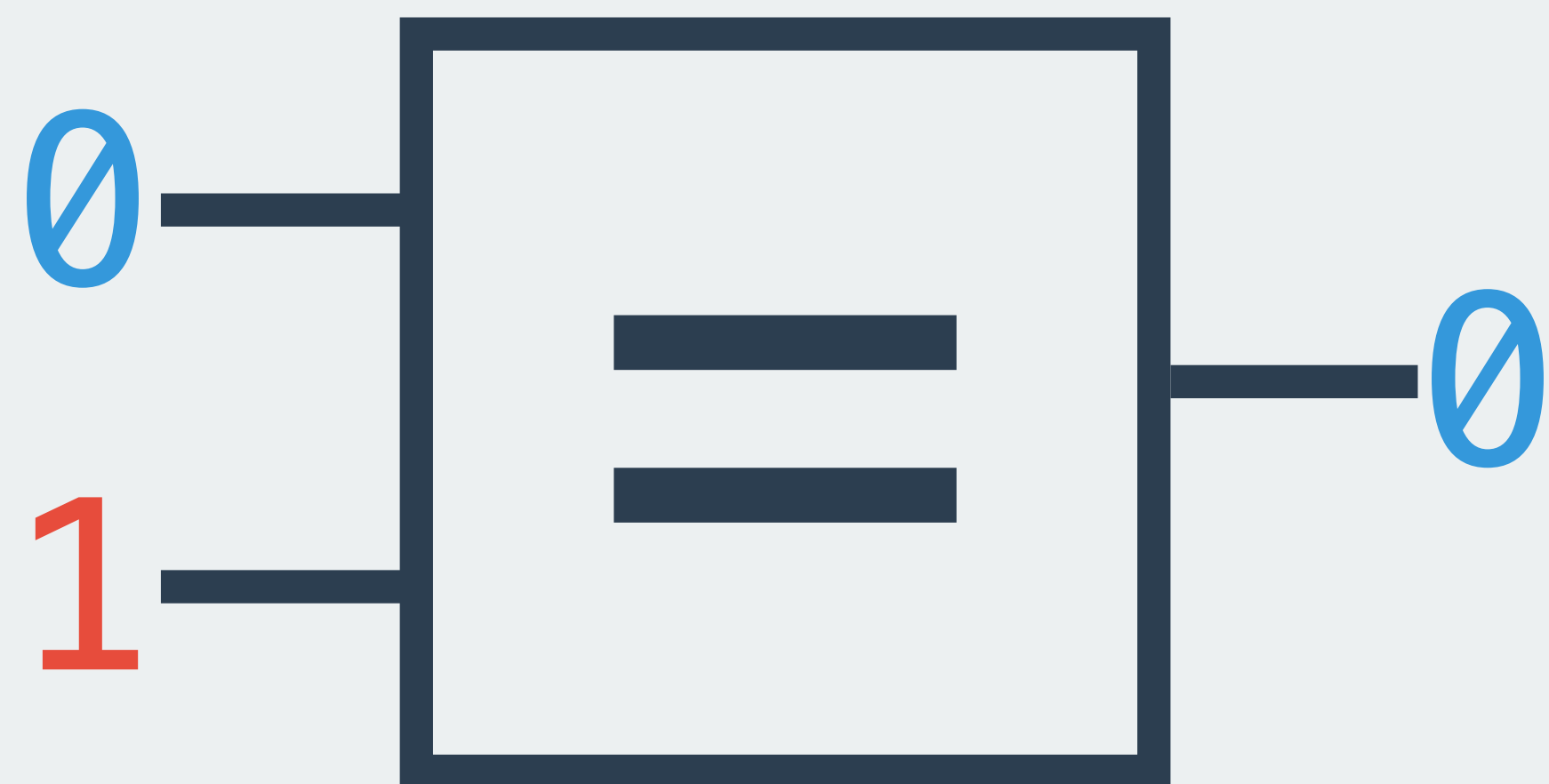


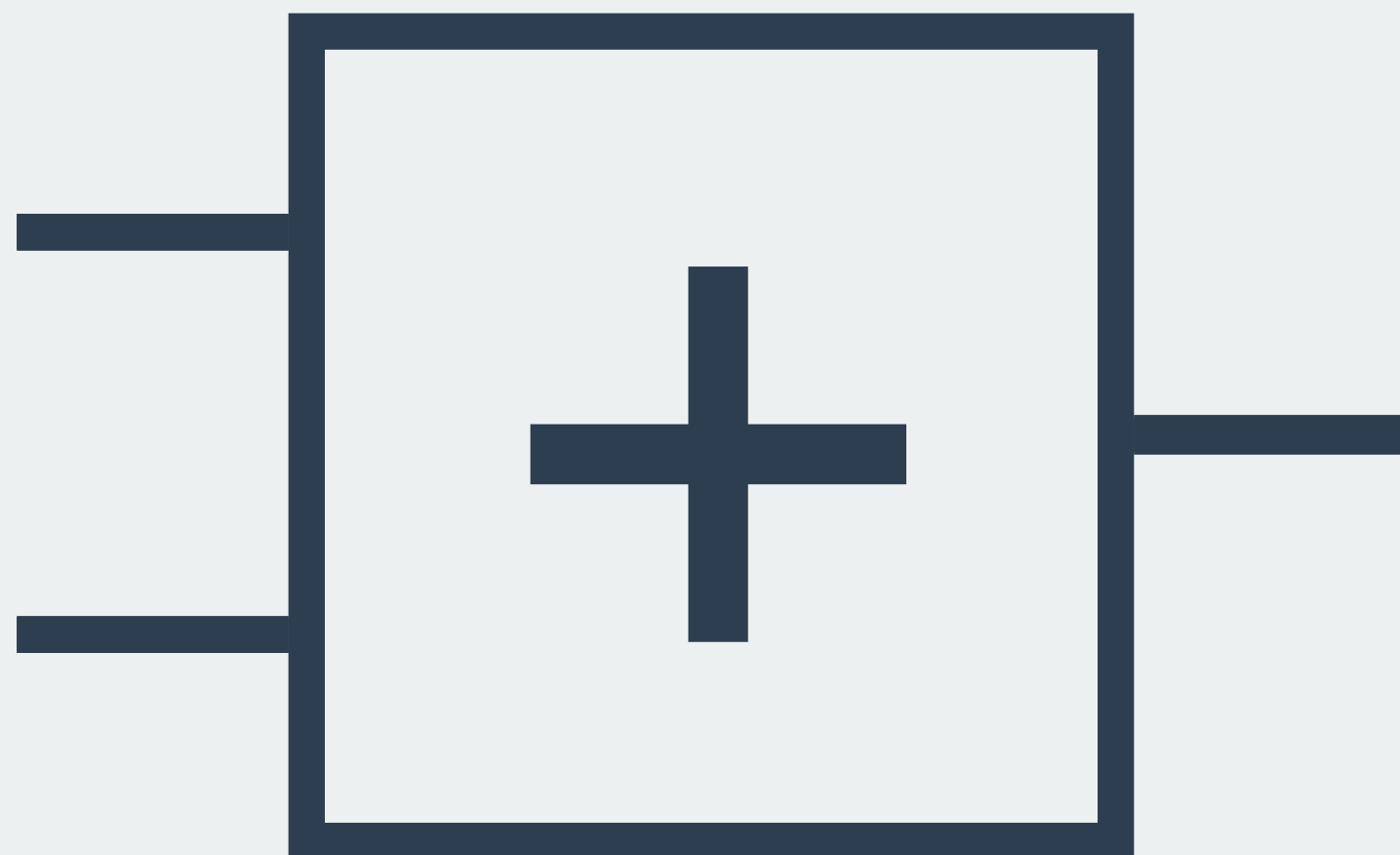


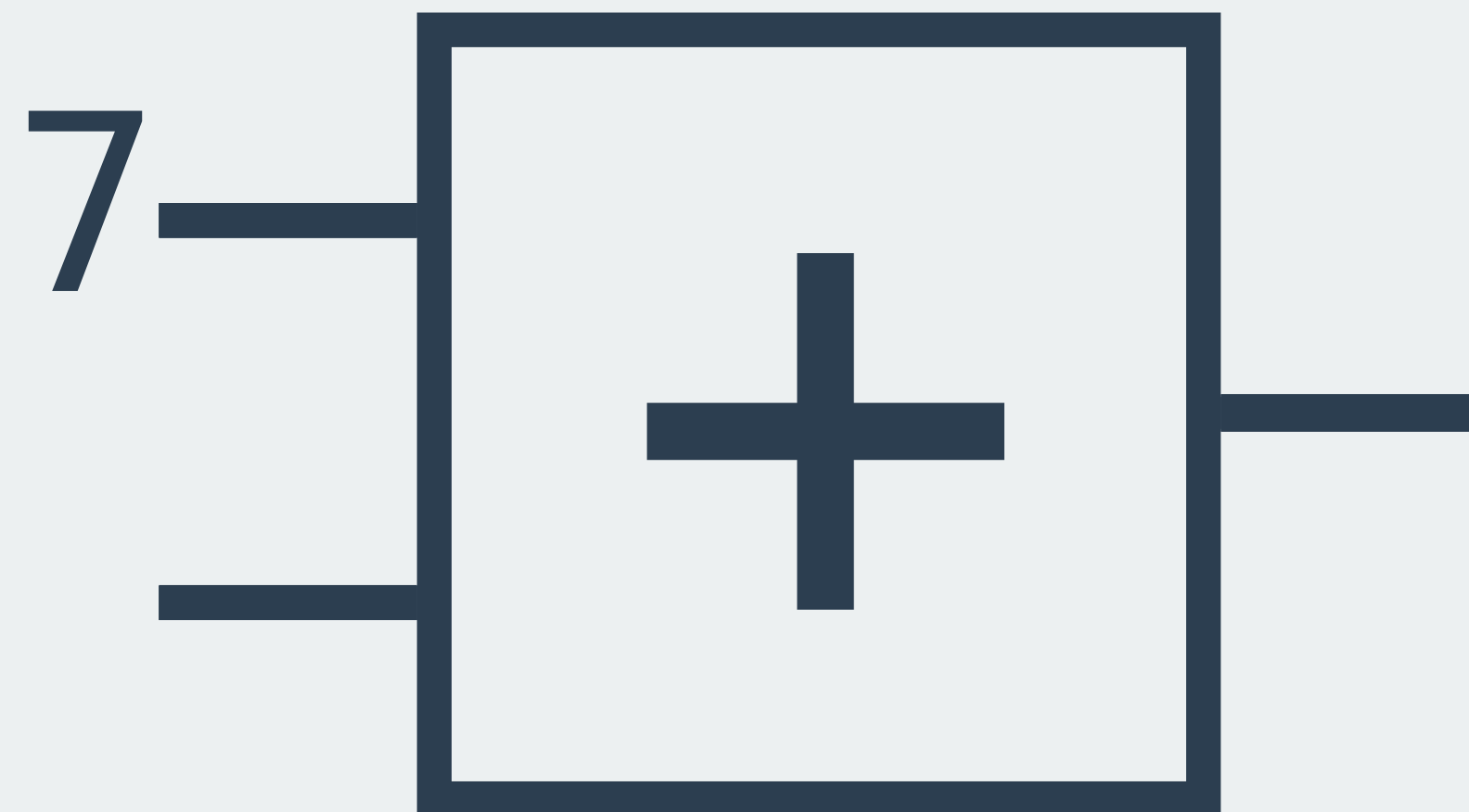


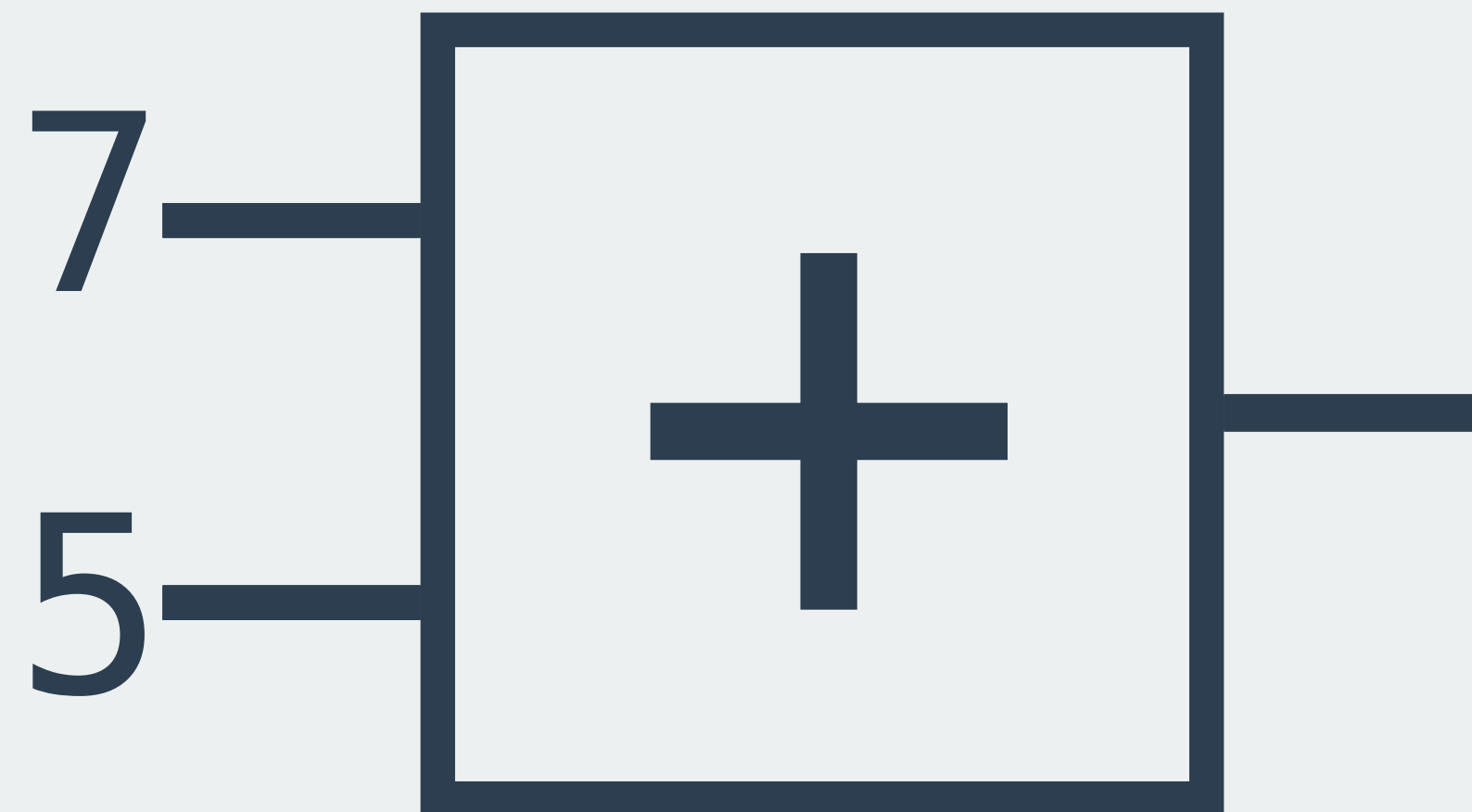










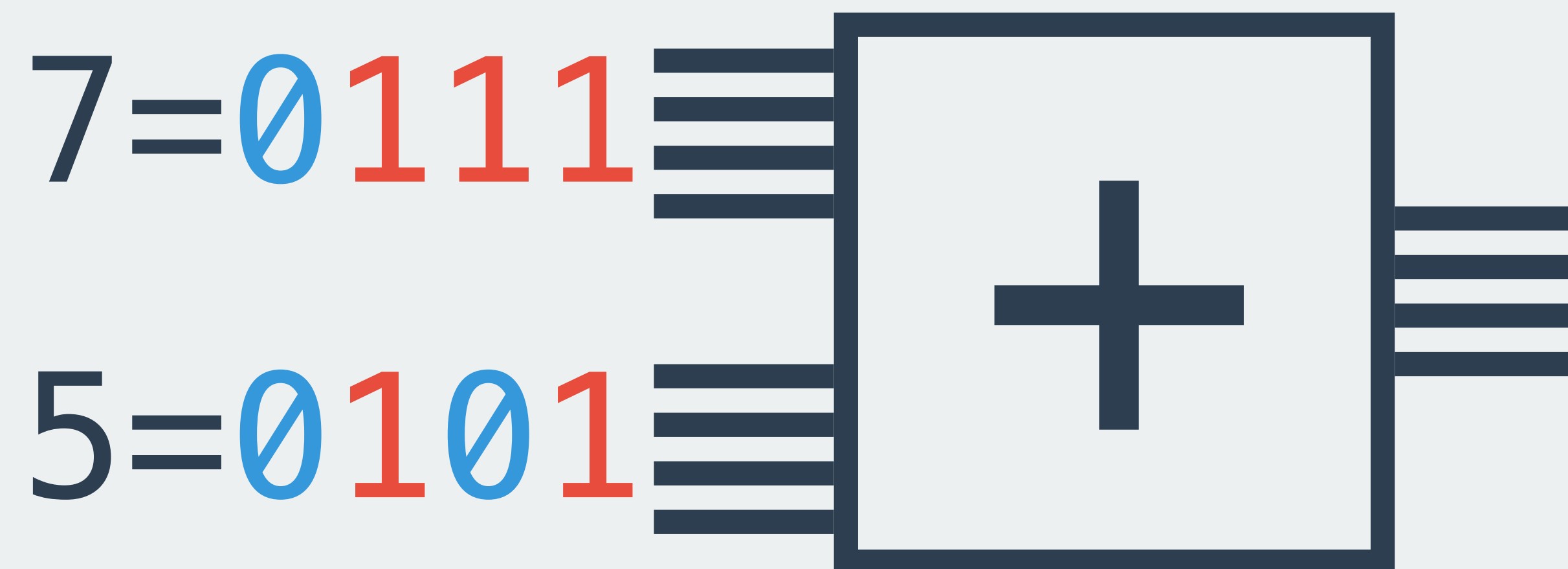


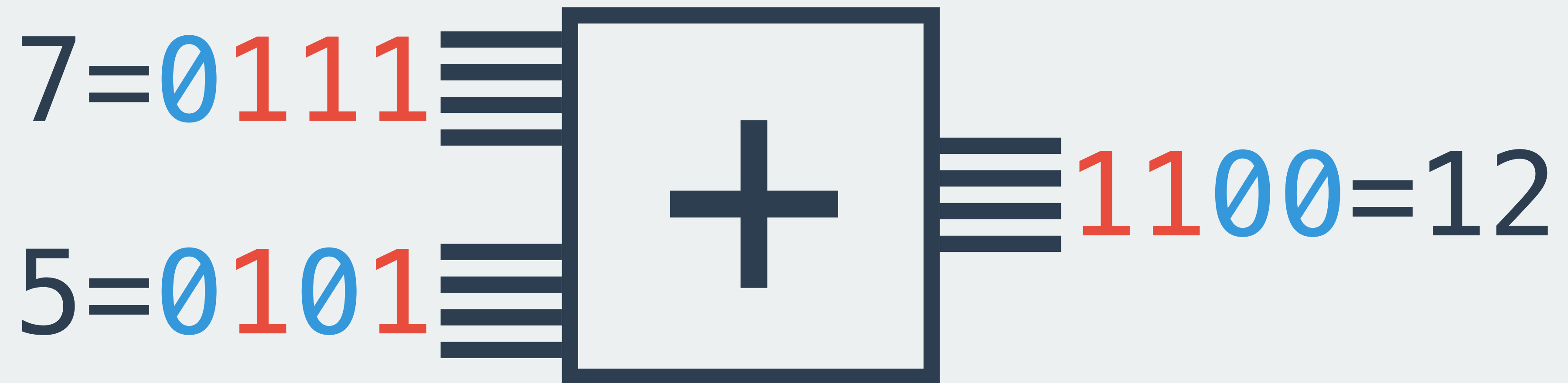












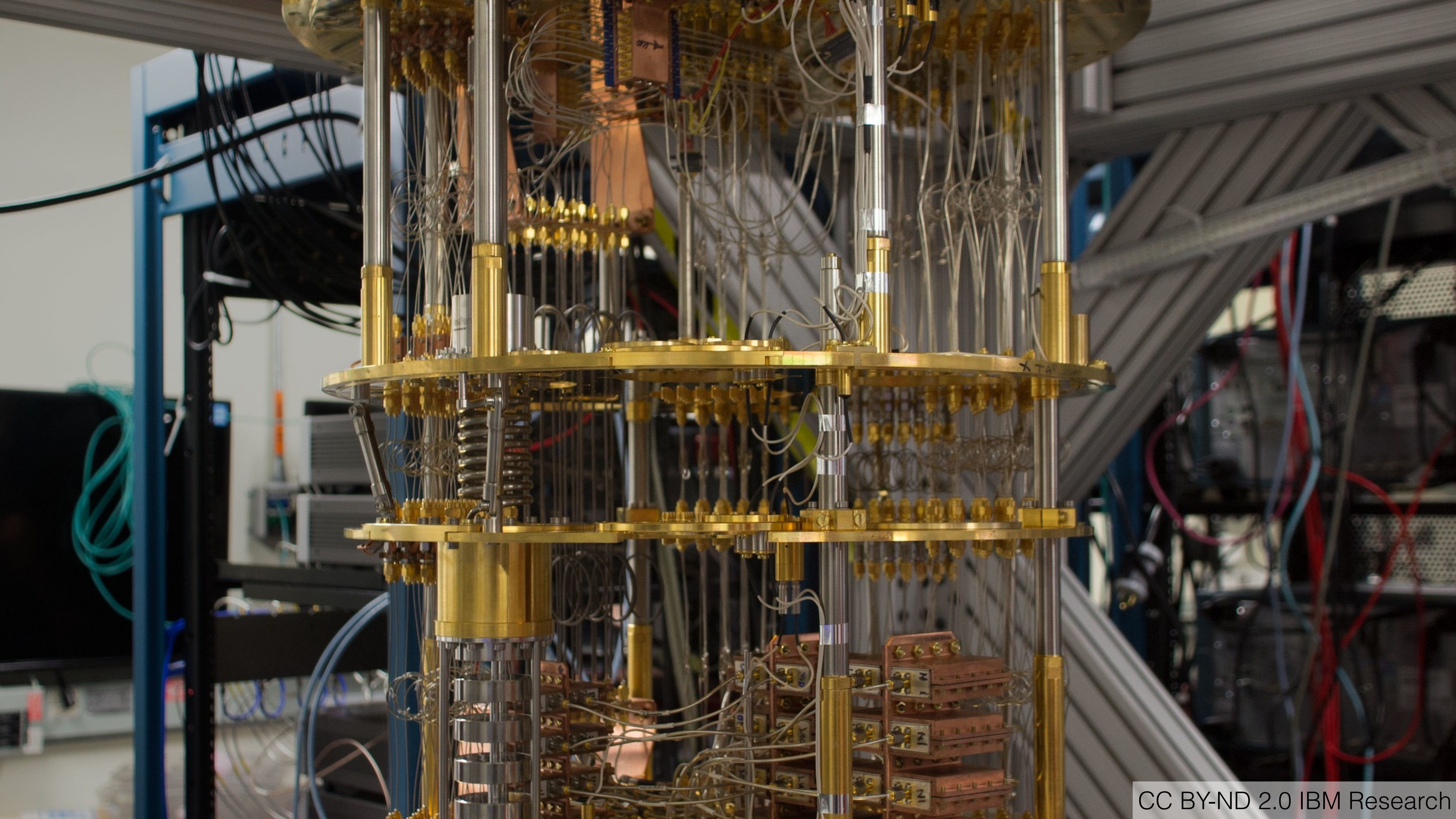


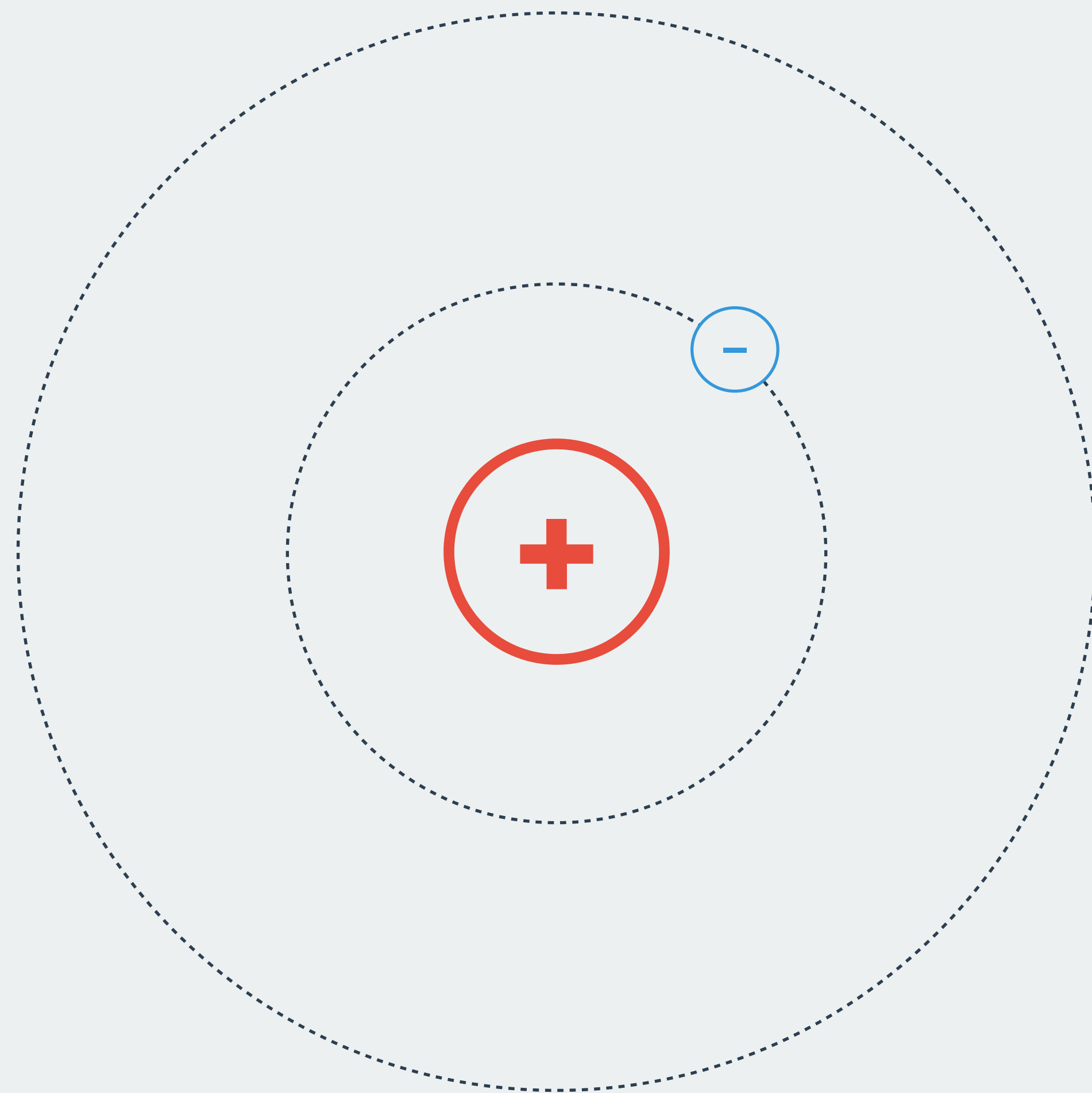
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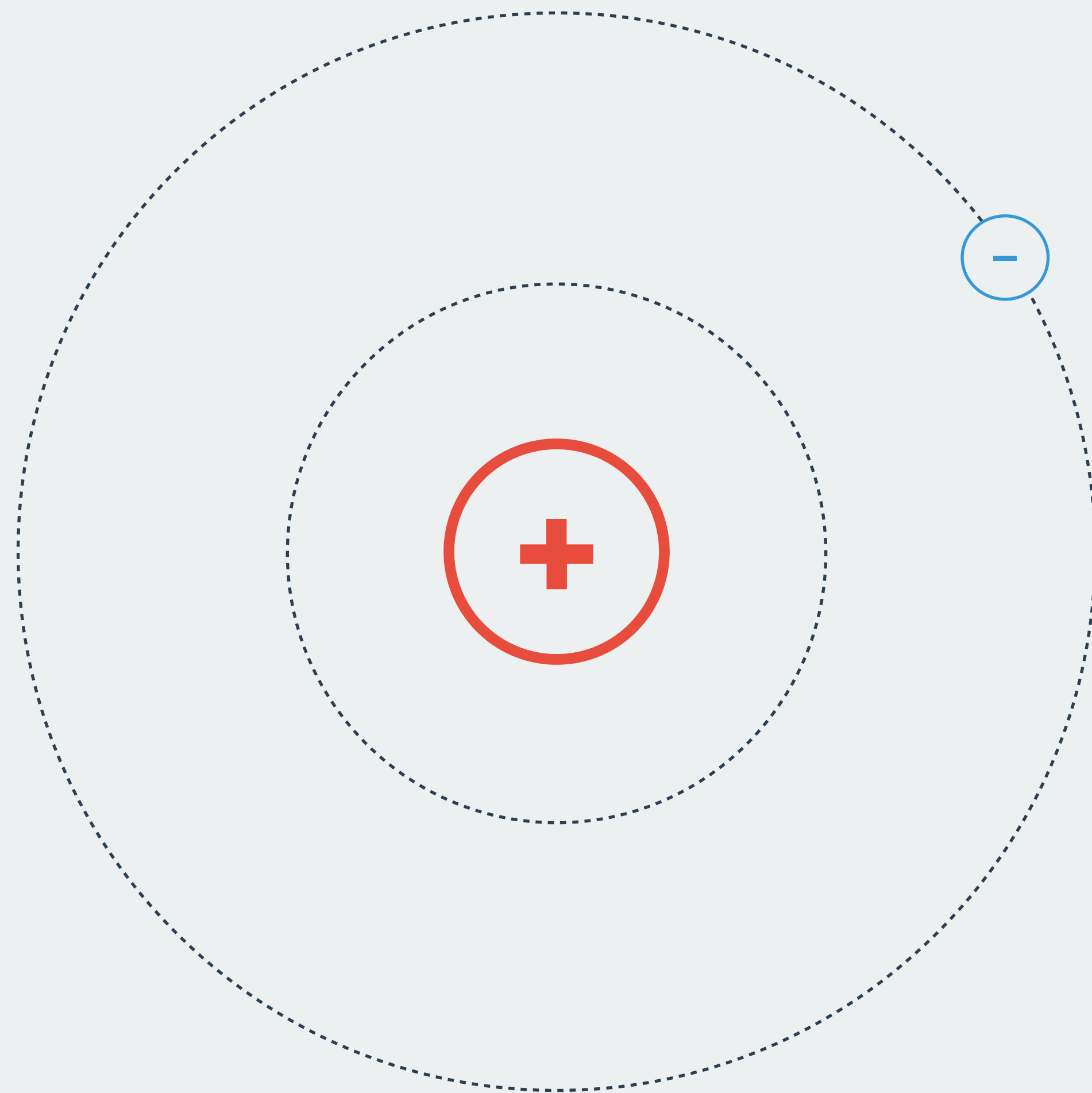
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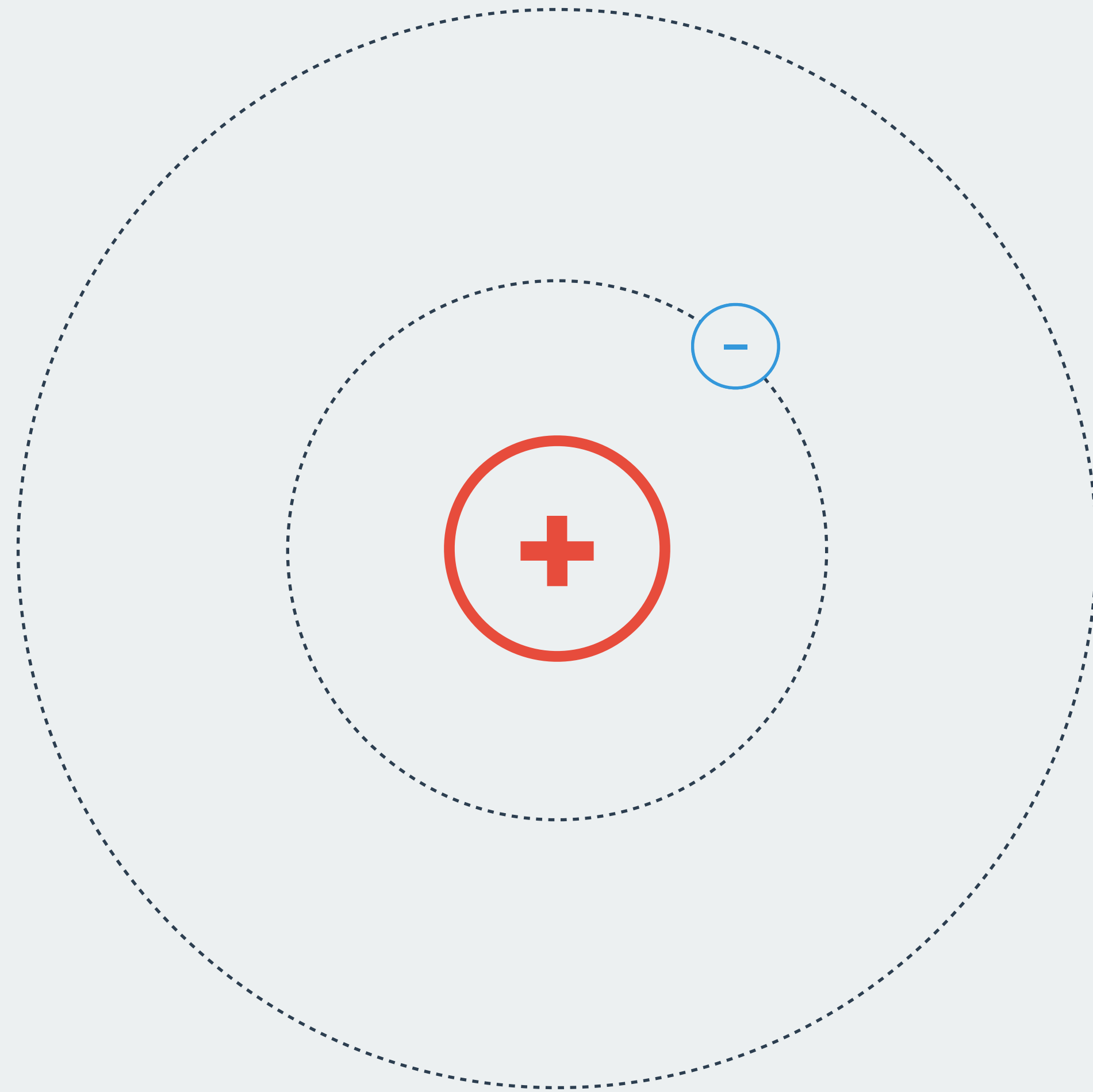
The background image shows a complex quantum computing setup. It features several vertical metal rods supporting horizontal plates with numerous gold-colored electrical contacts. A dense network of white and grey cables is connected to these contacts. The entire setup is housed within a blue metal frame. The image is dimmed with a dark blue overlay to make the yellow text stand out.

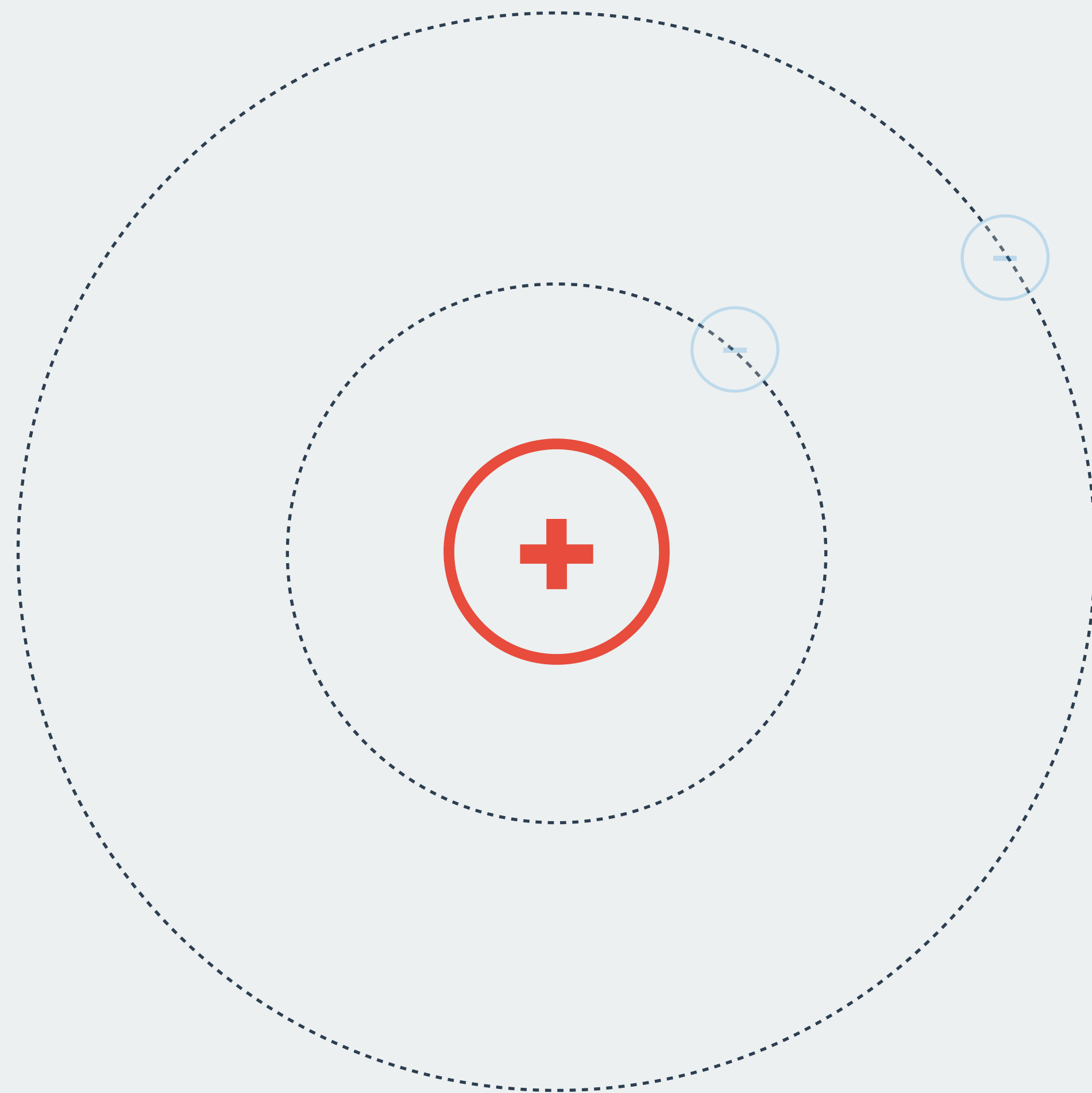
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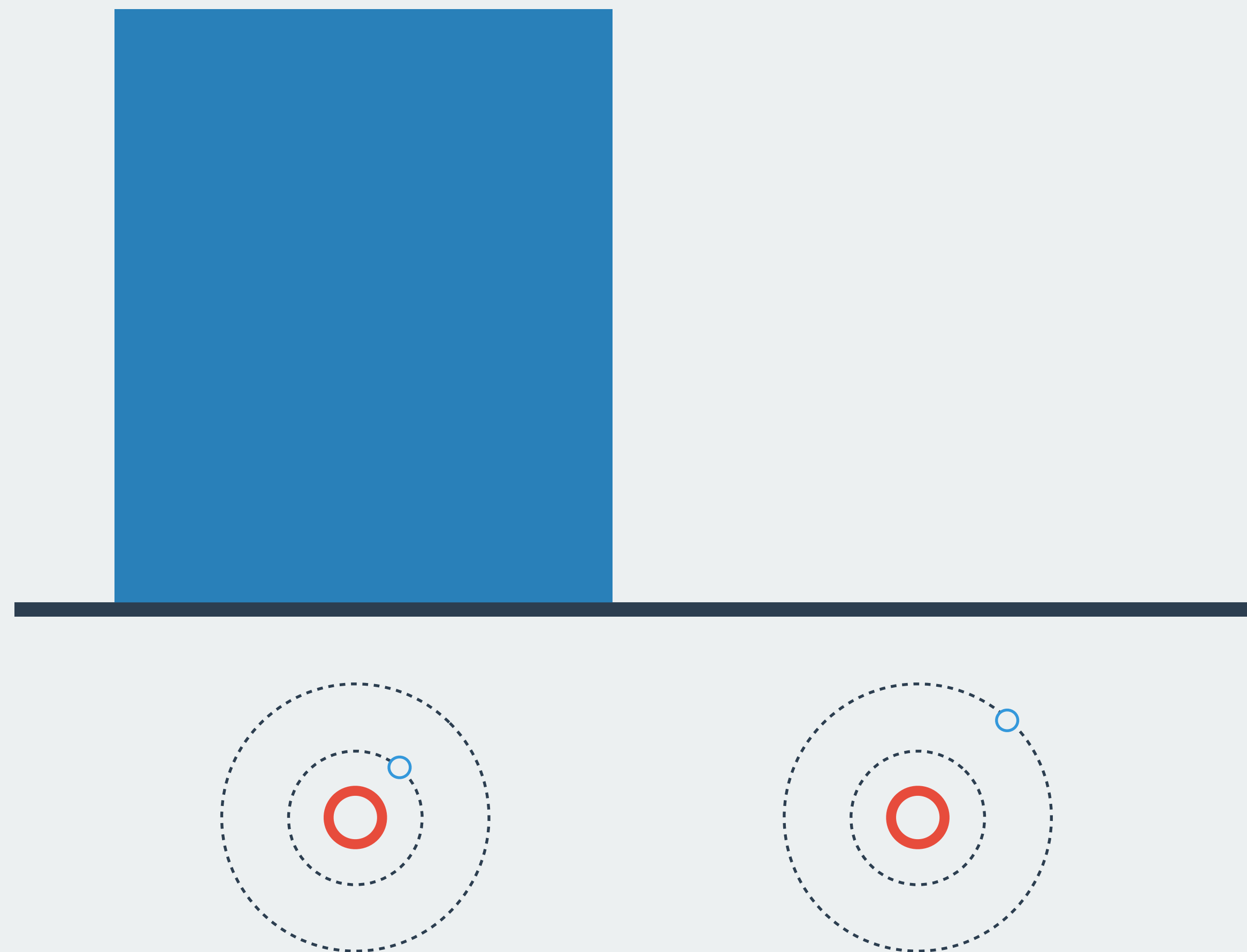
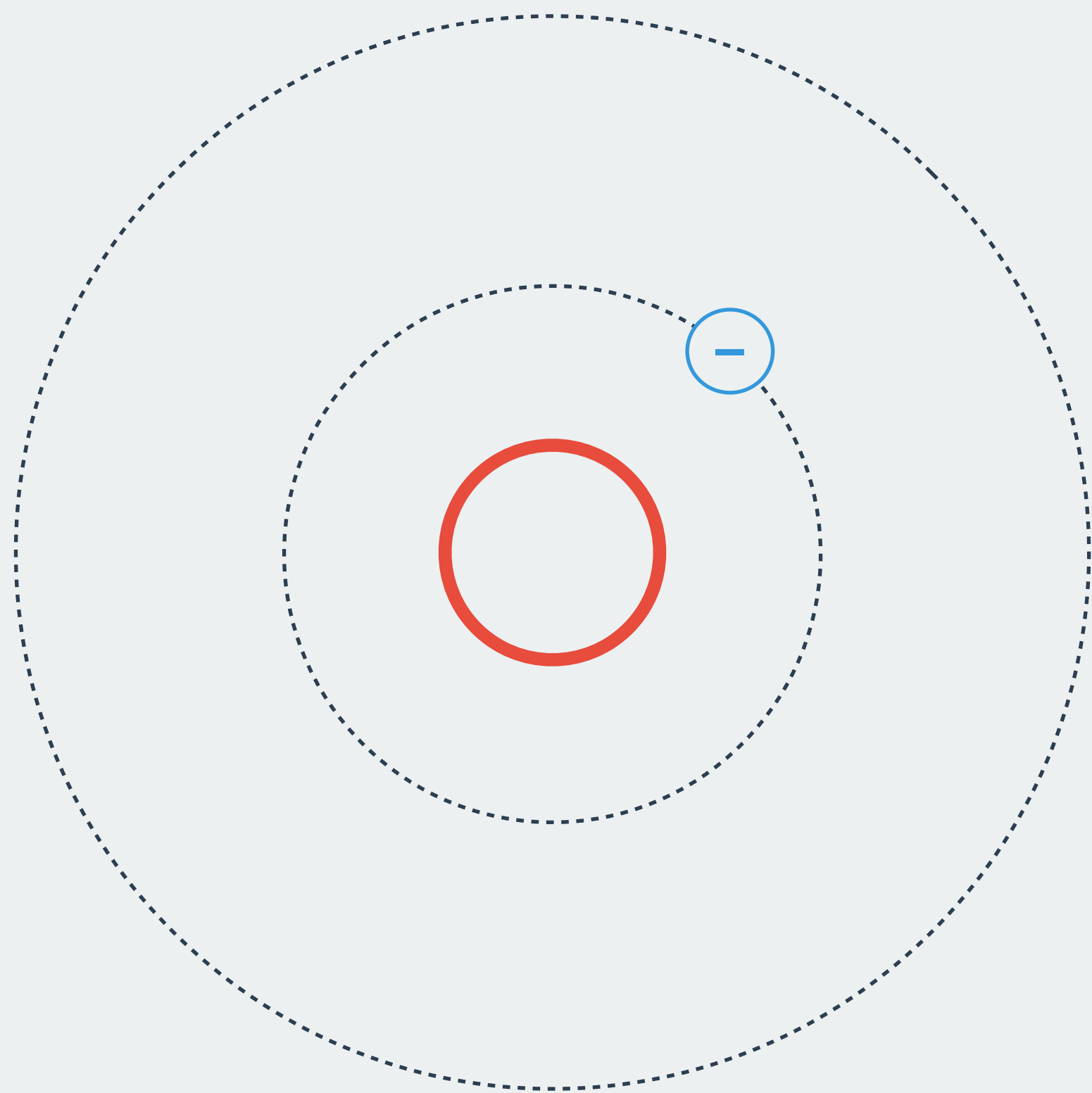


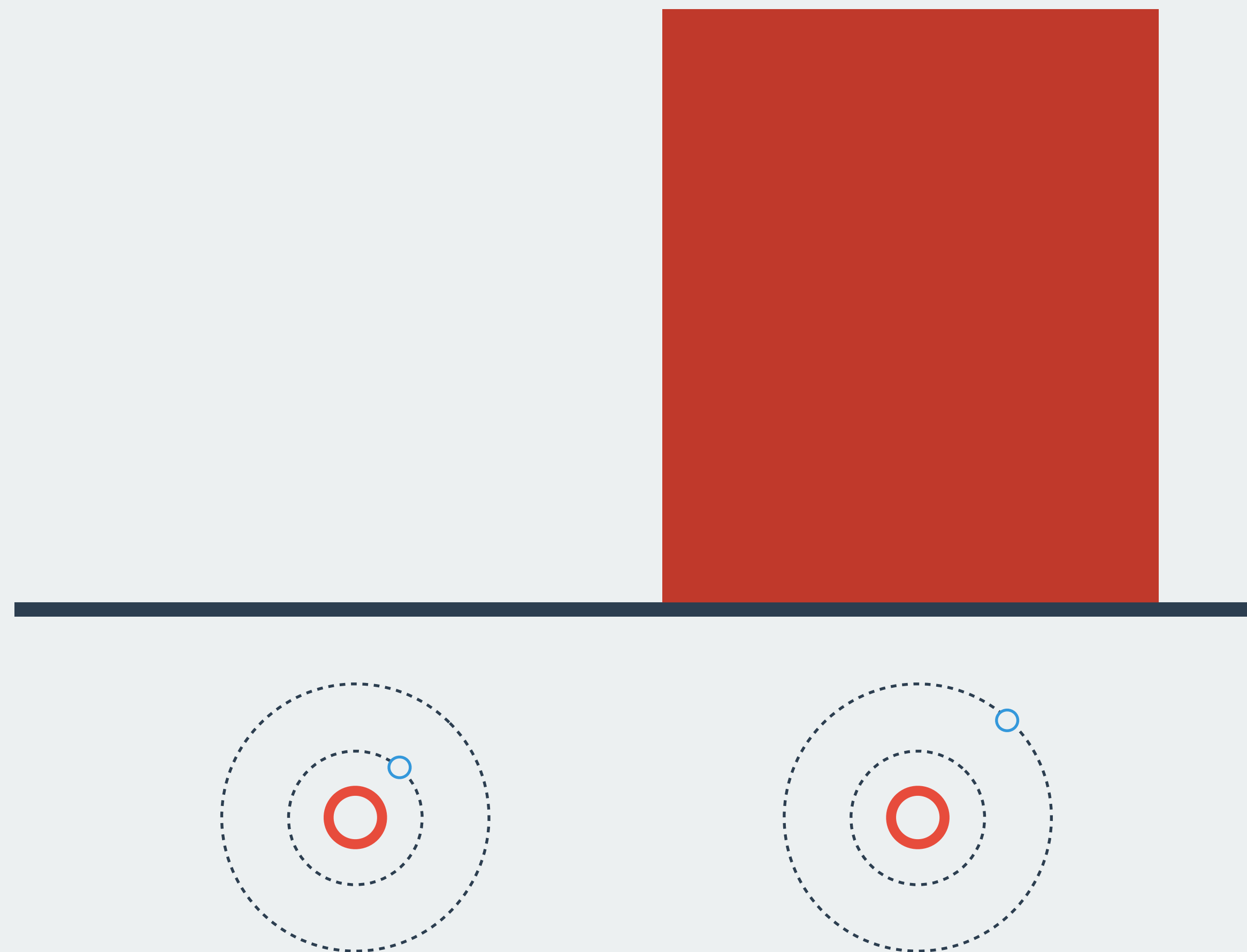
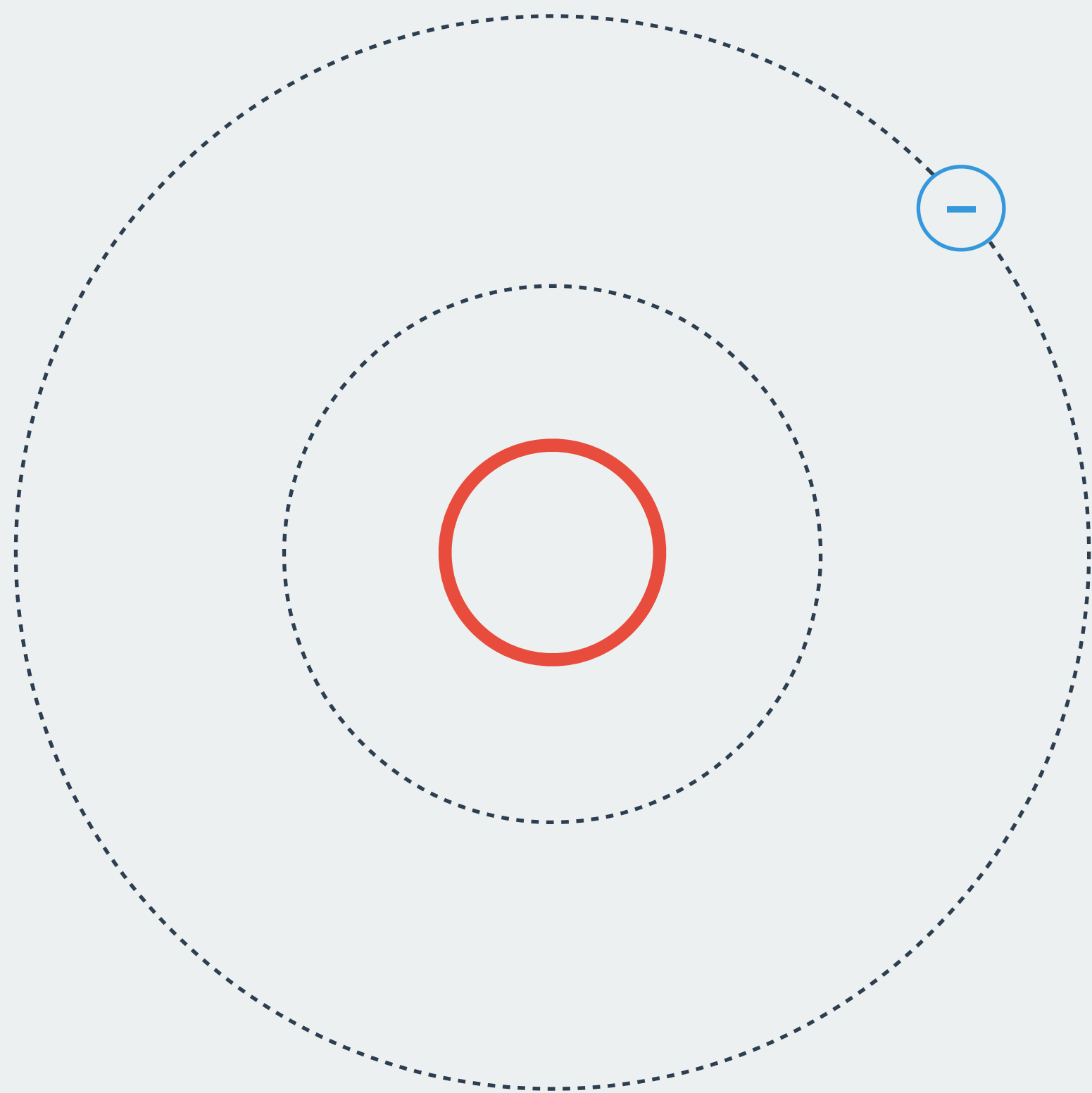


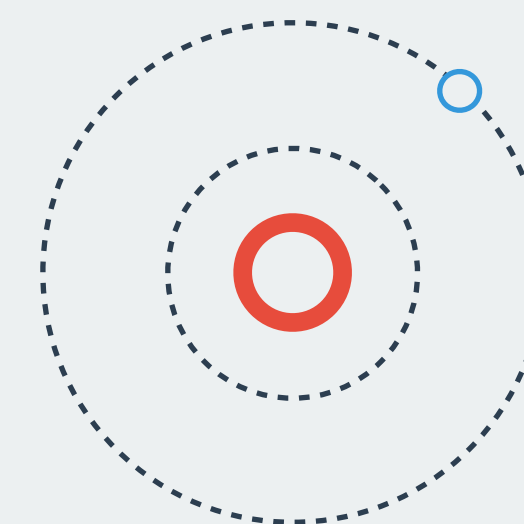
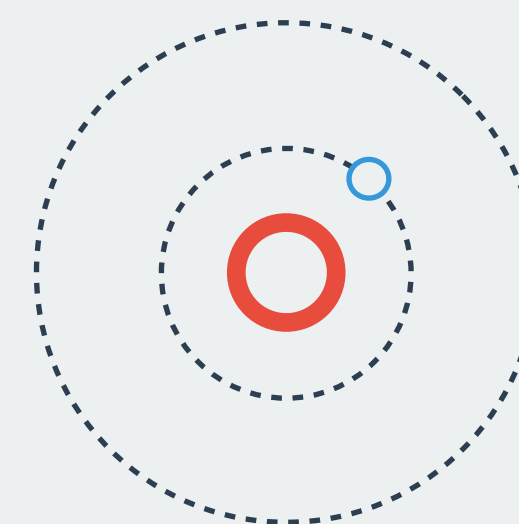
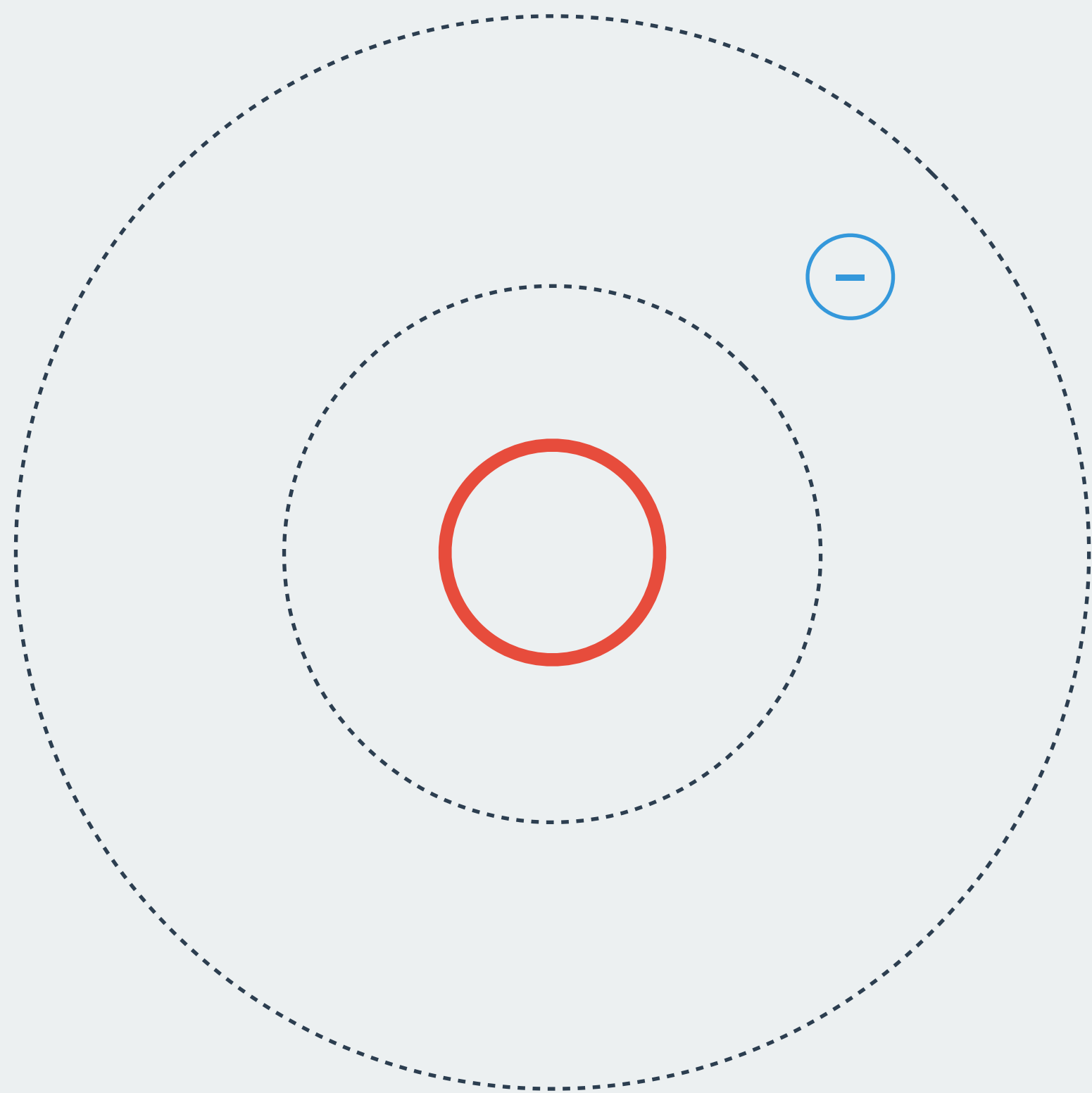


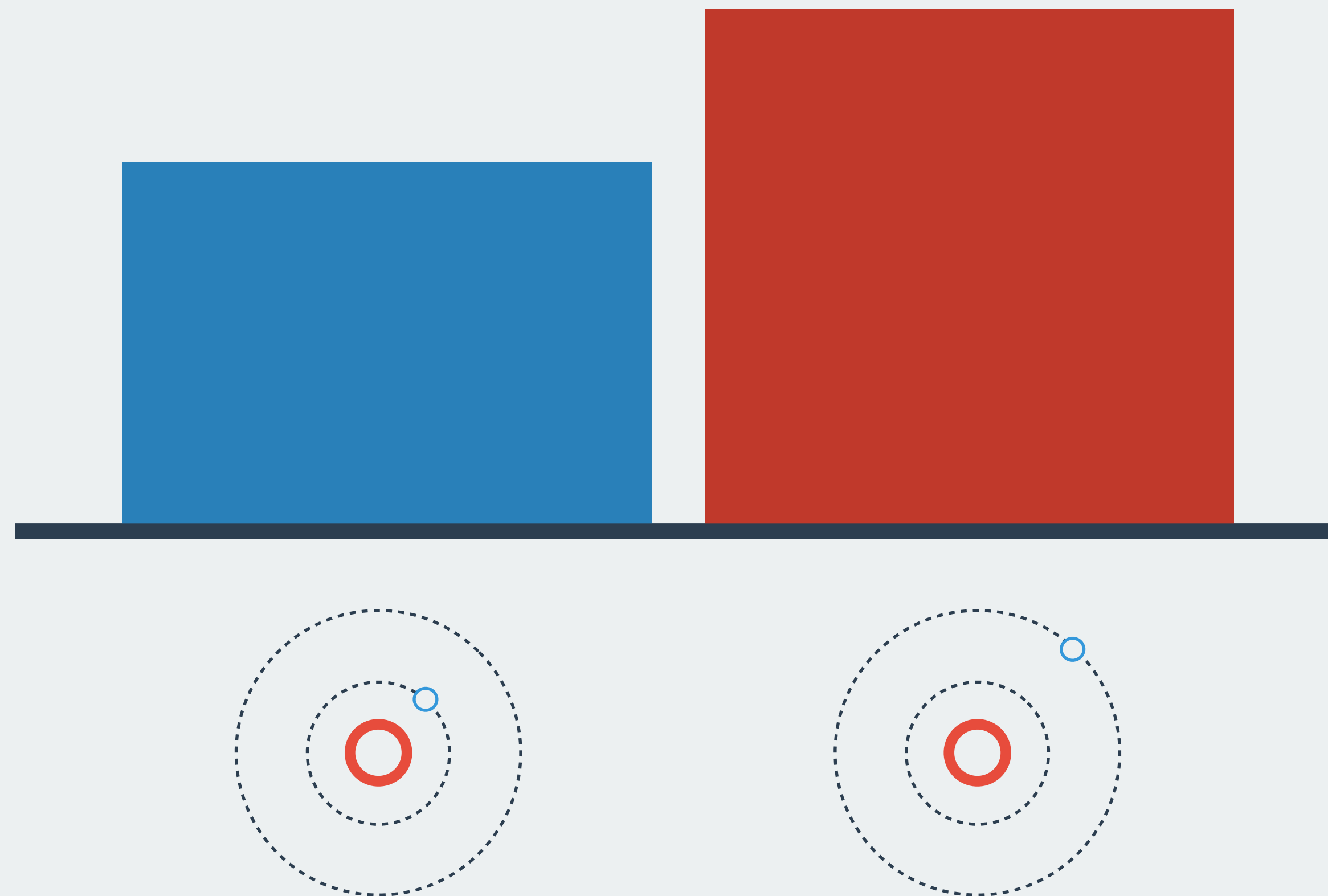
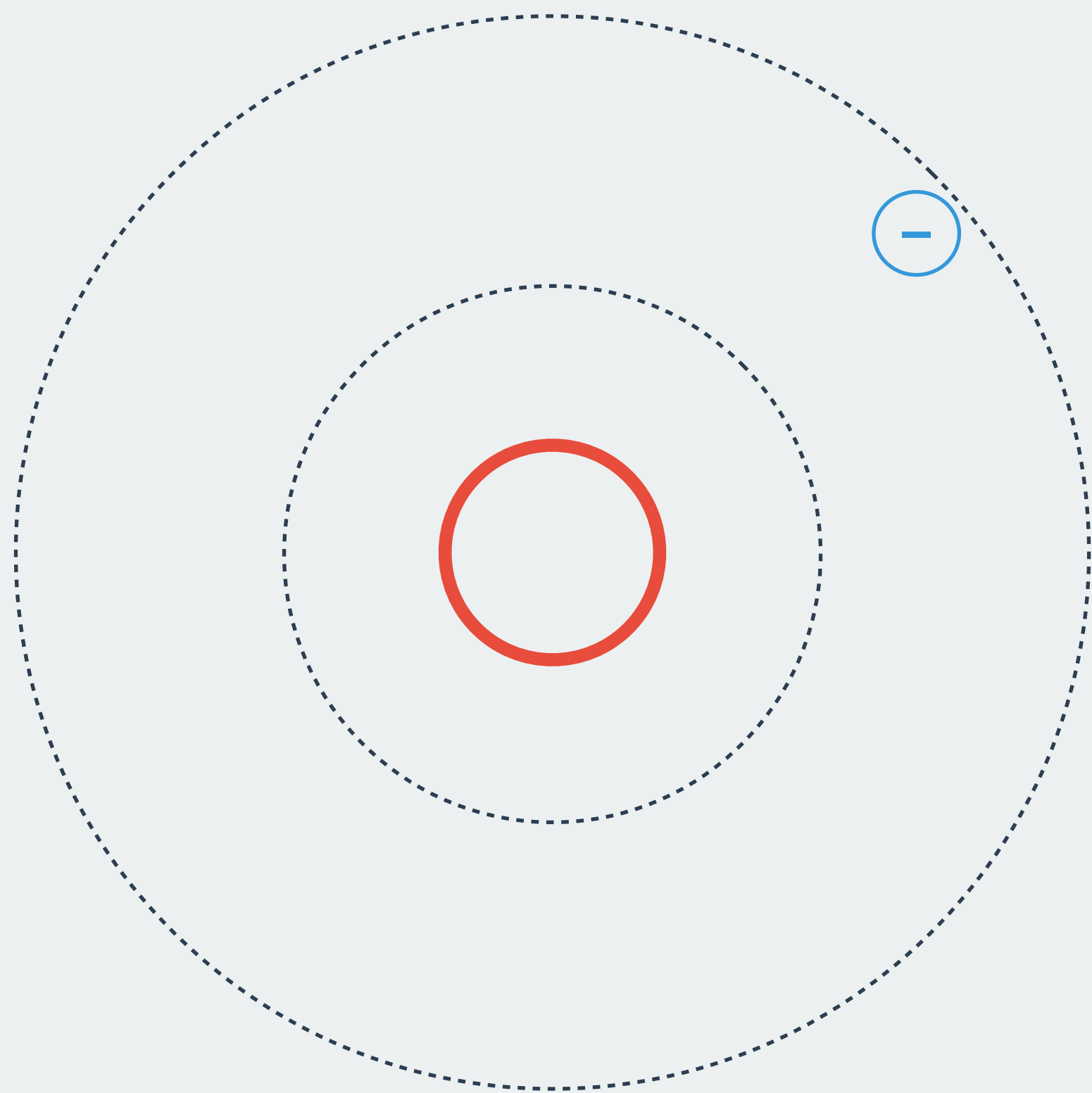


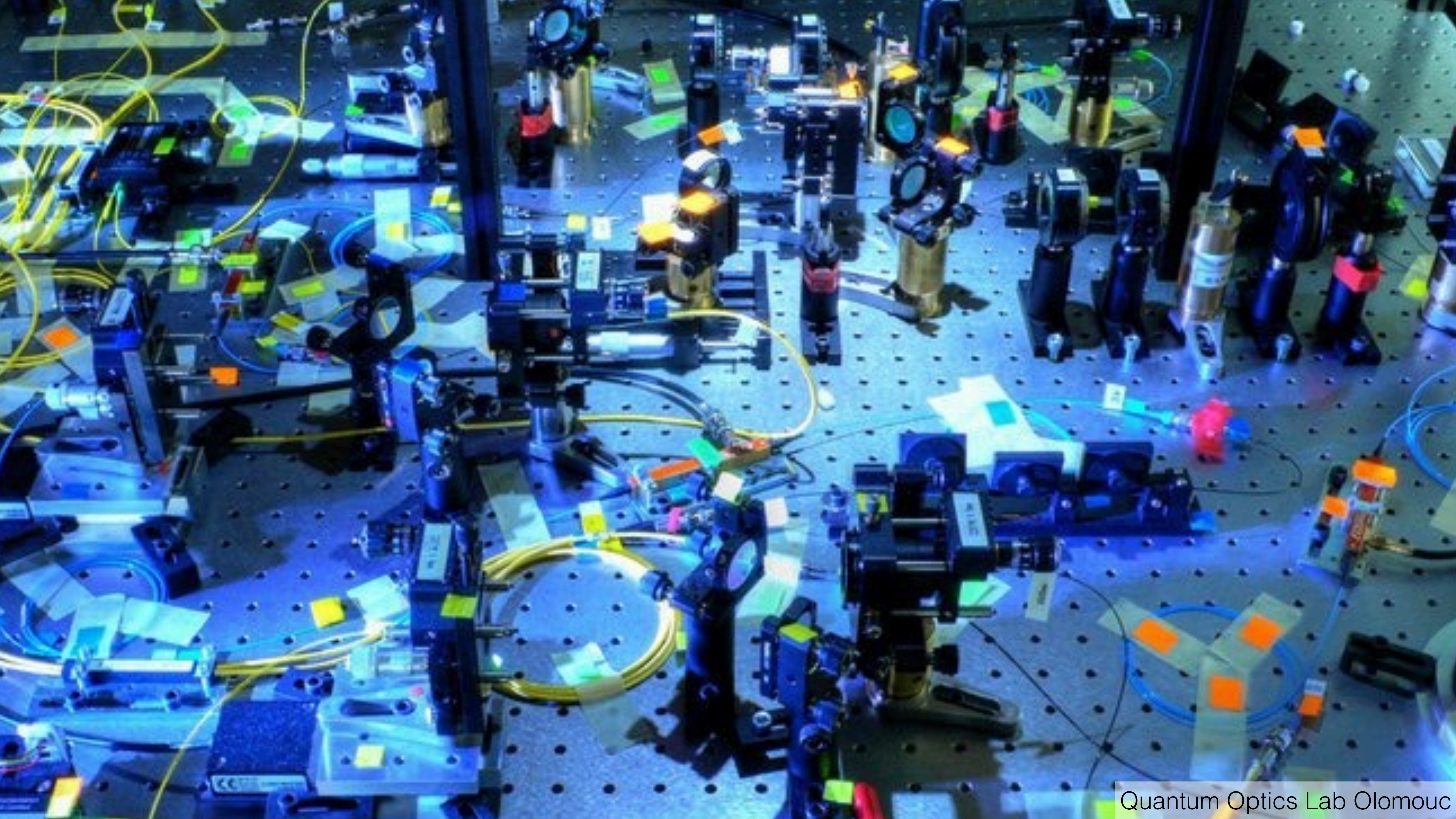






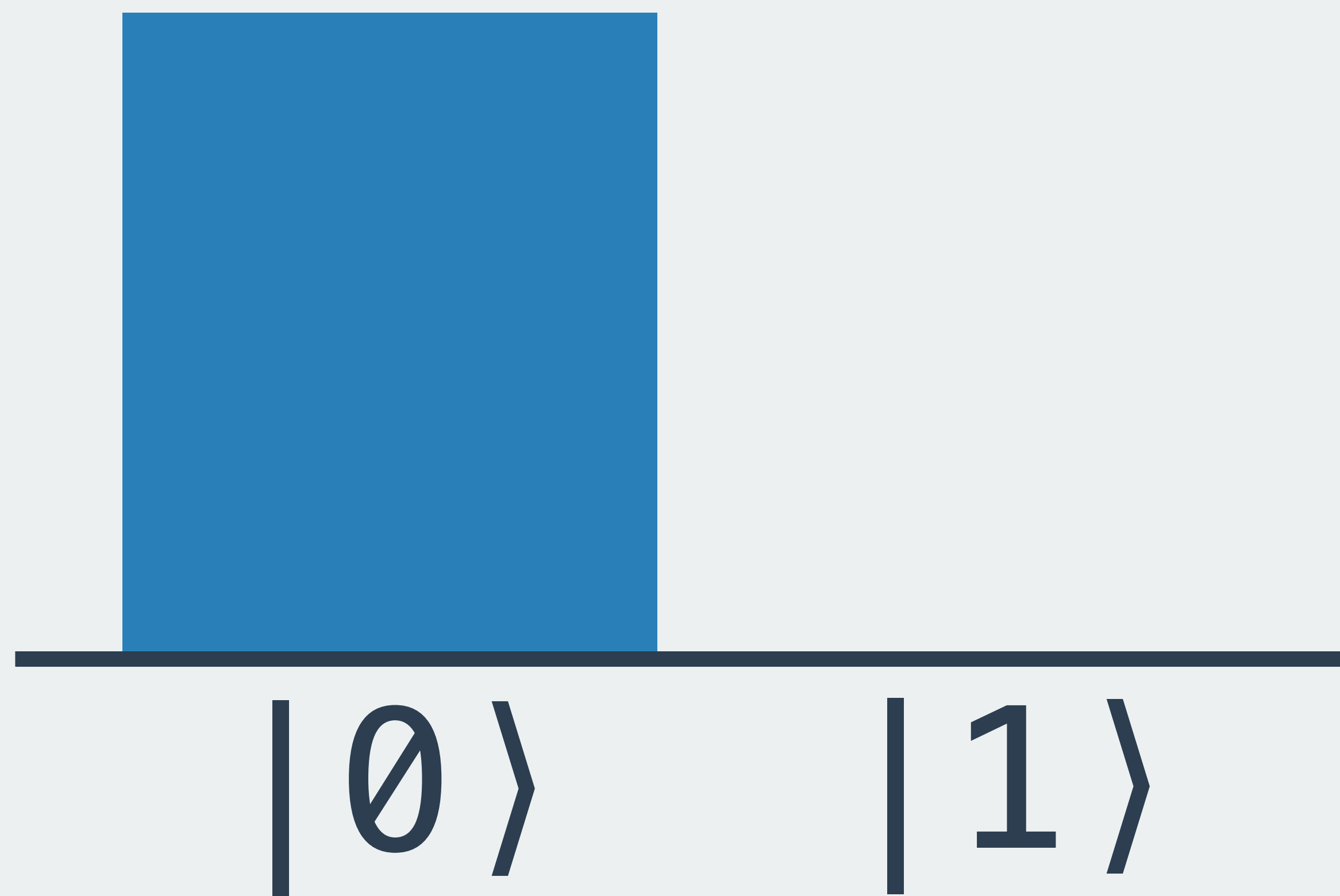


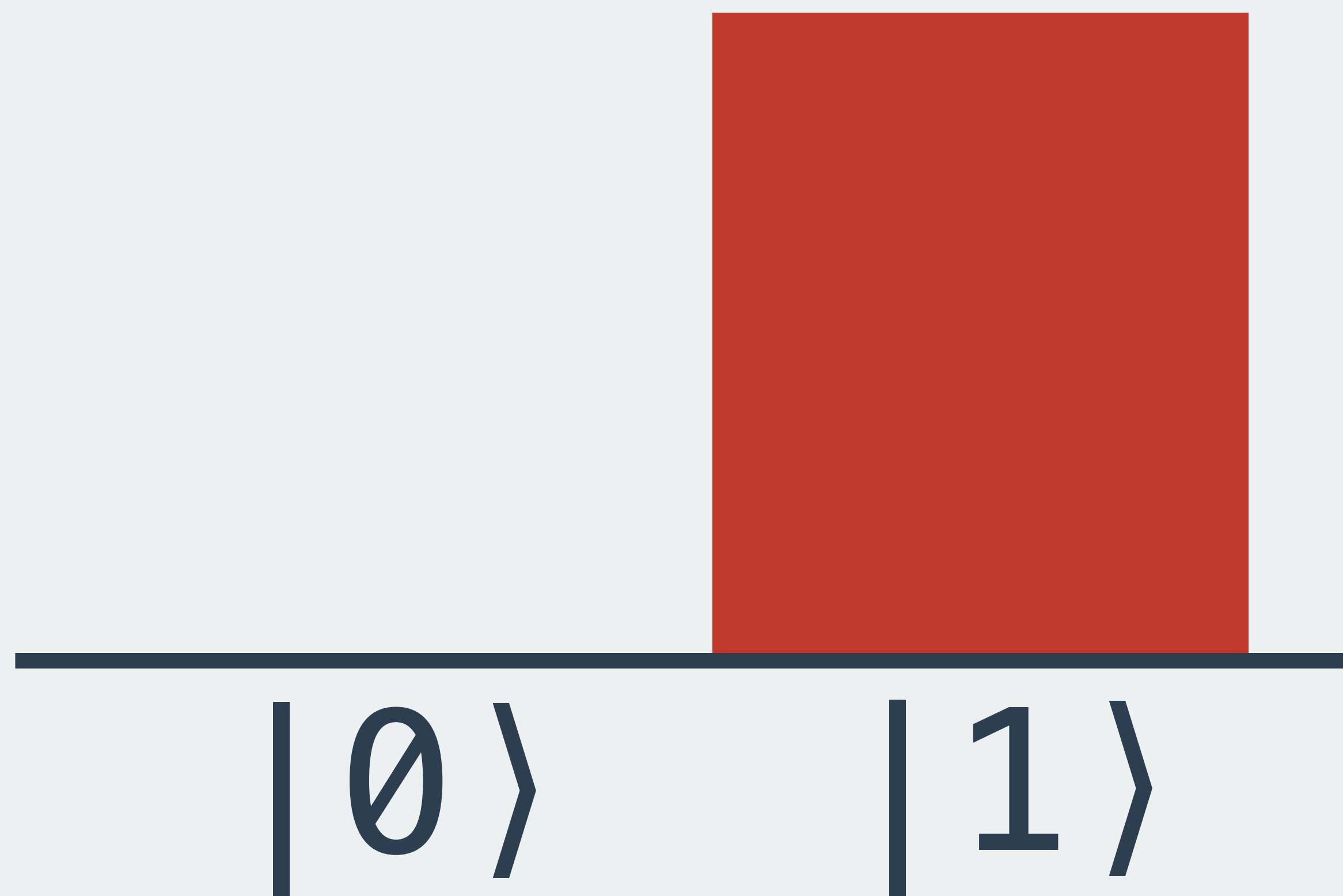


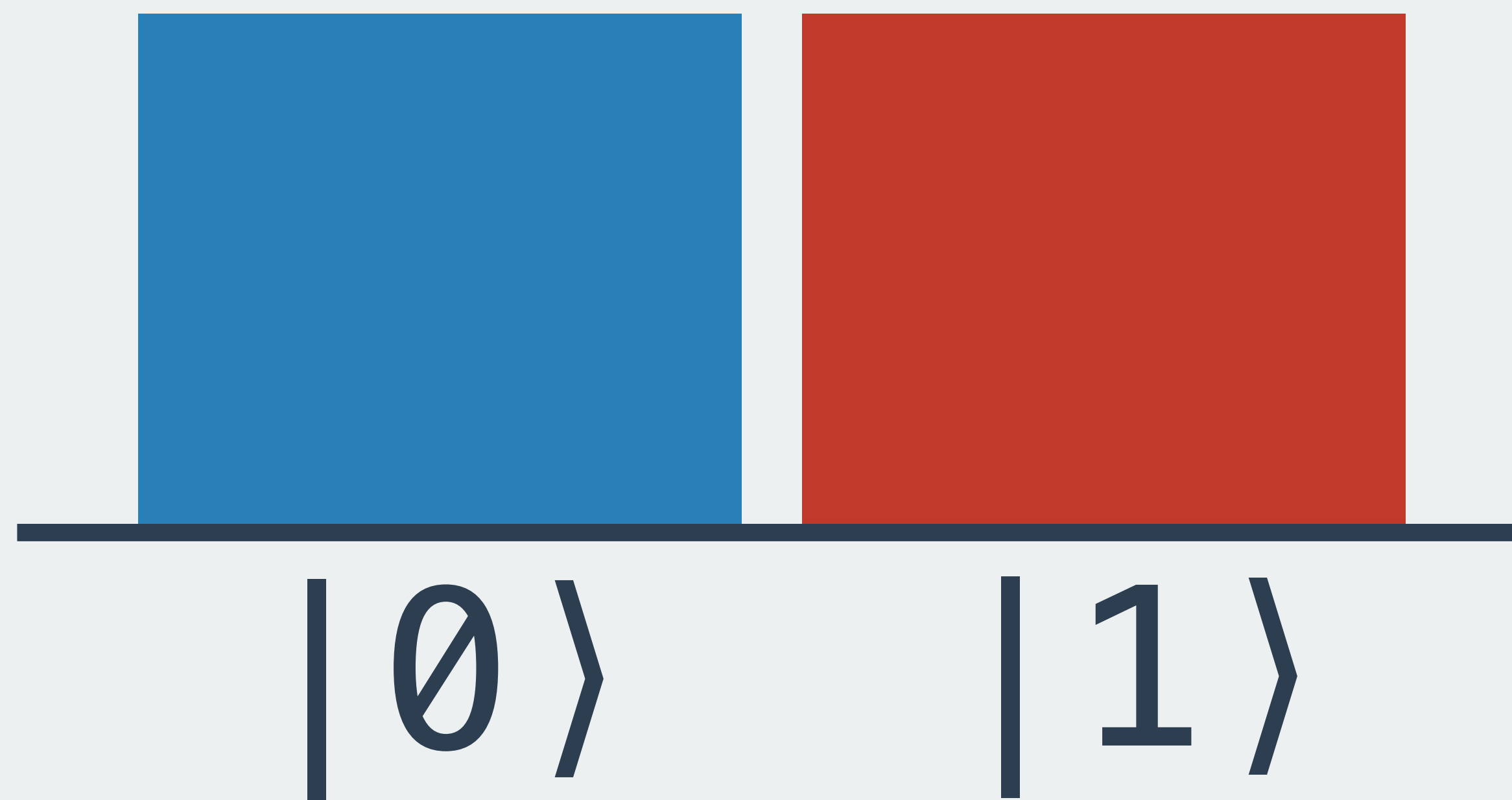


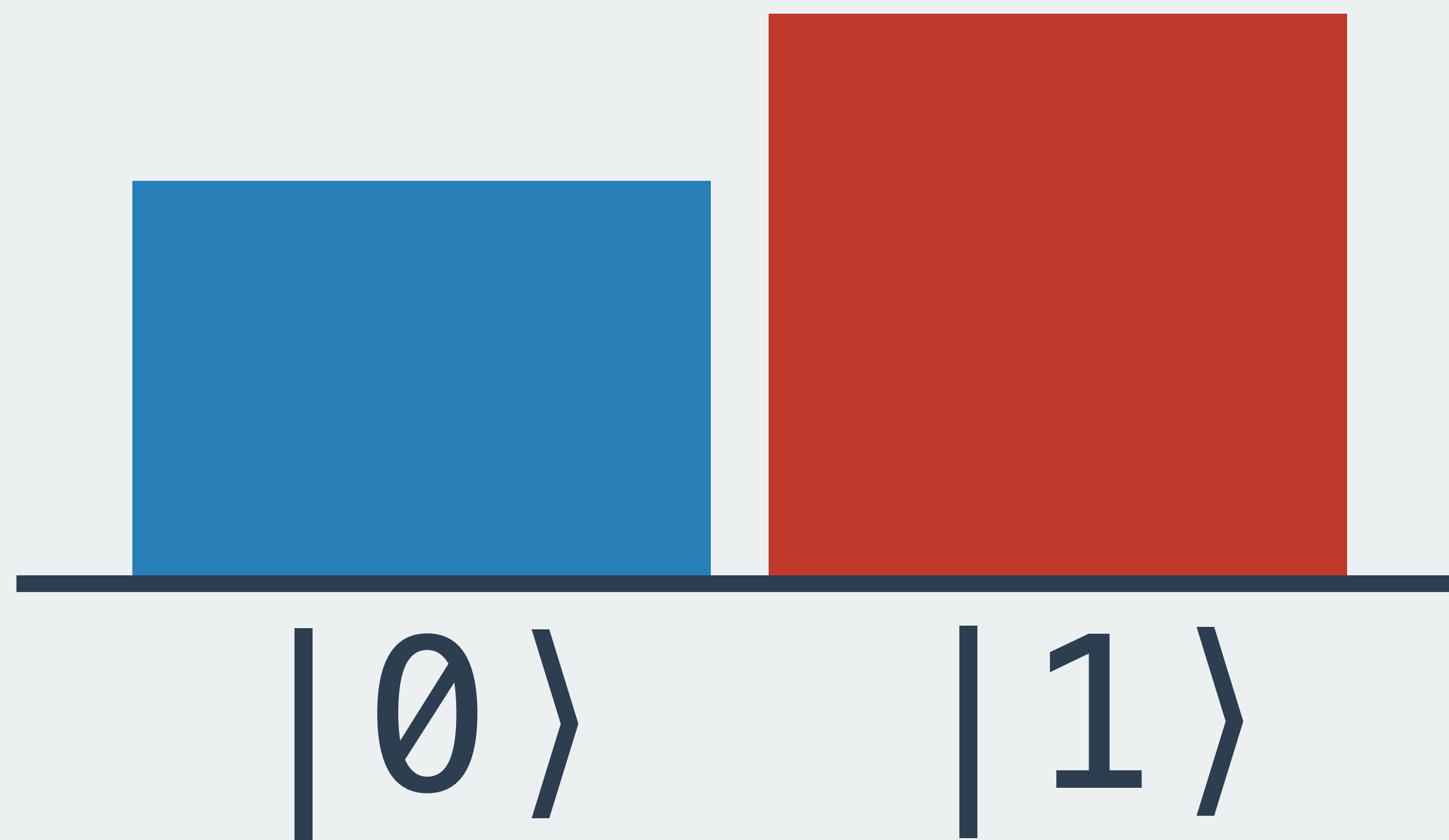


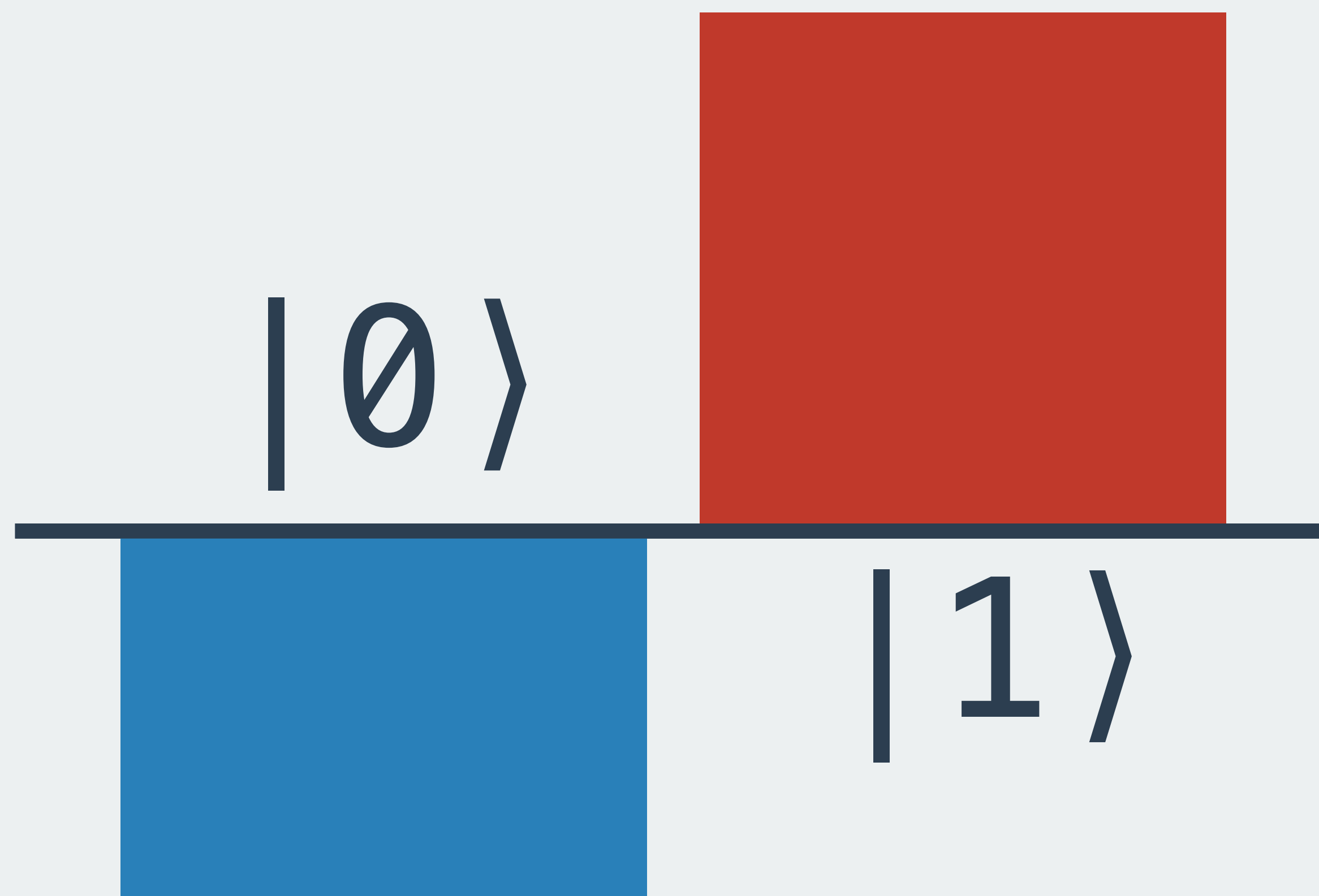
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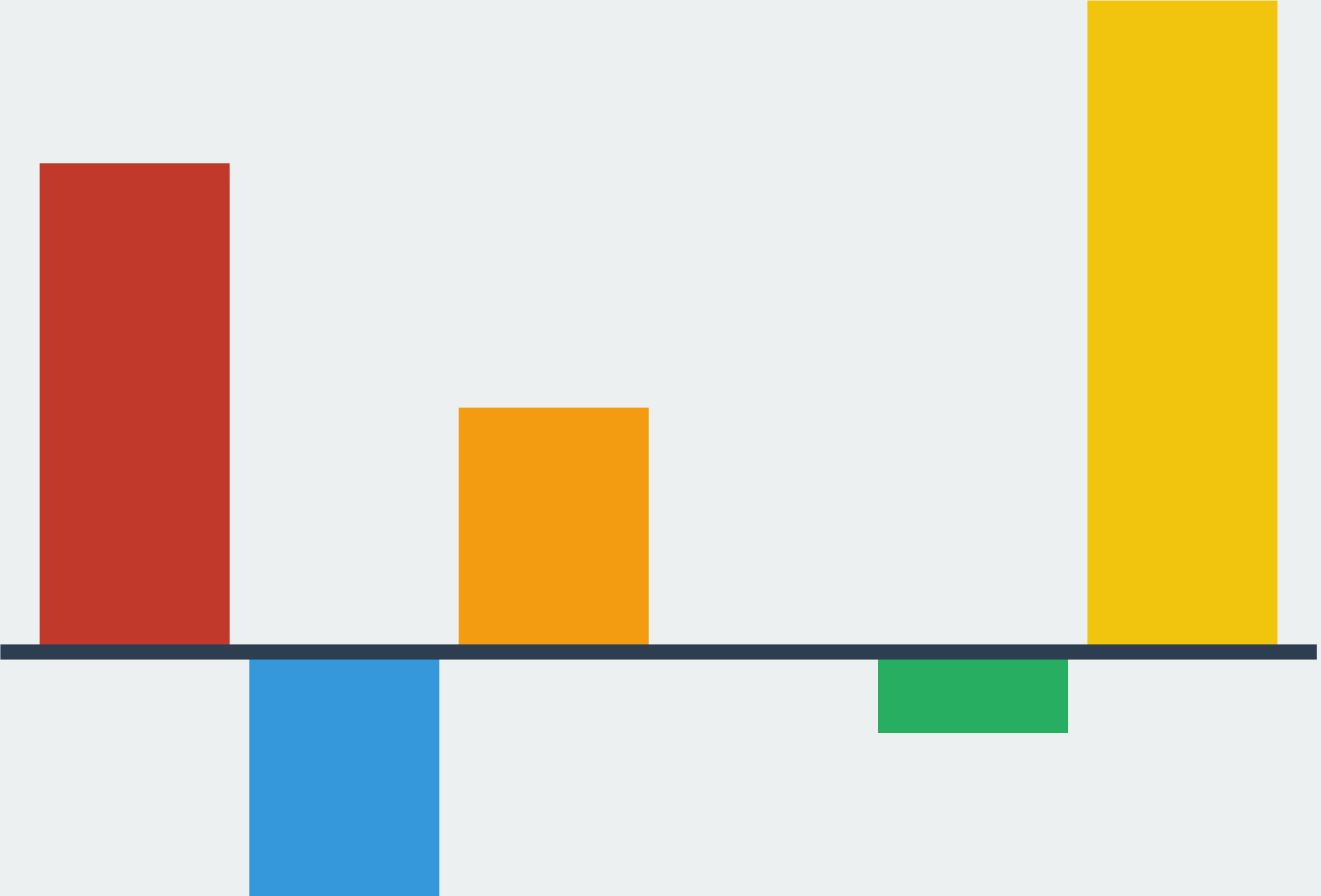




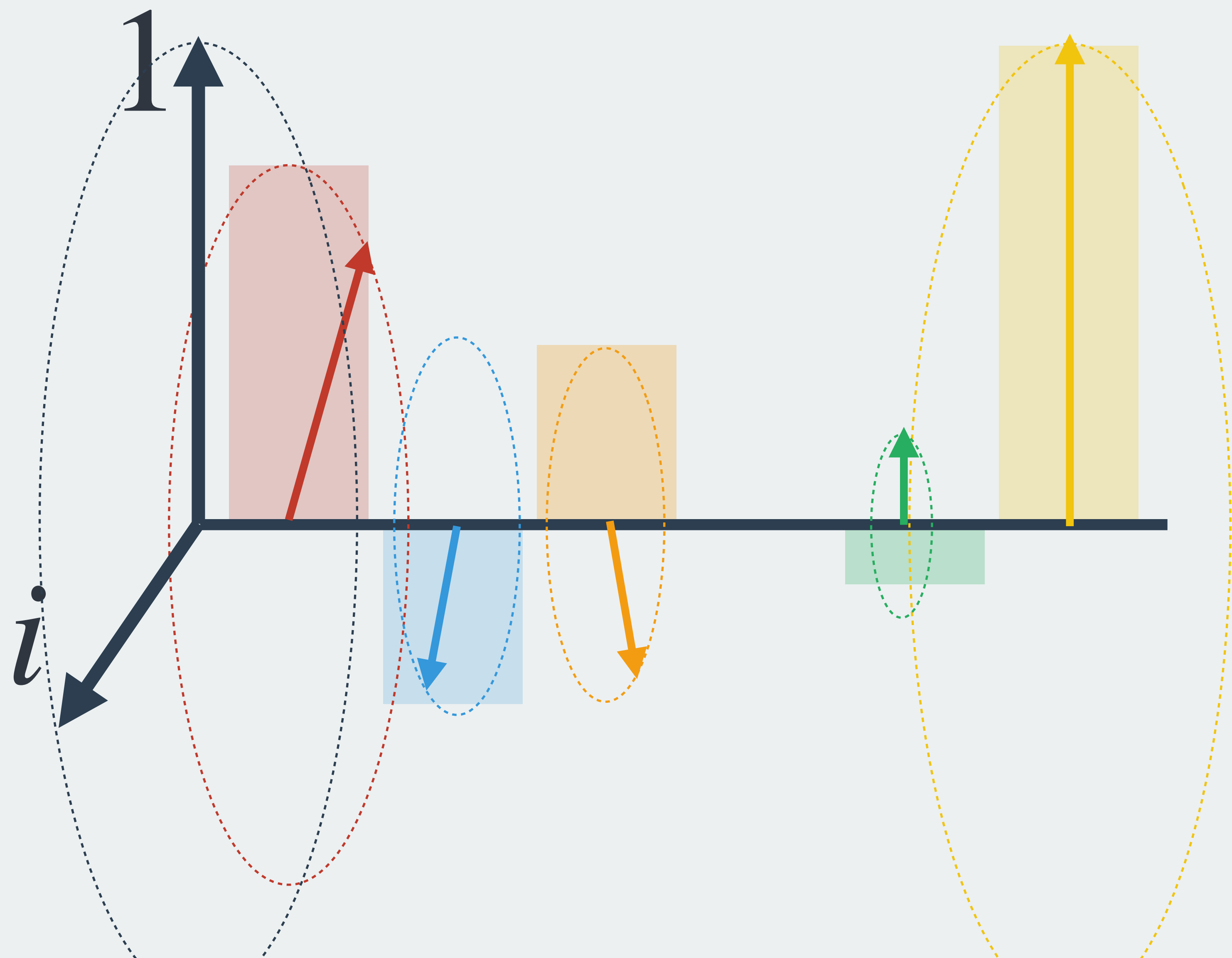




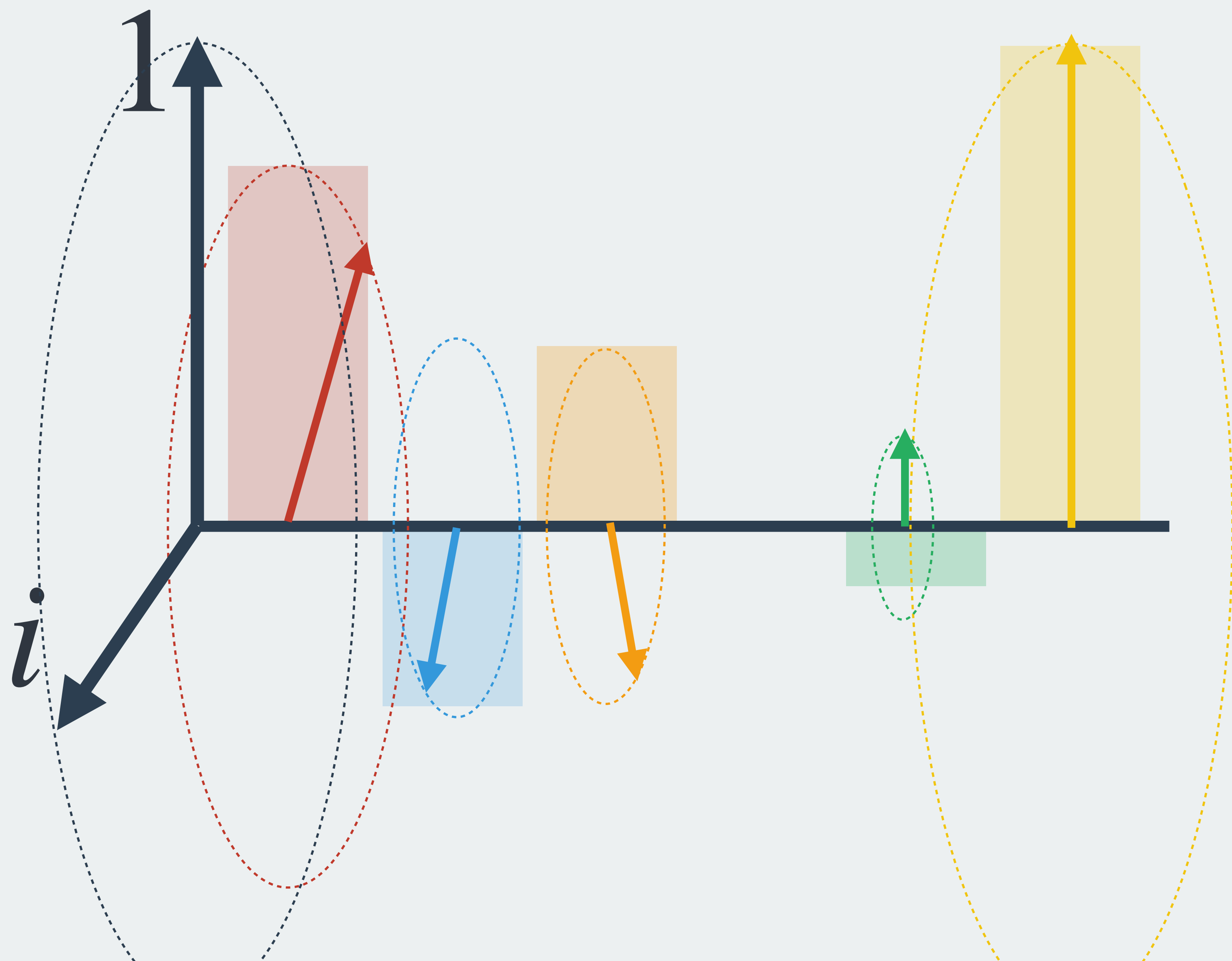


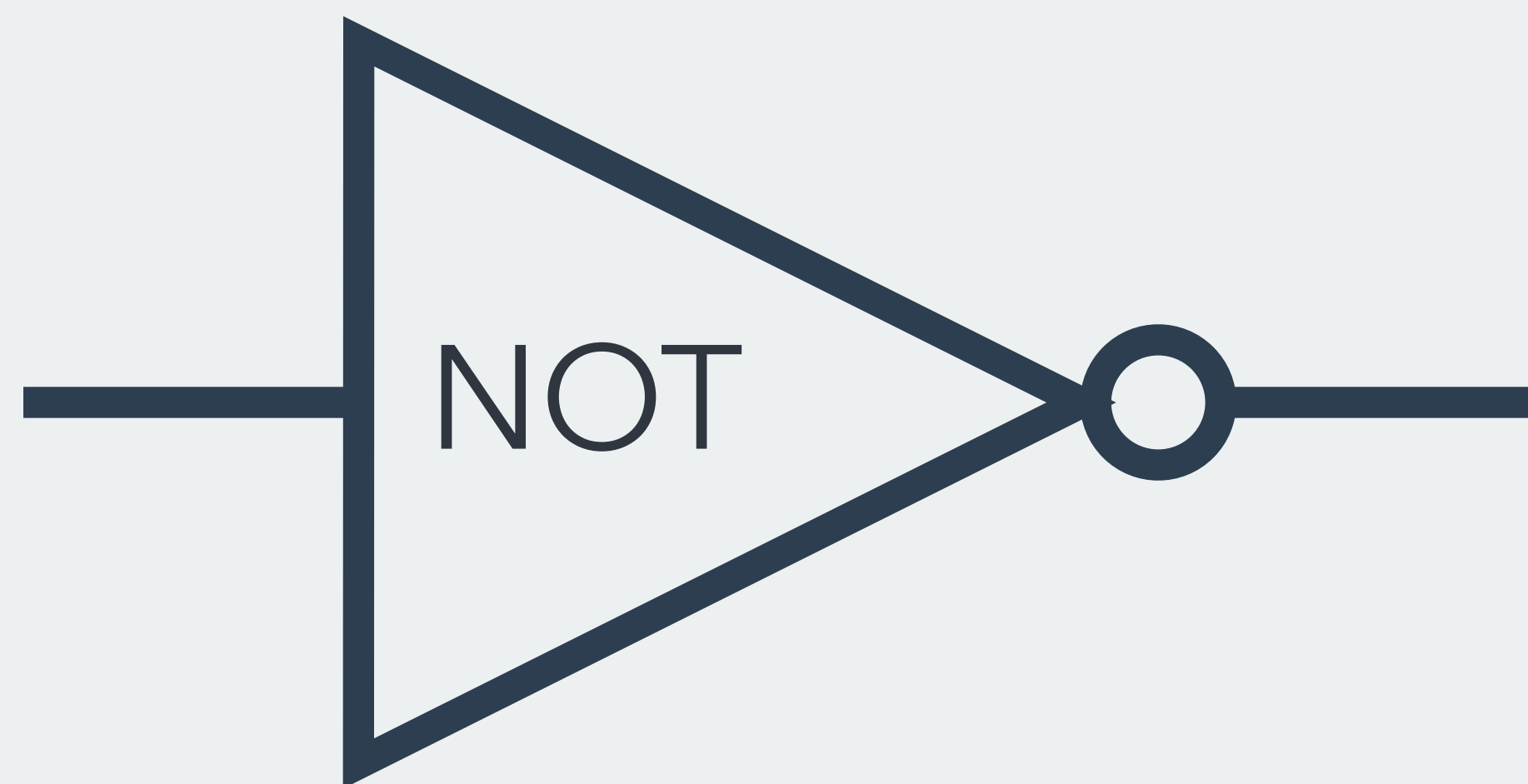


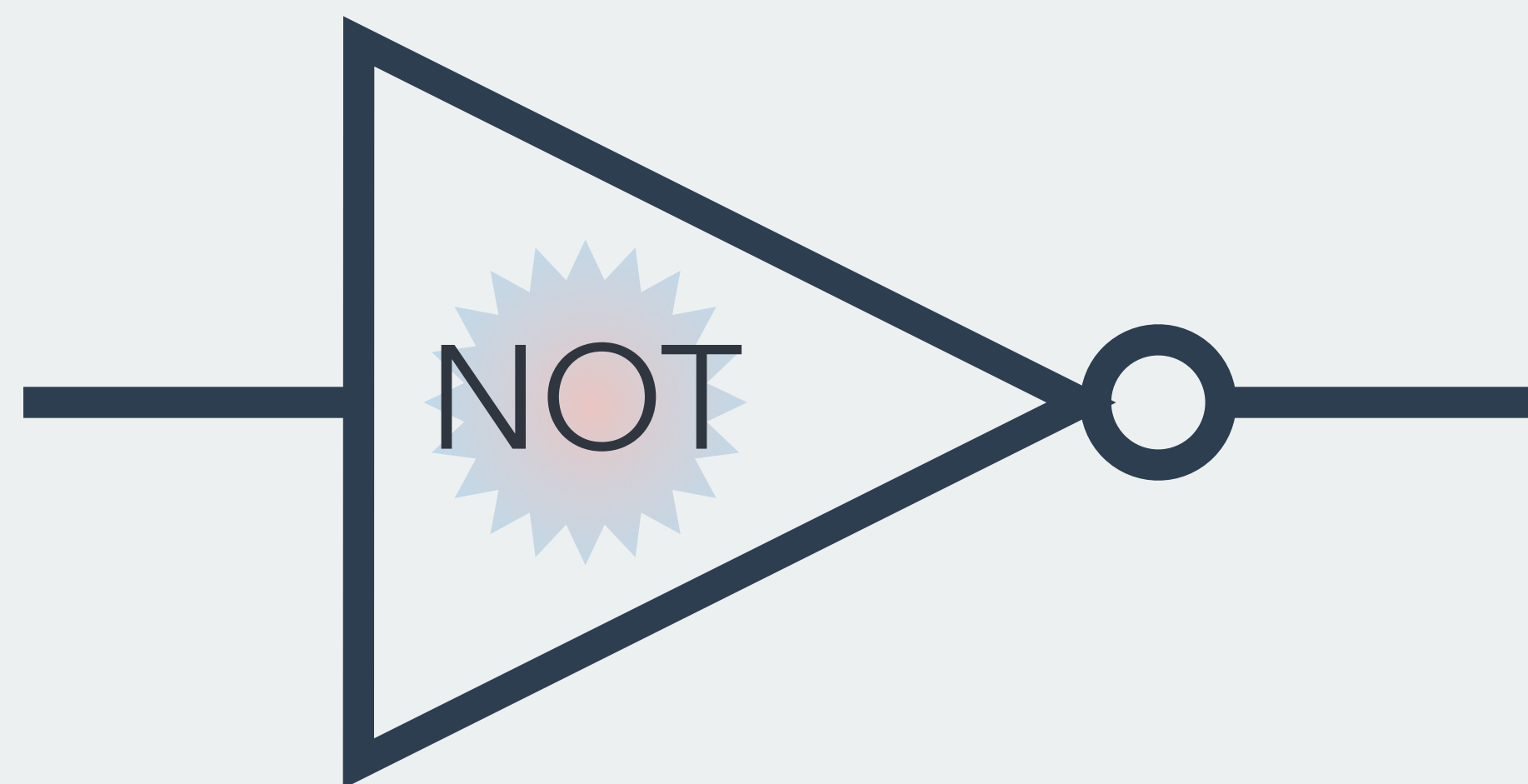


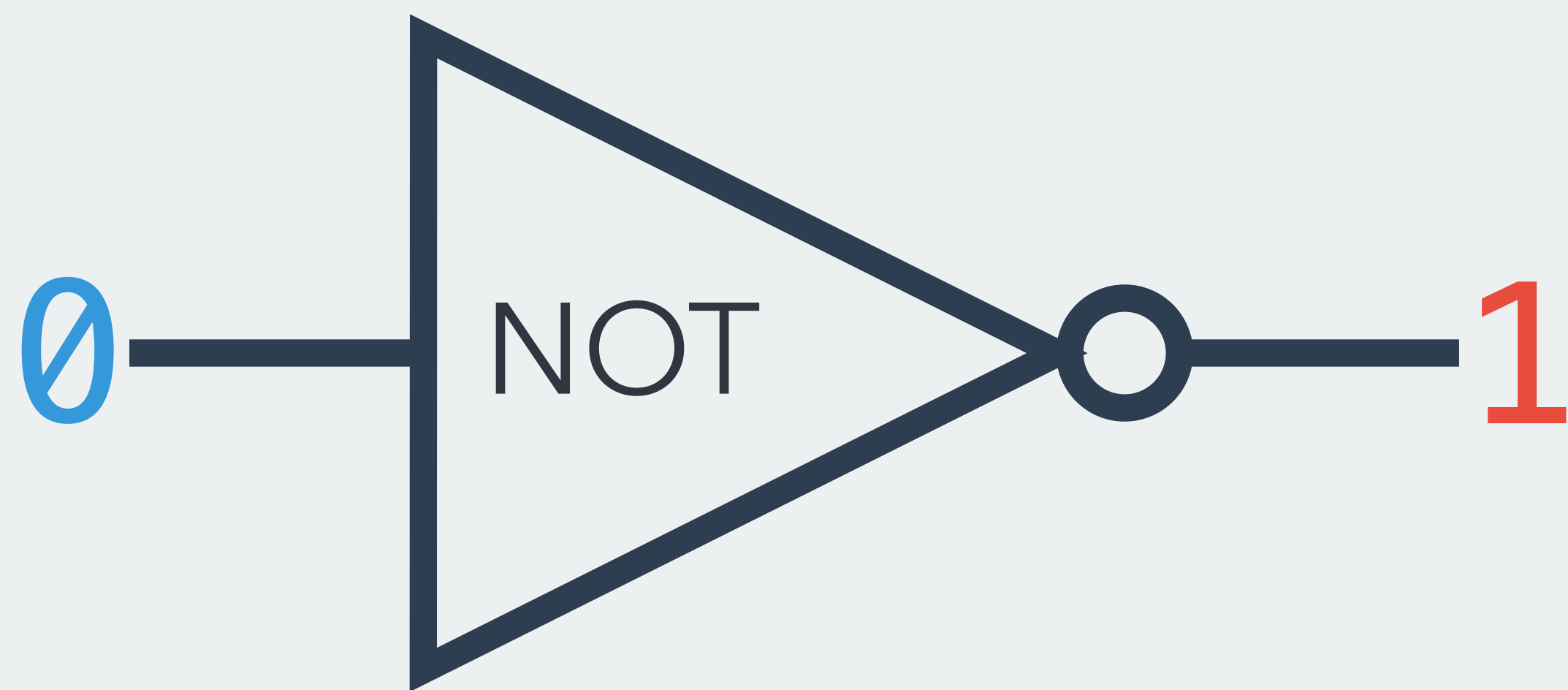


$$|\alpha_1|^2 + \dots + |\alpha_n|^2 = 1$$



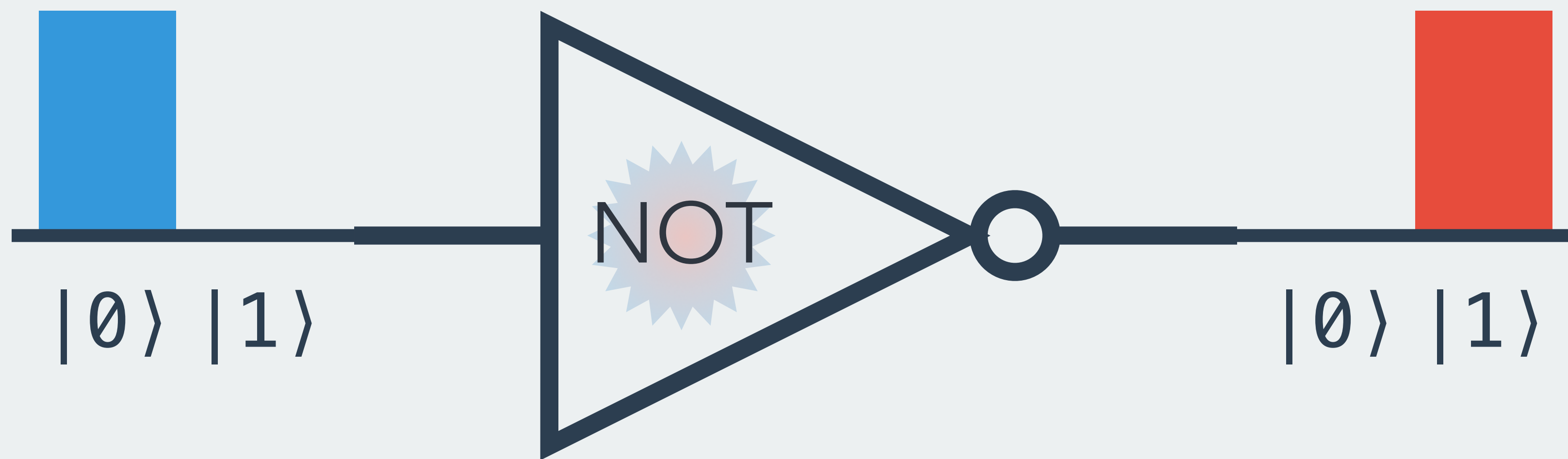


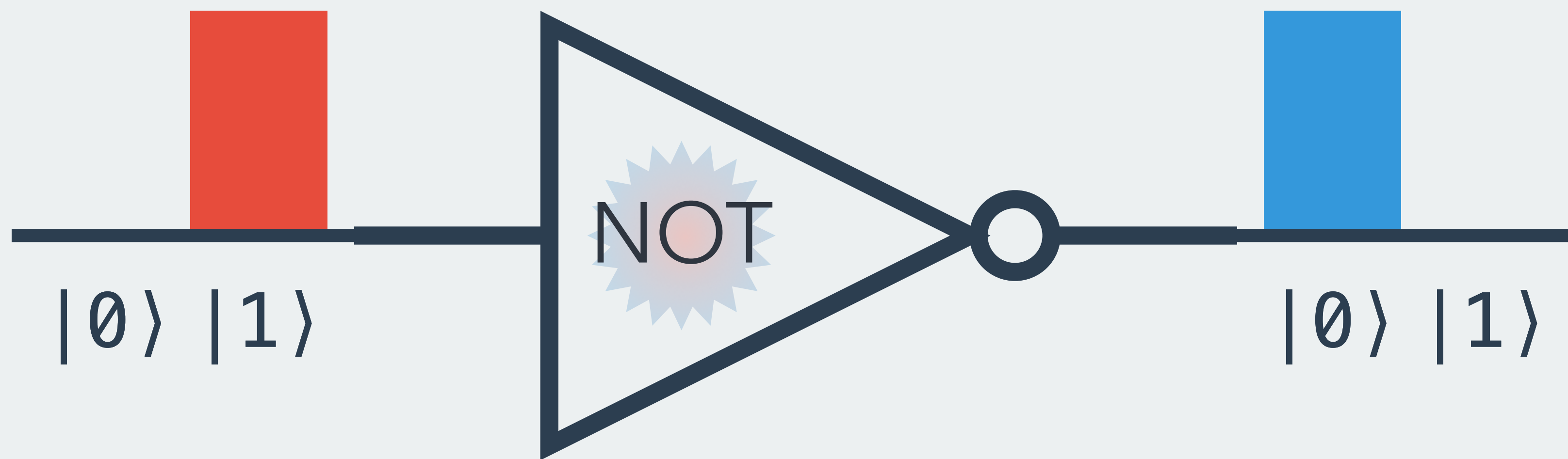


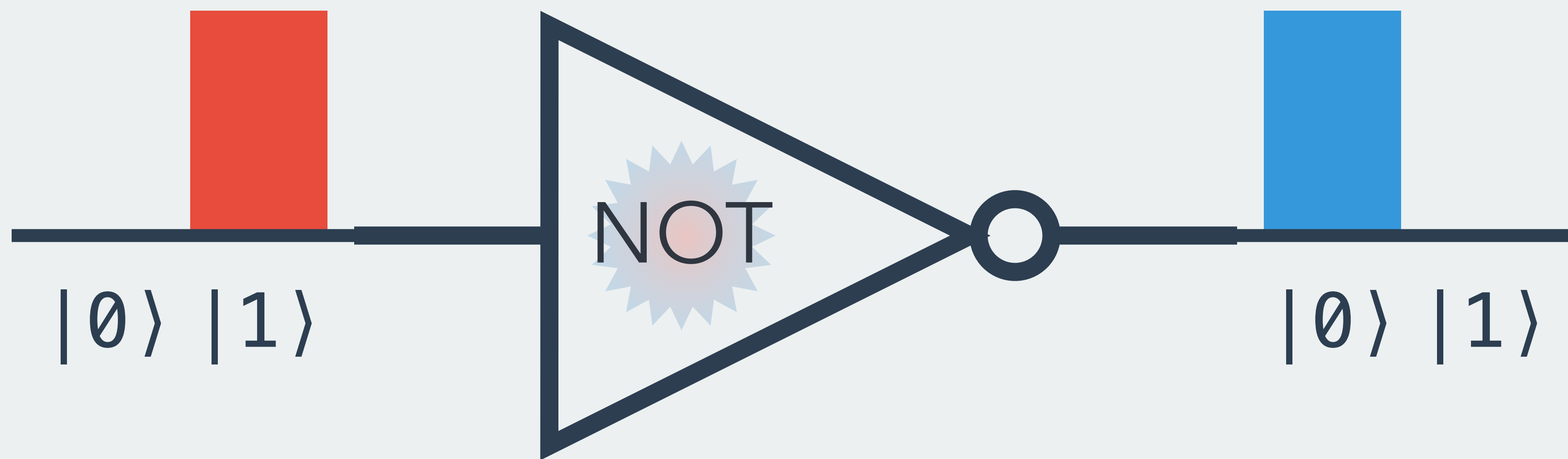


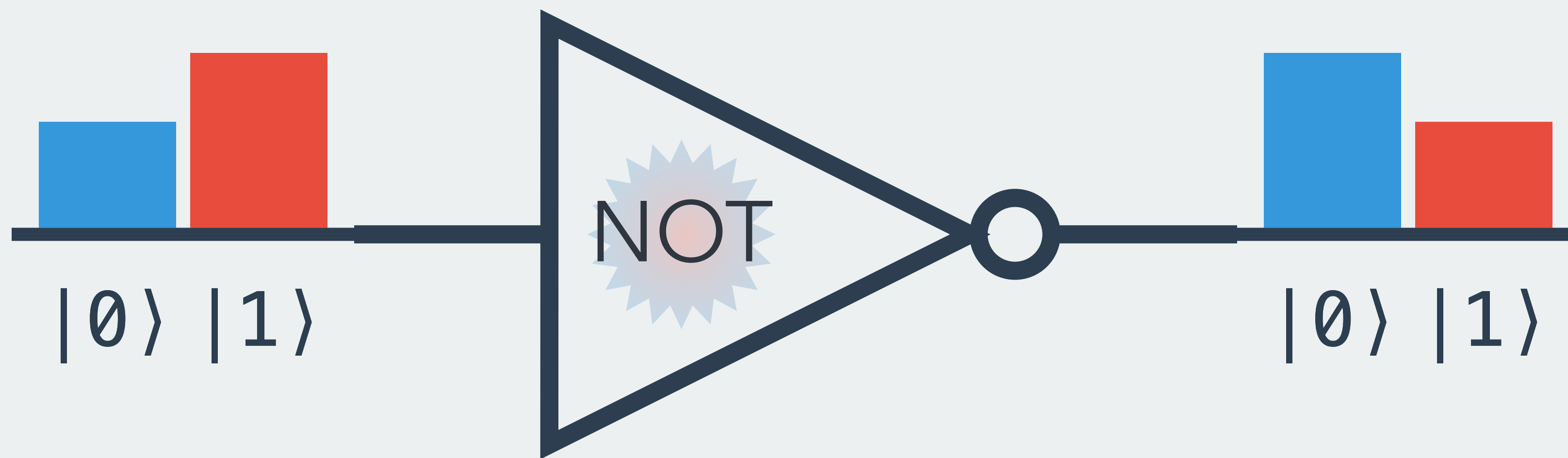


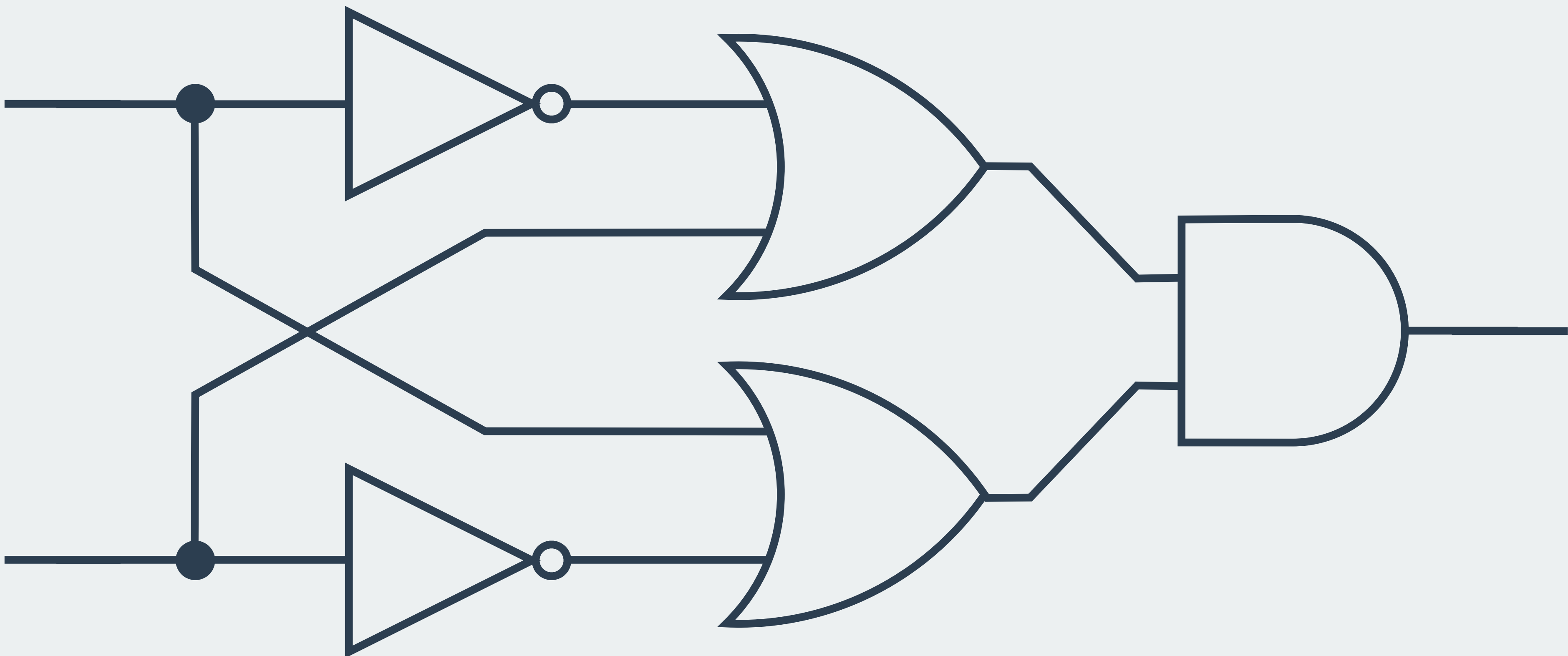
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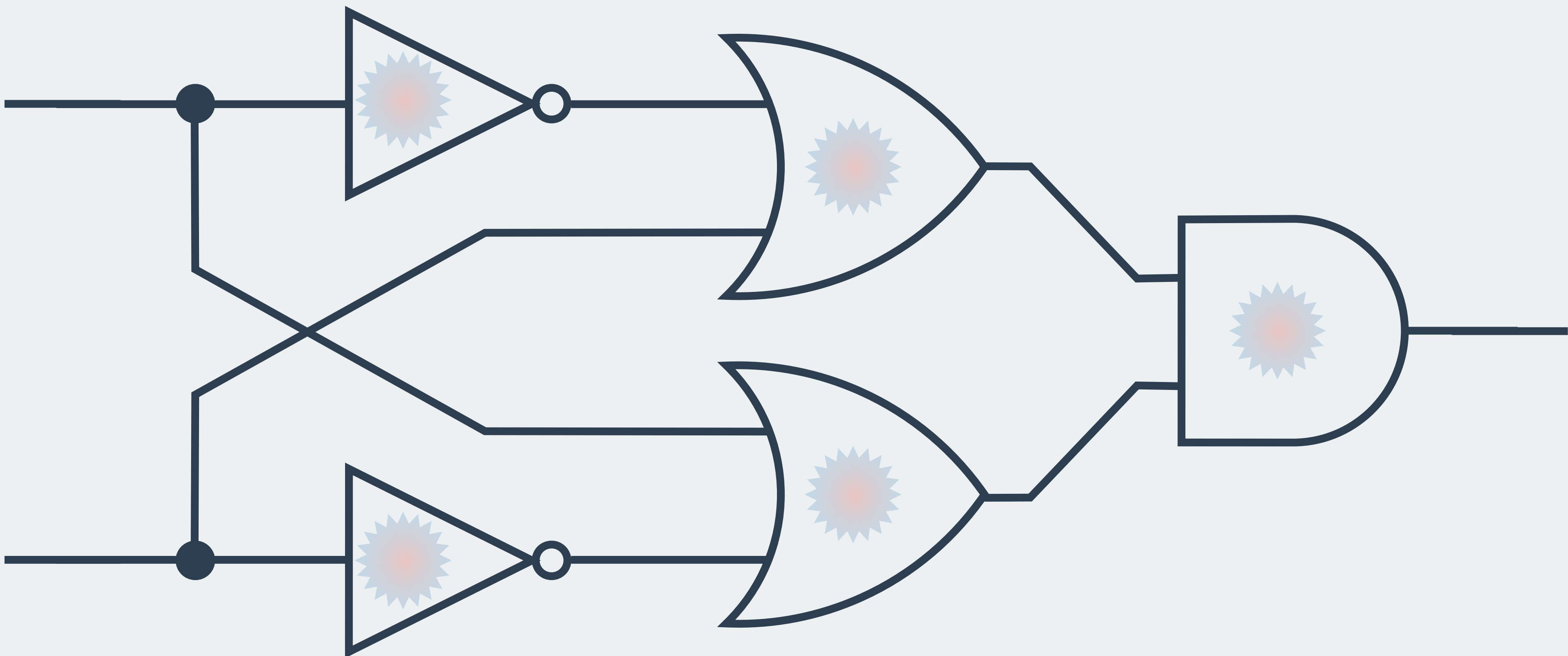


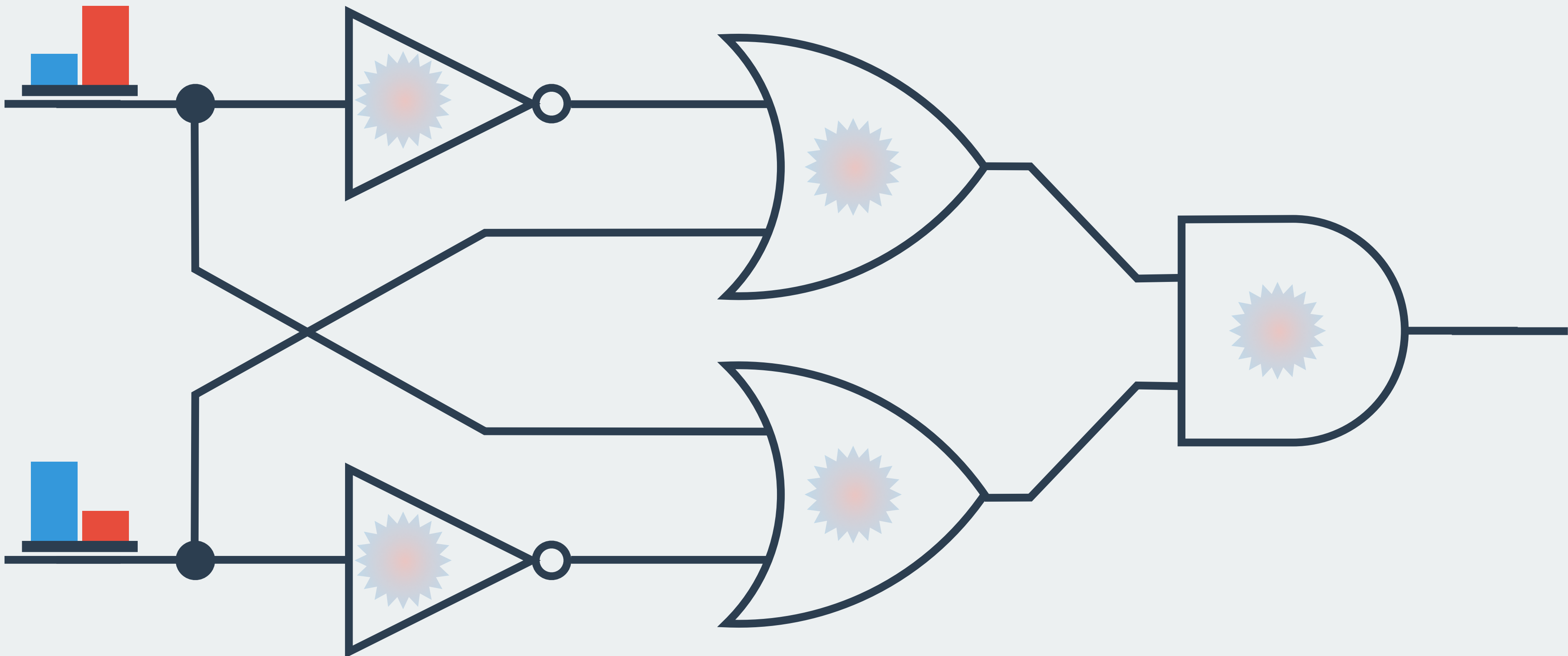


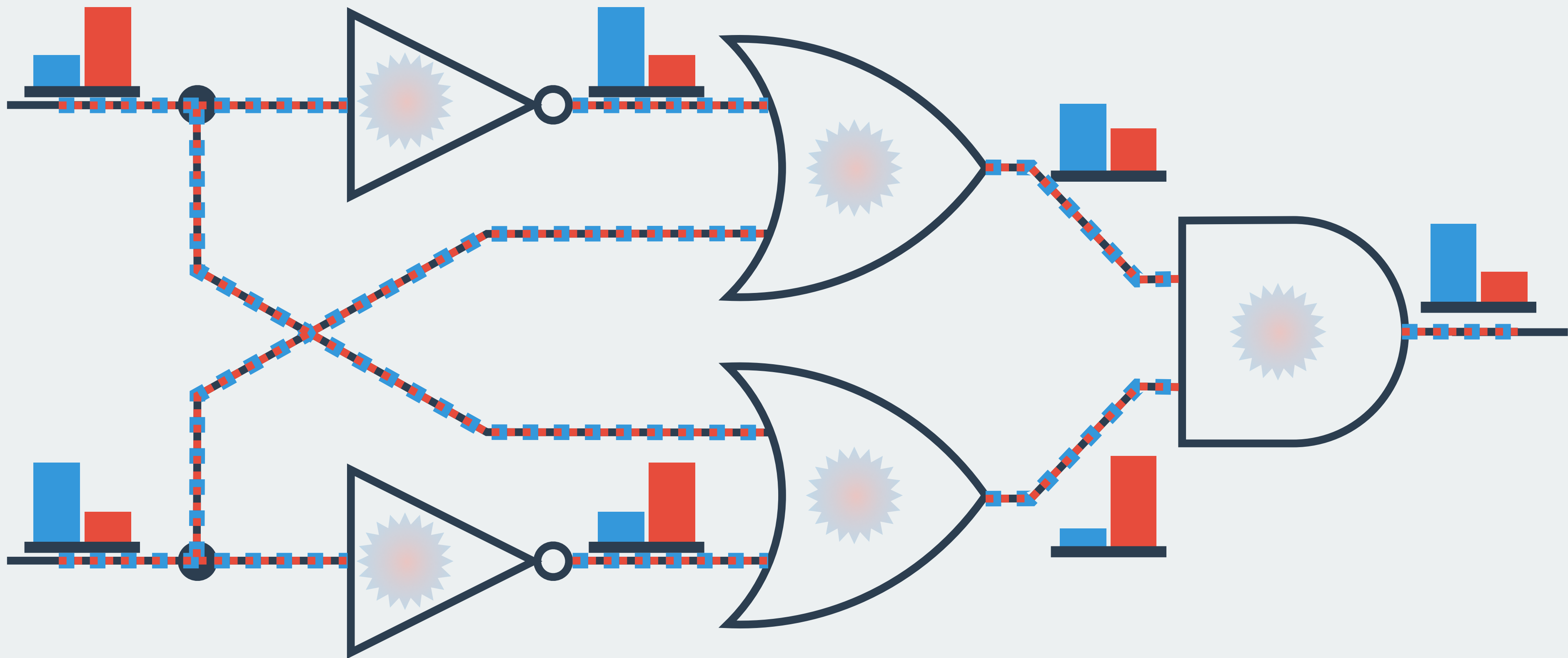








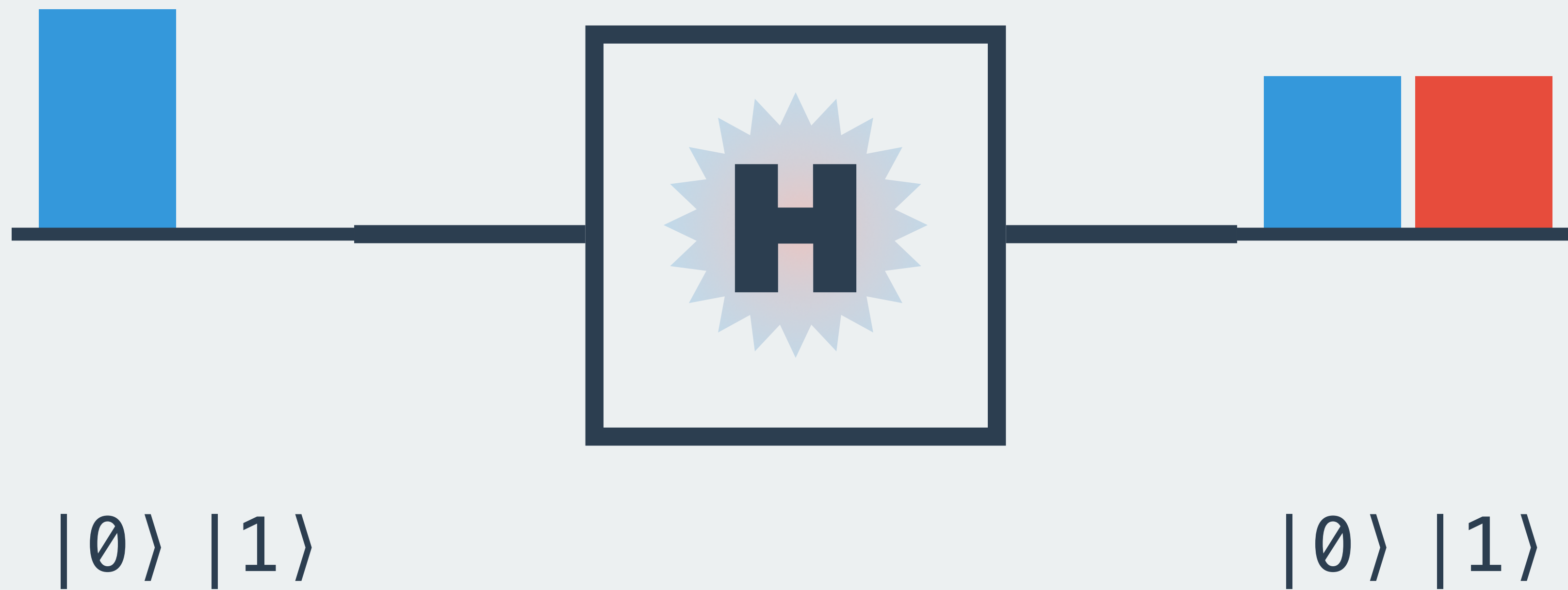


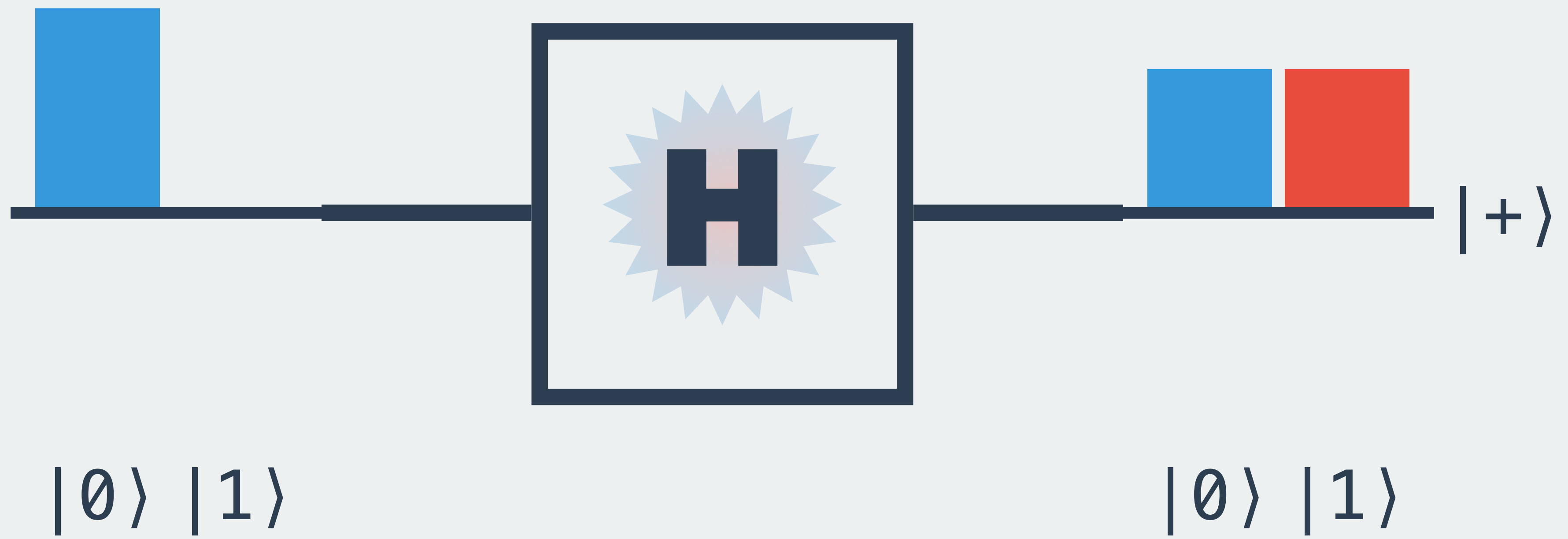


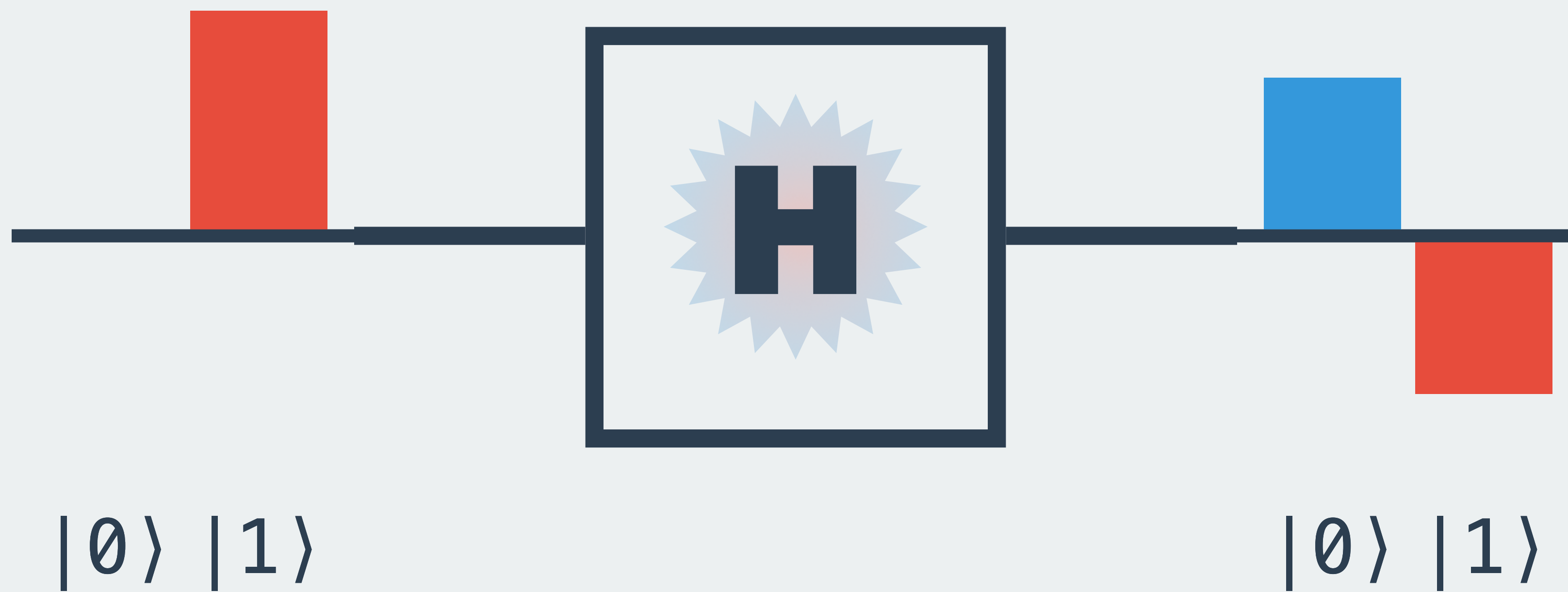


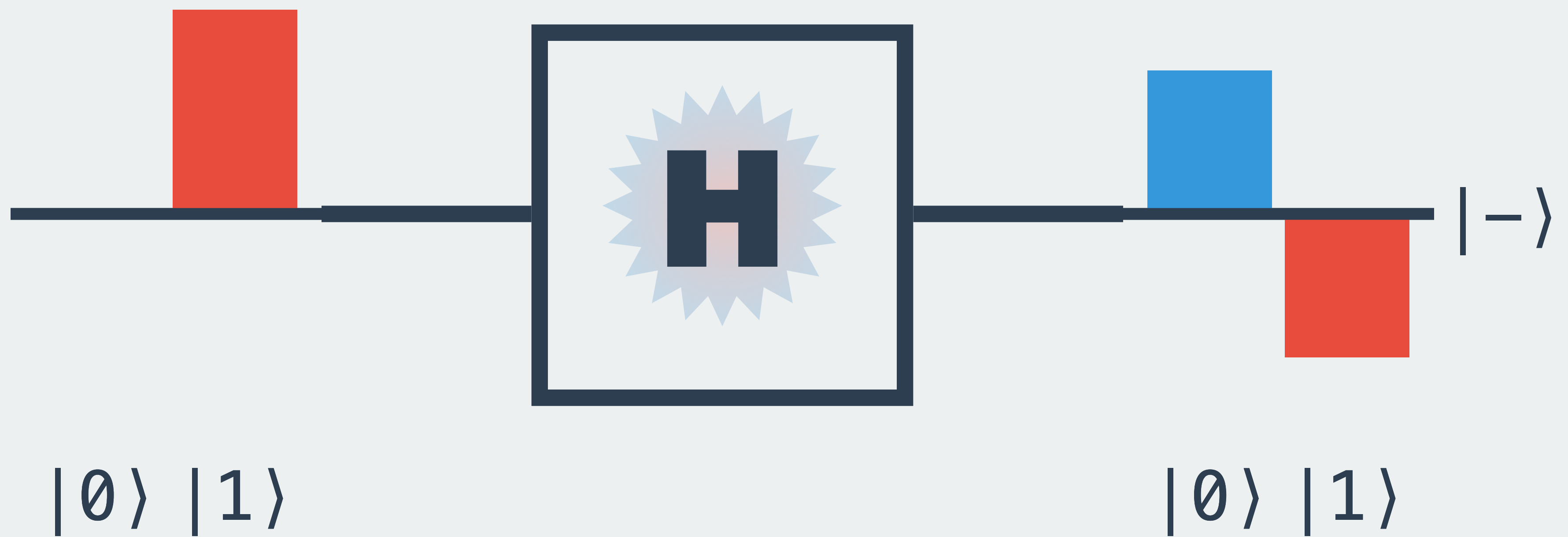


Jacques Hadamard



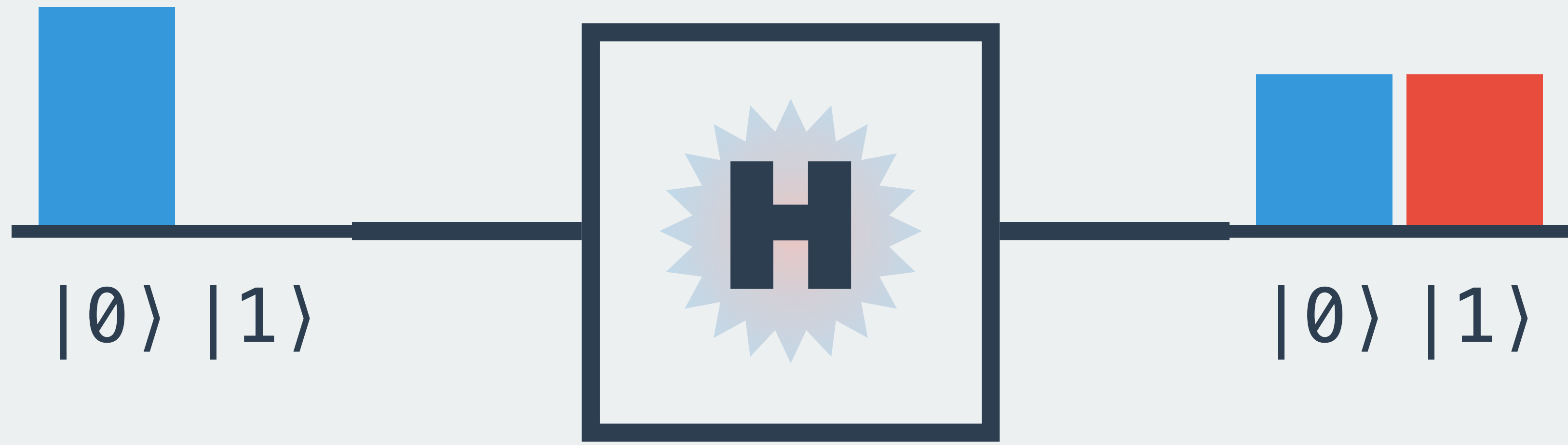
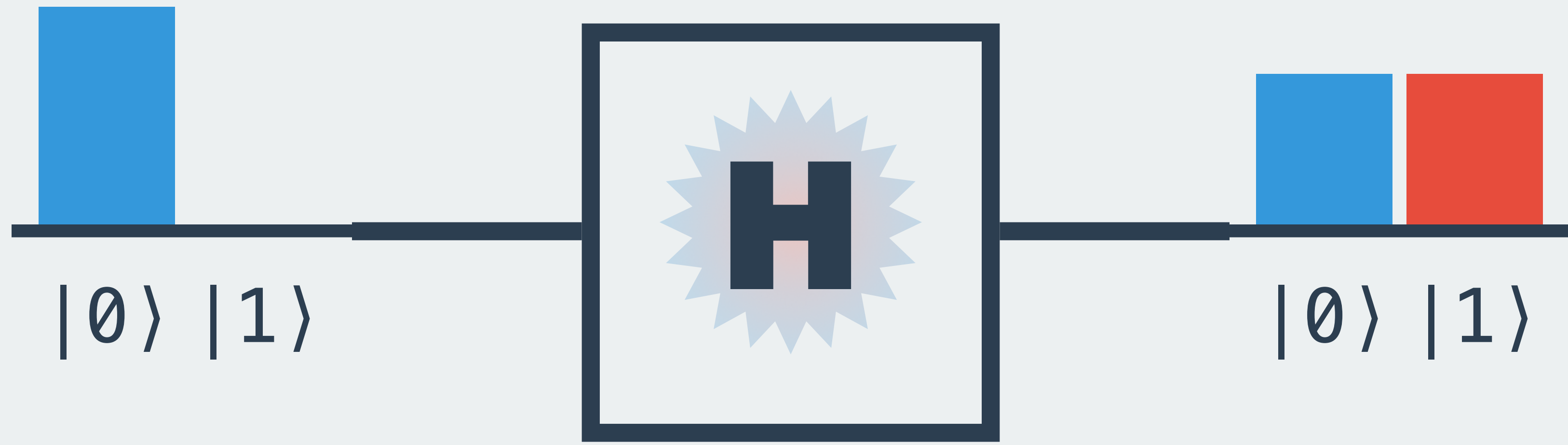


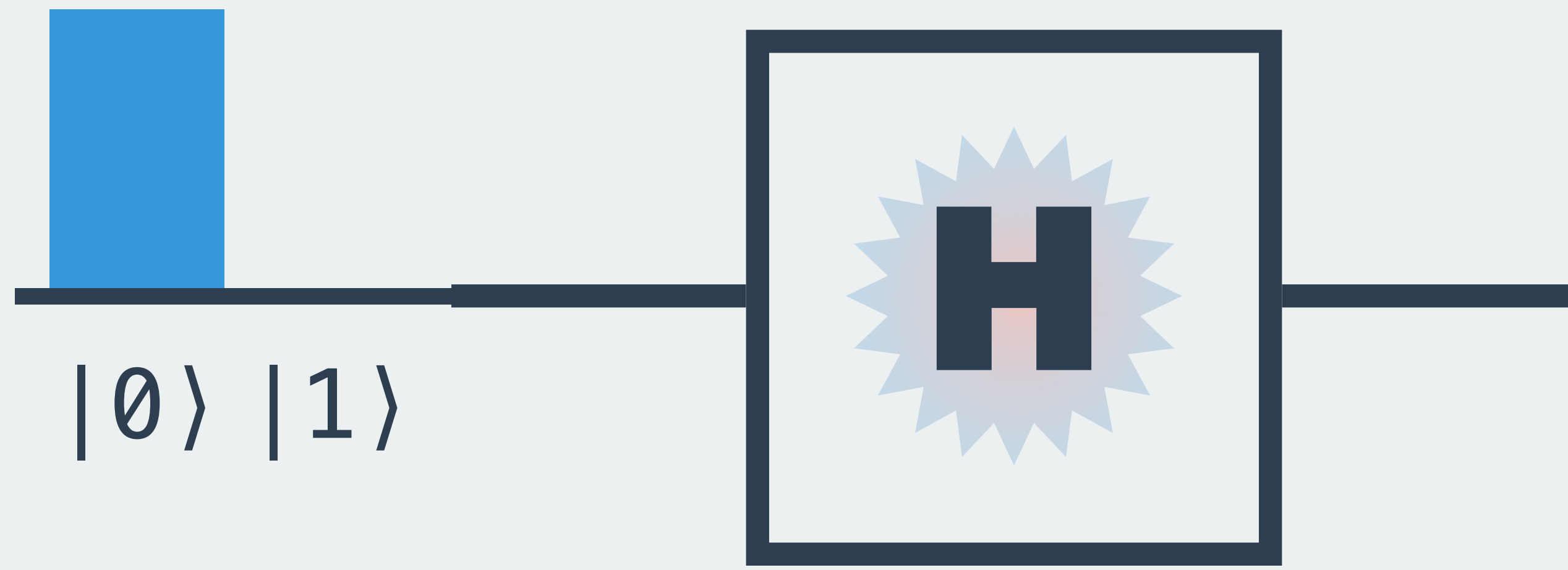
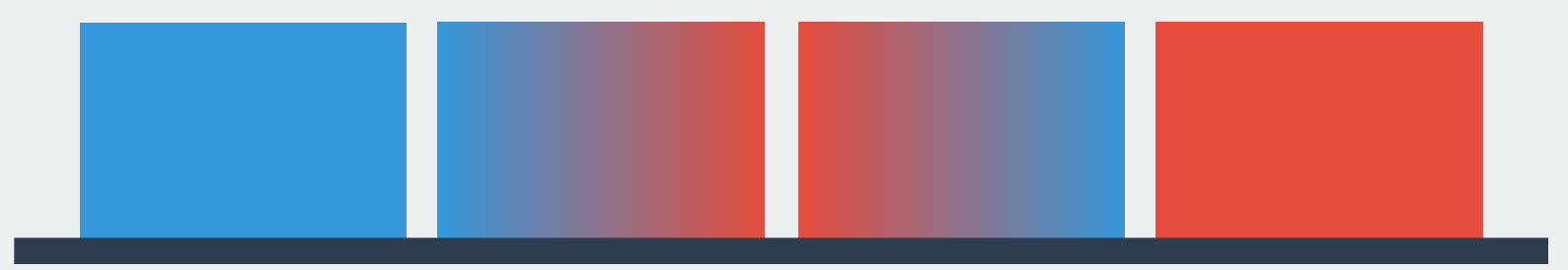
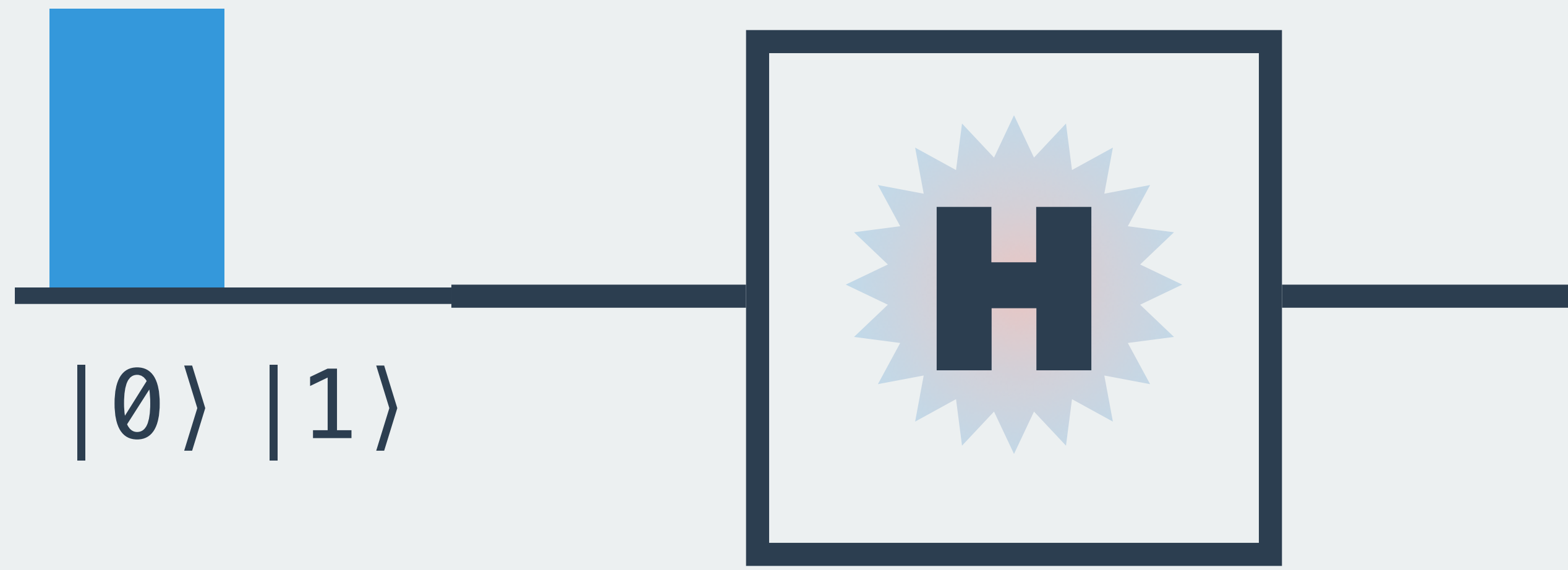




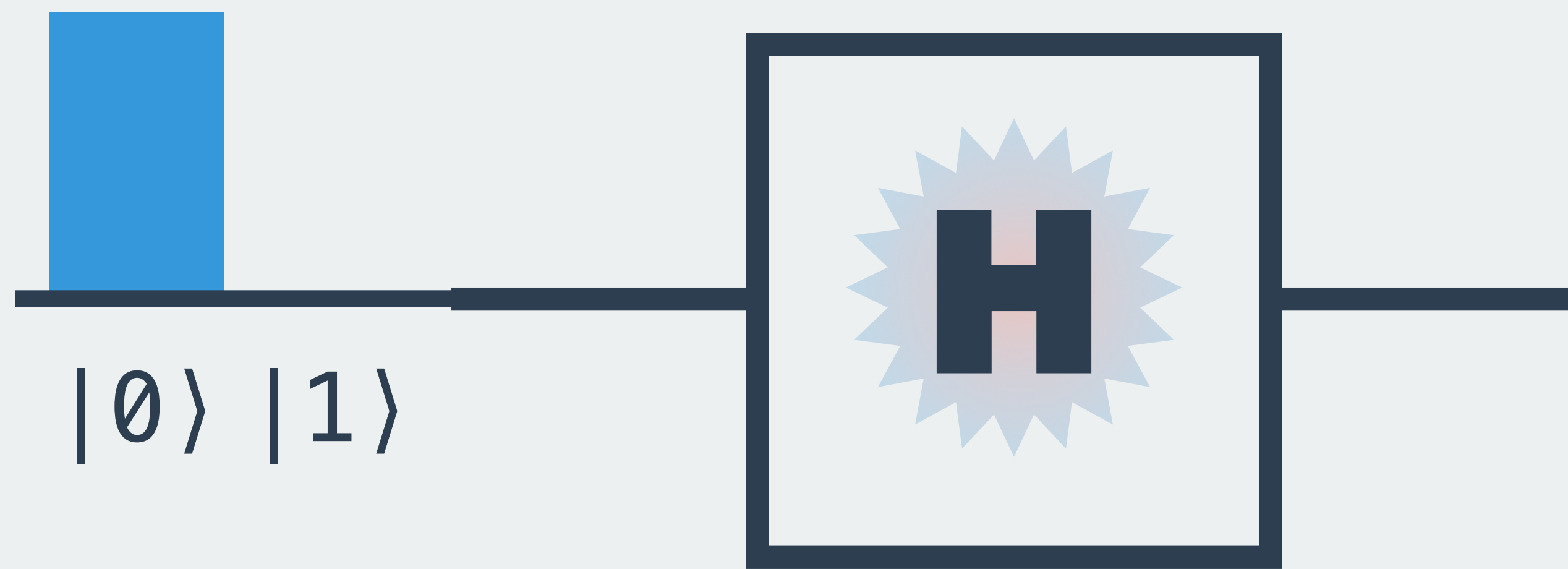
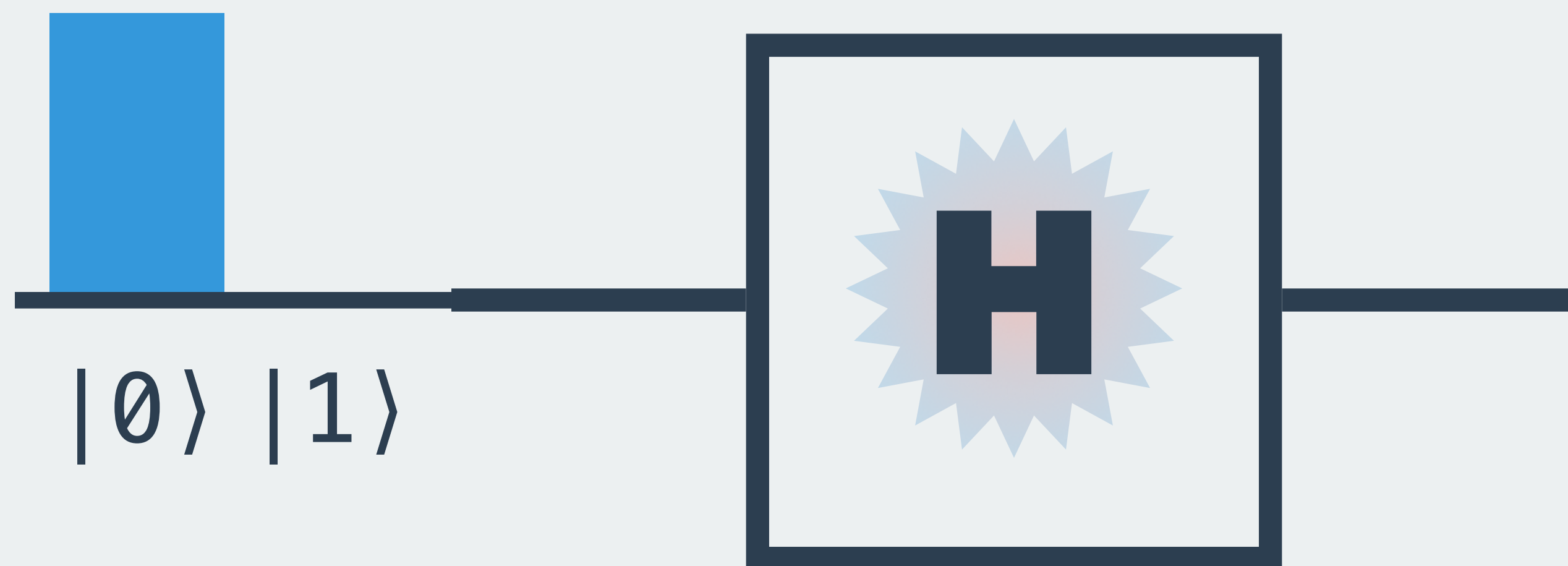
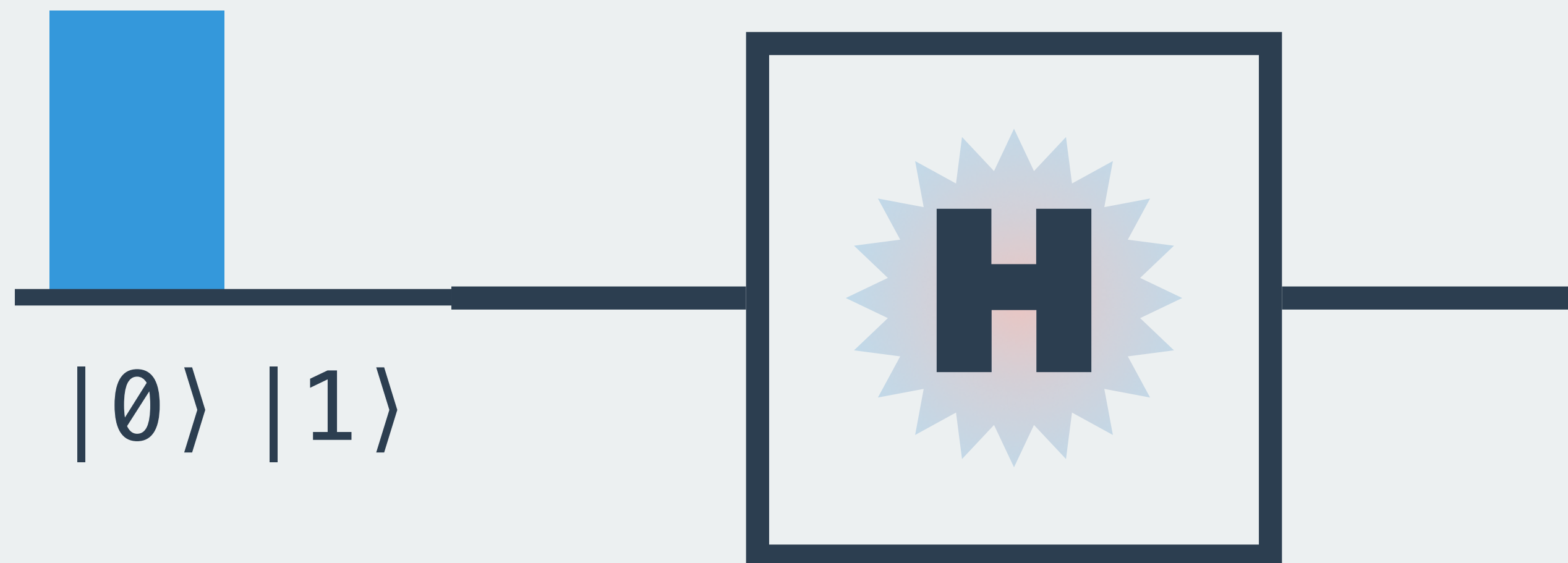


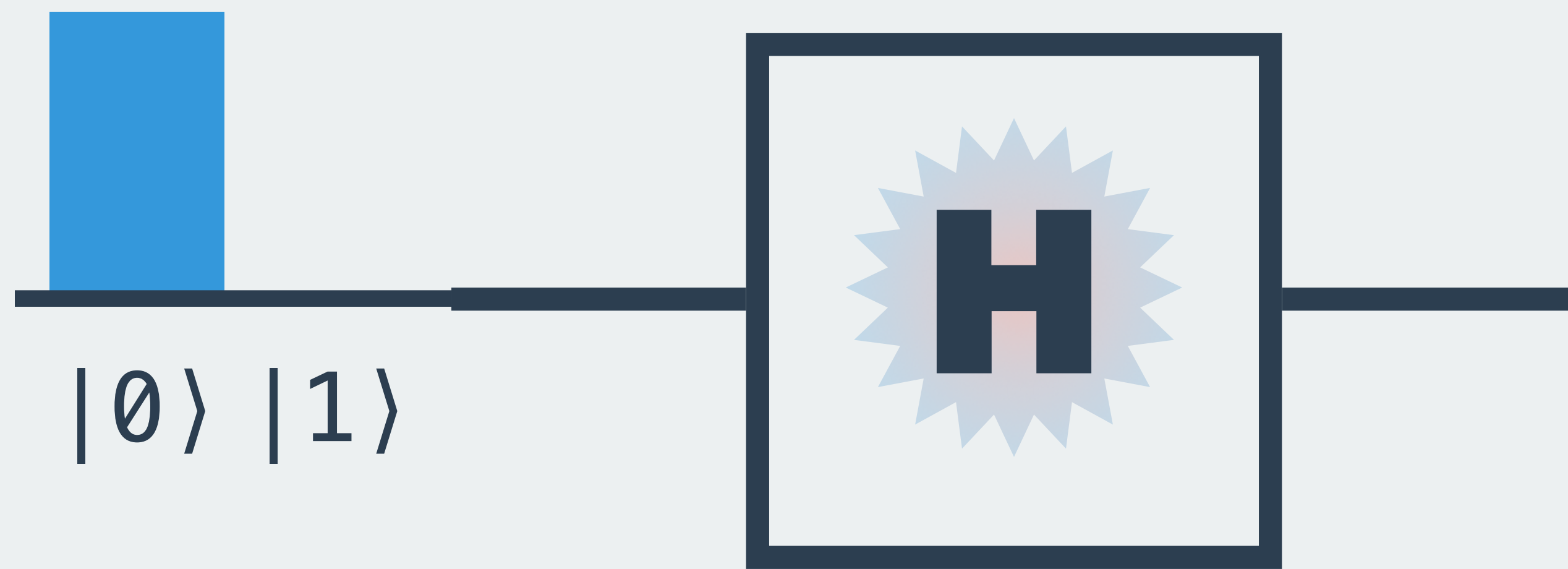
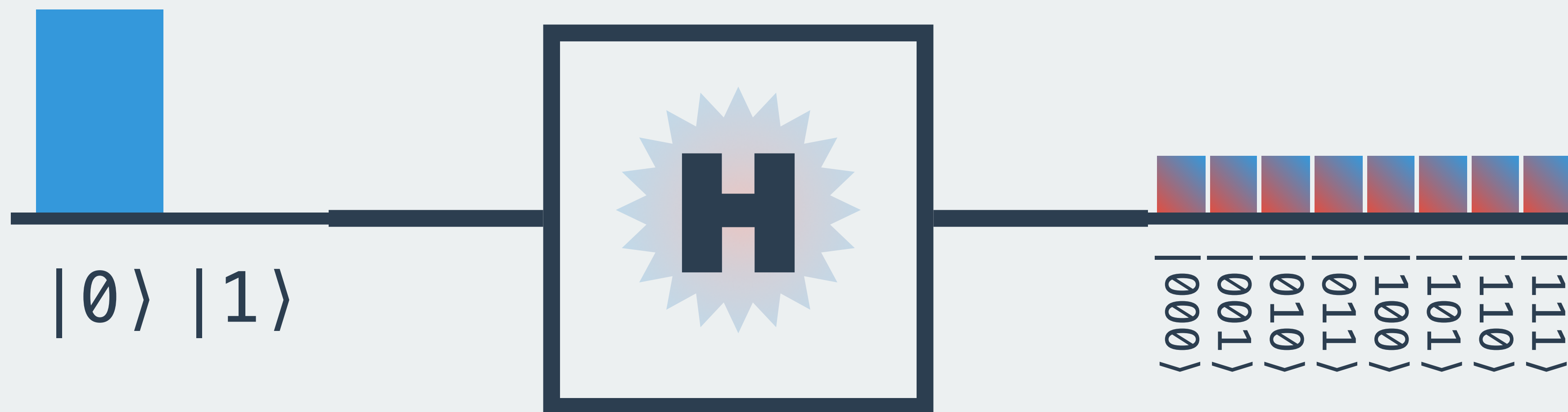
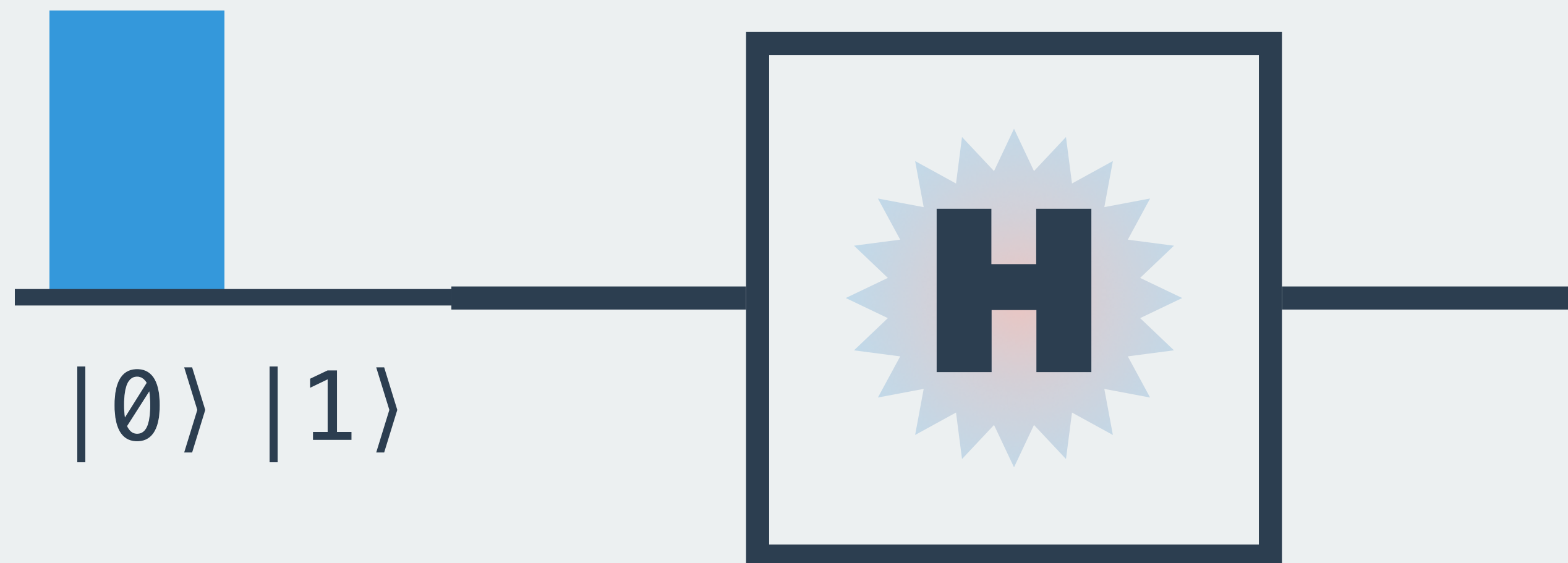
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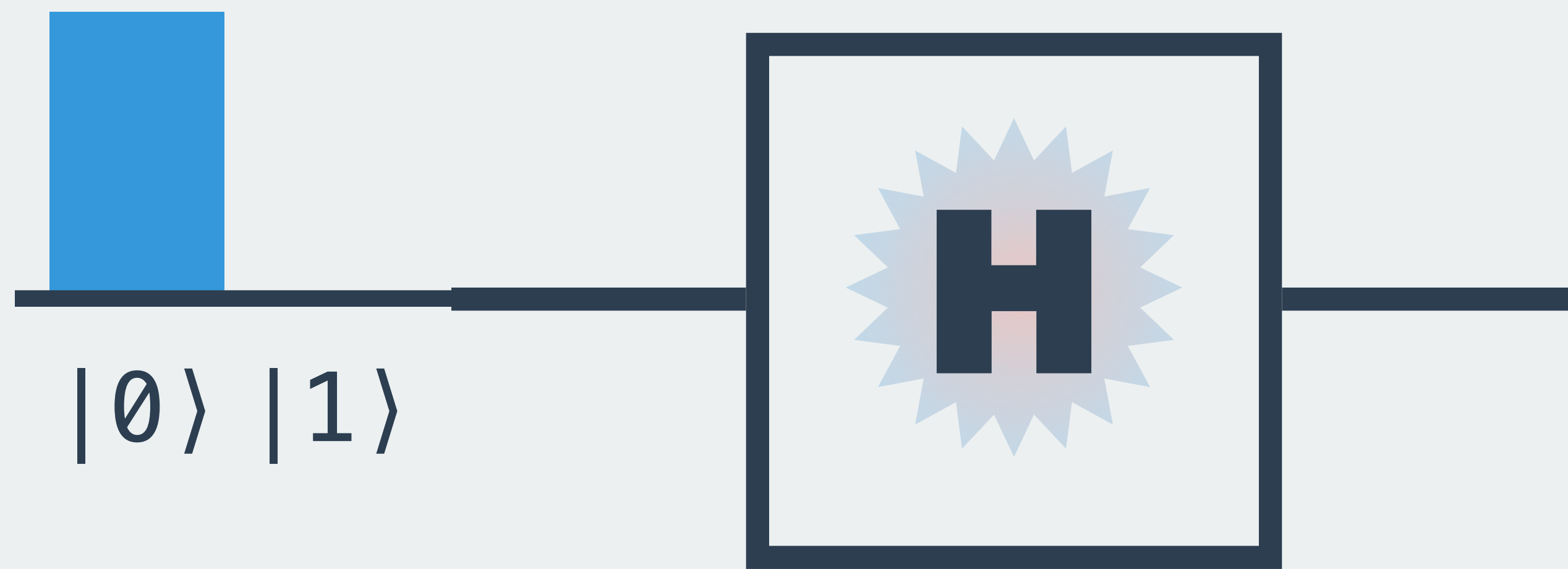
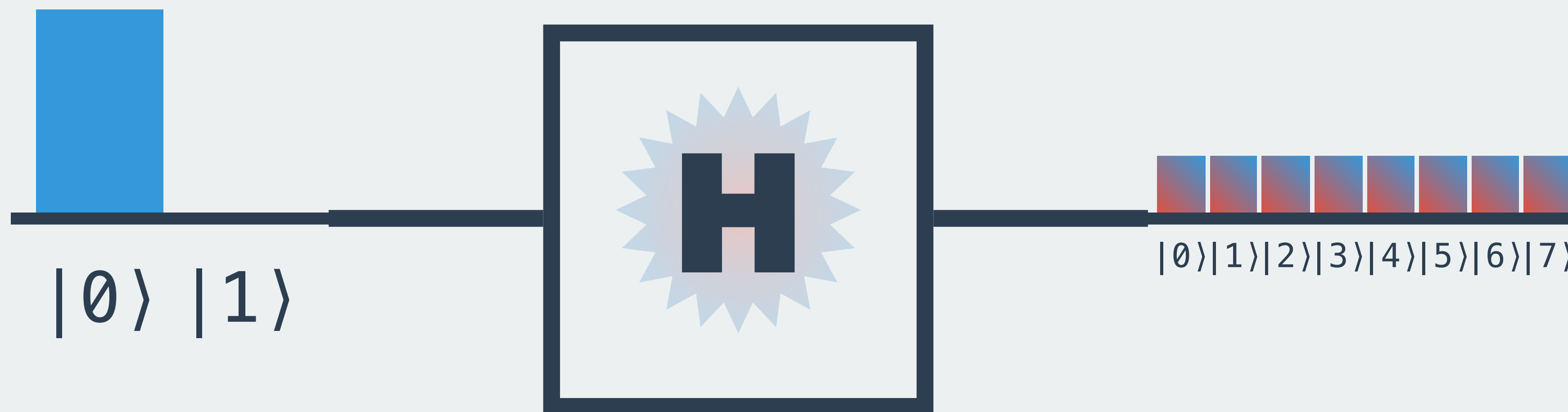
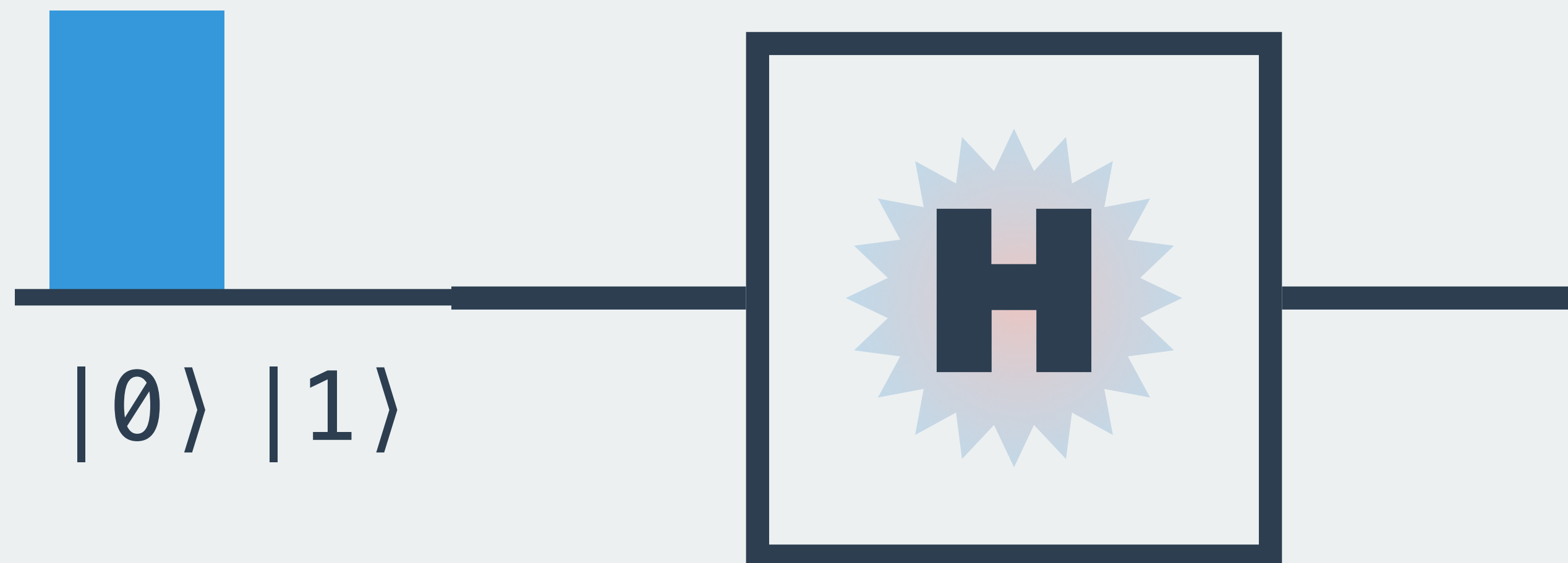




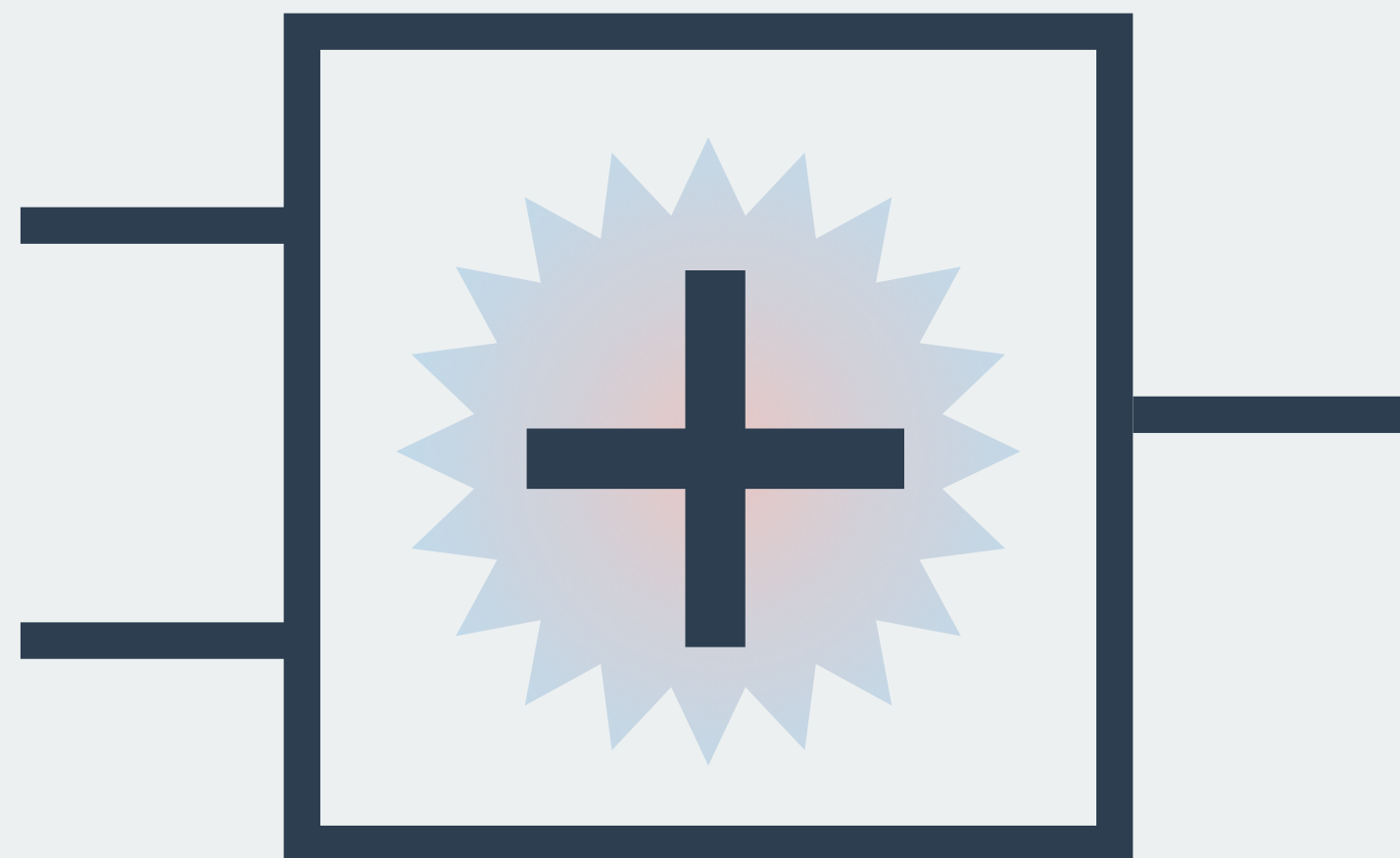
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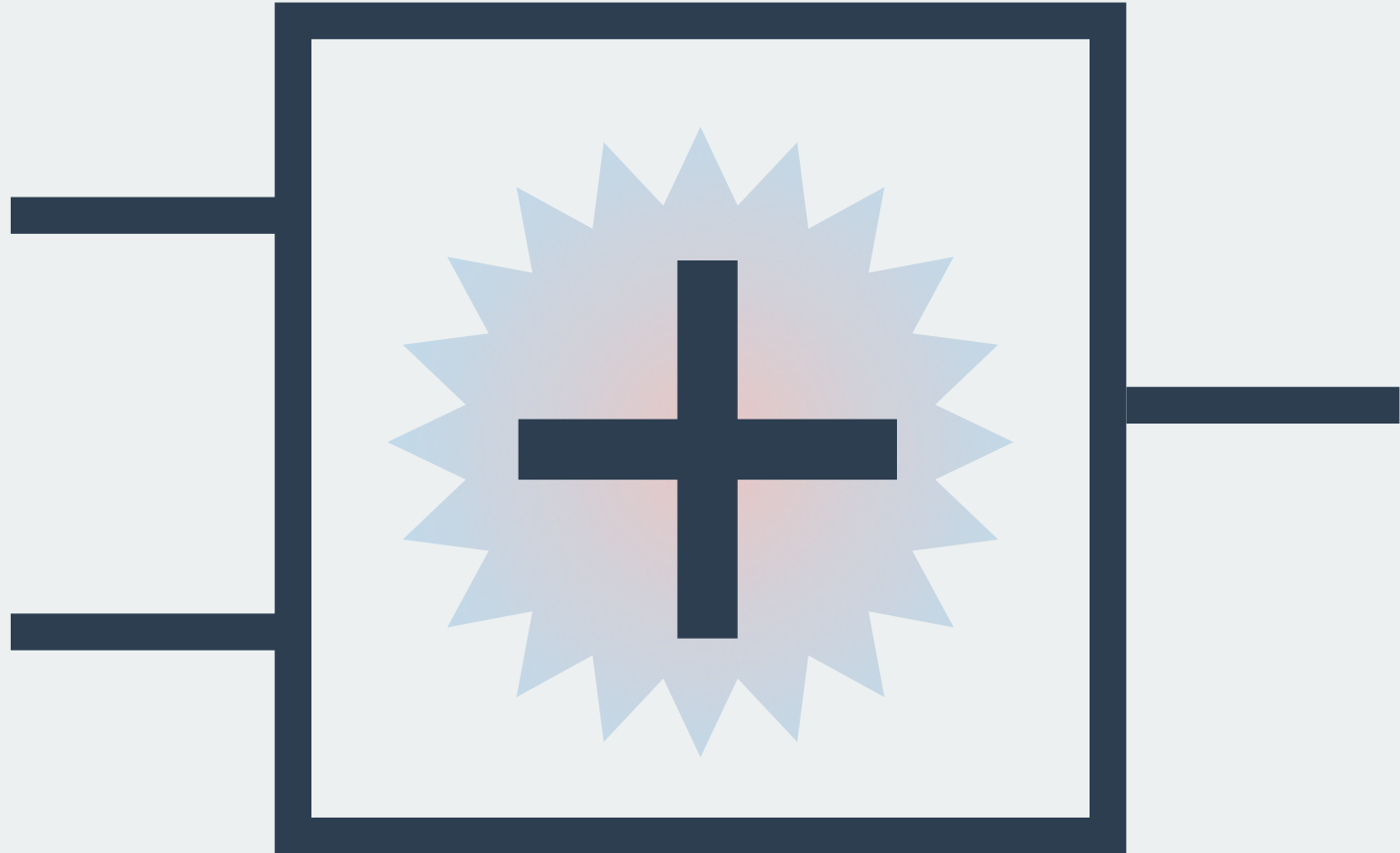


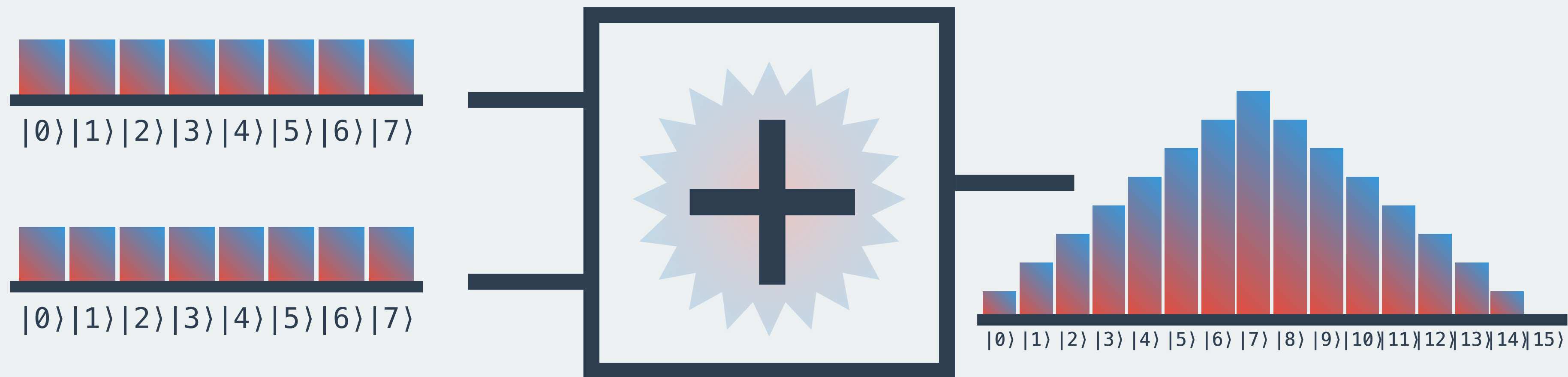


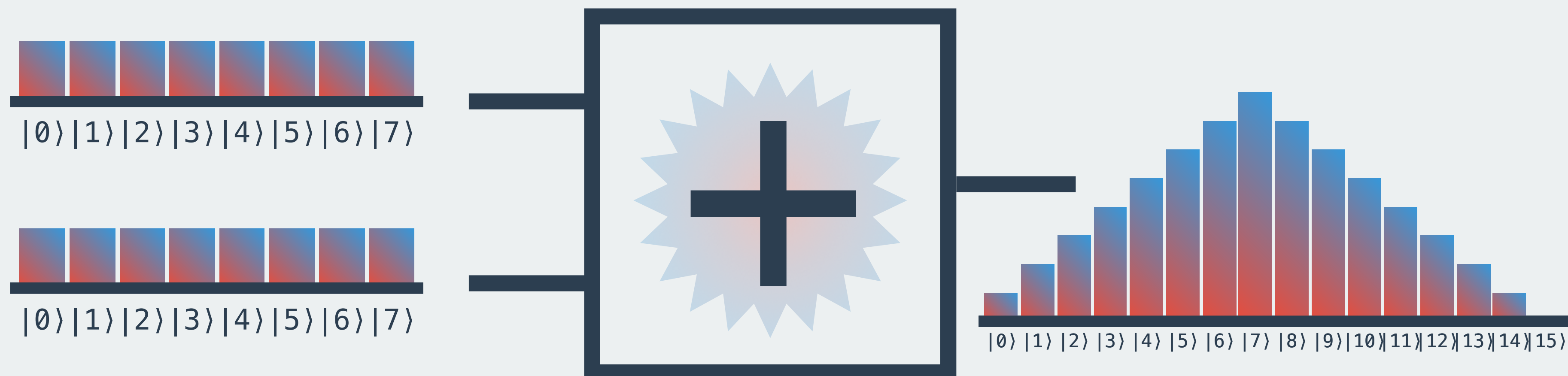



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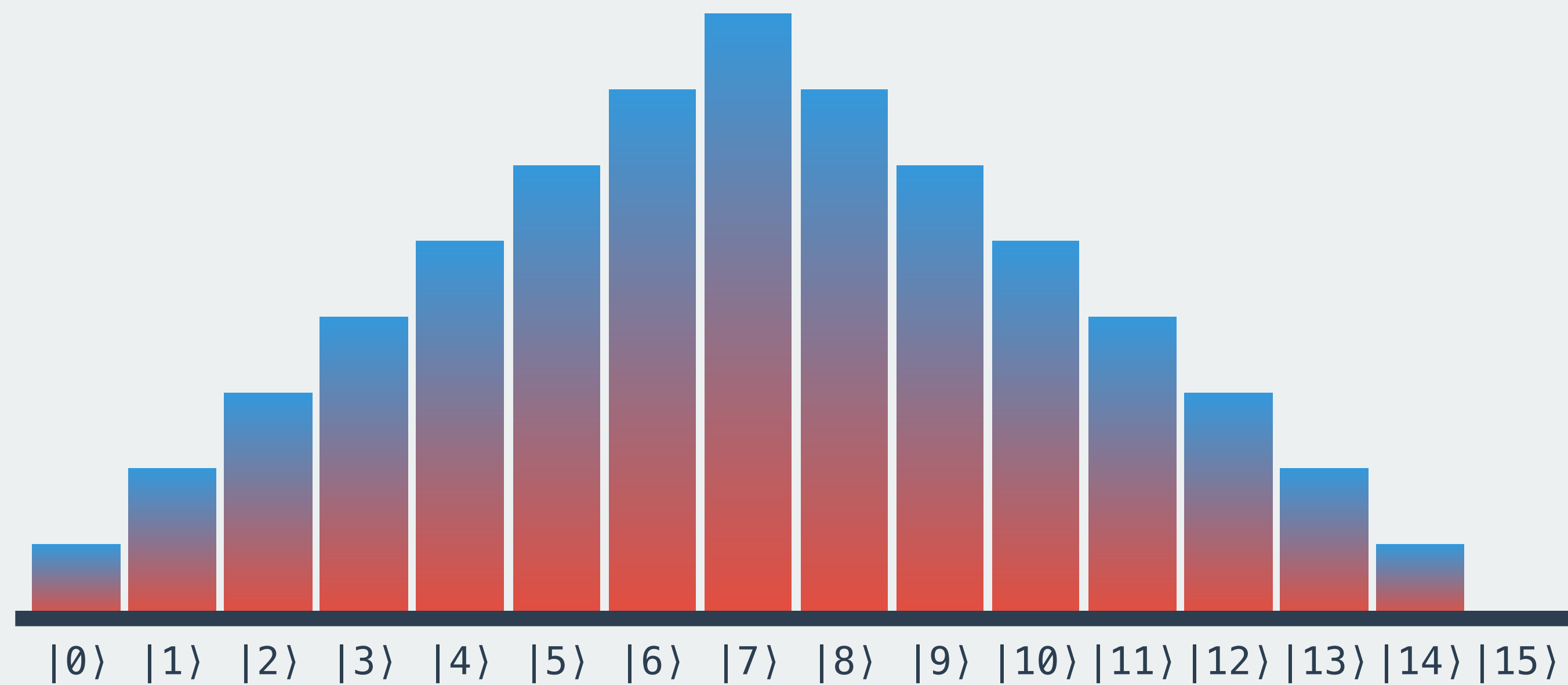


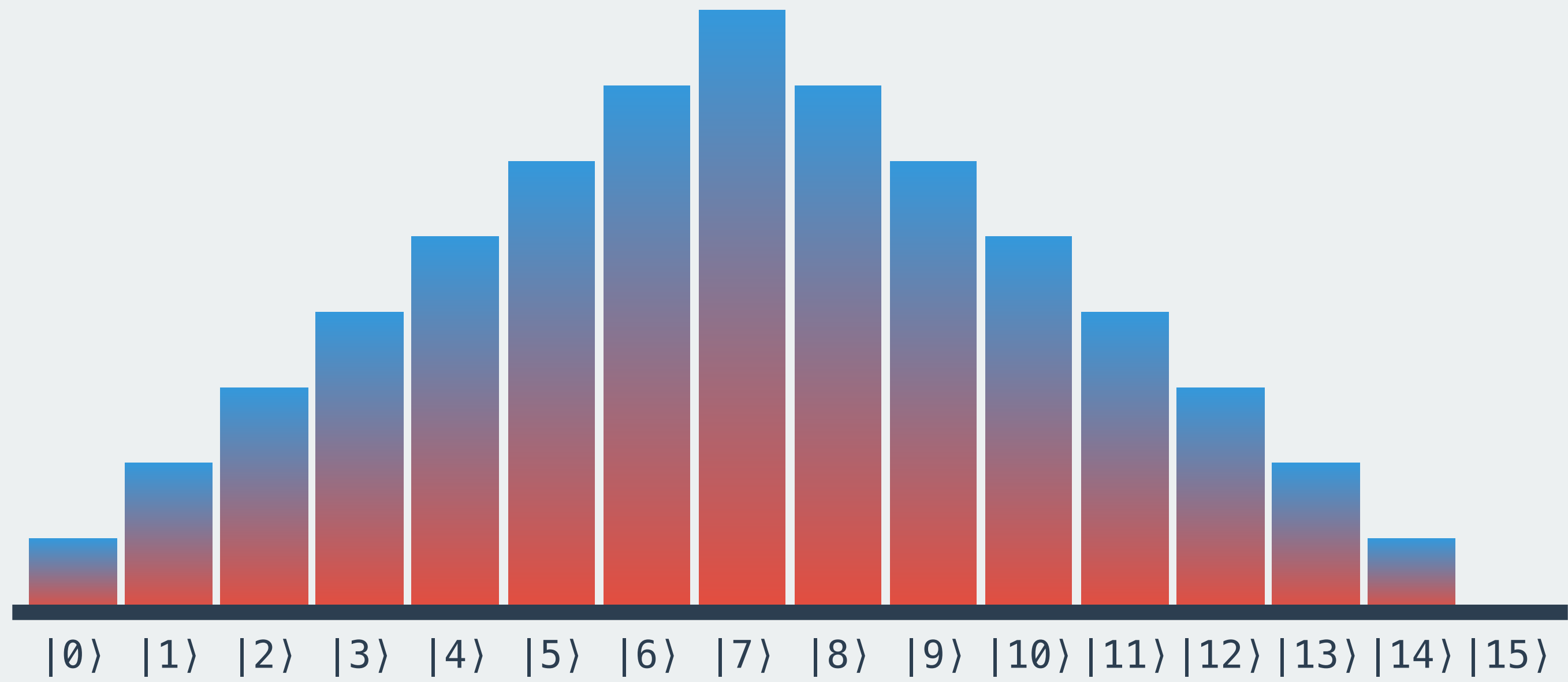
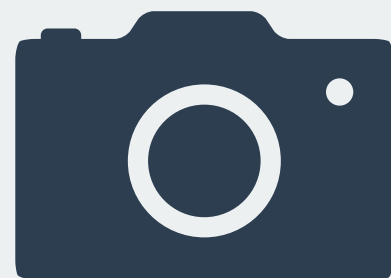


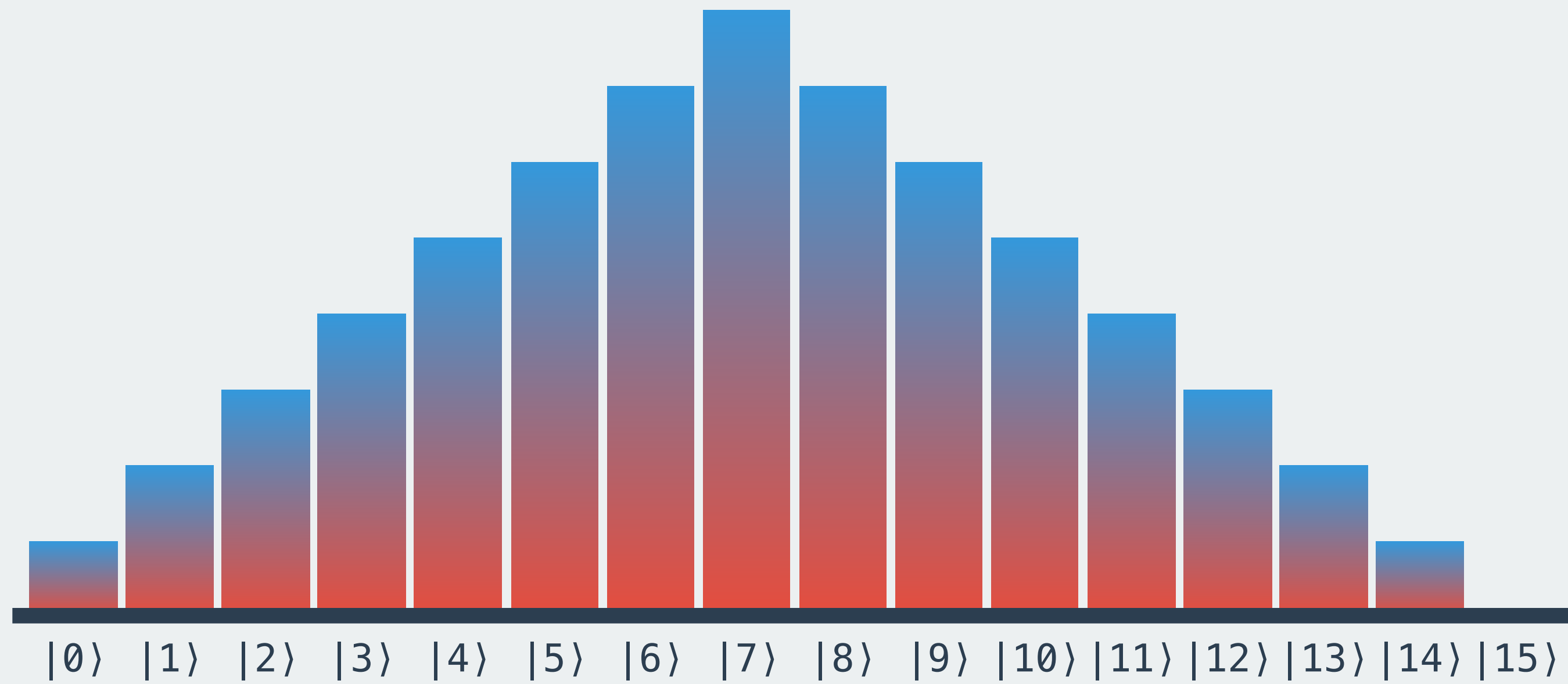
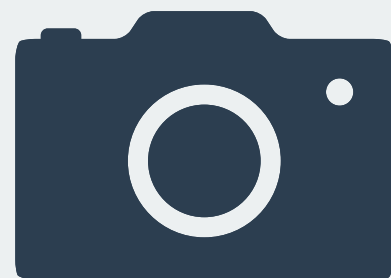


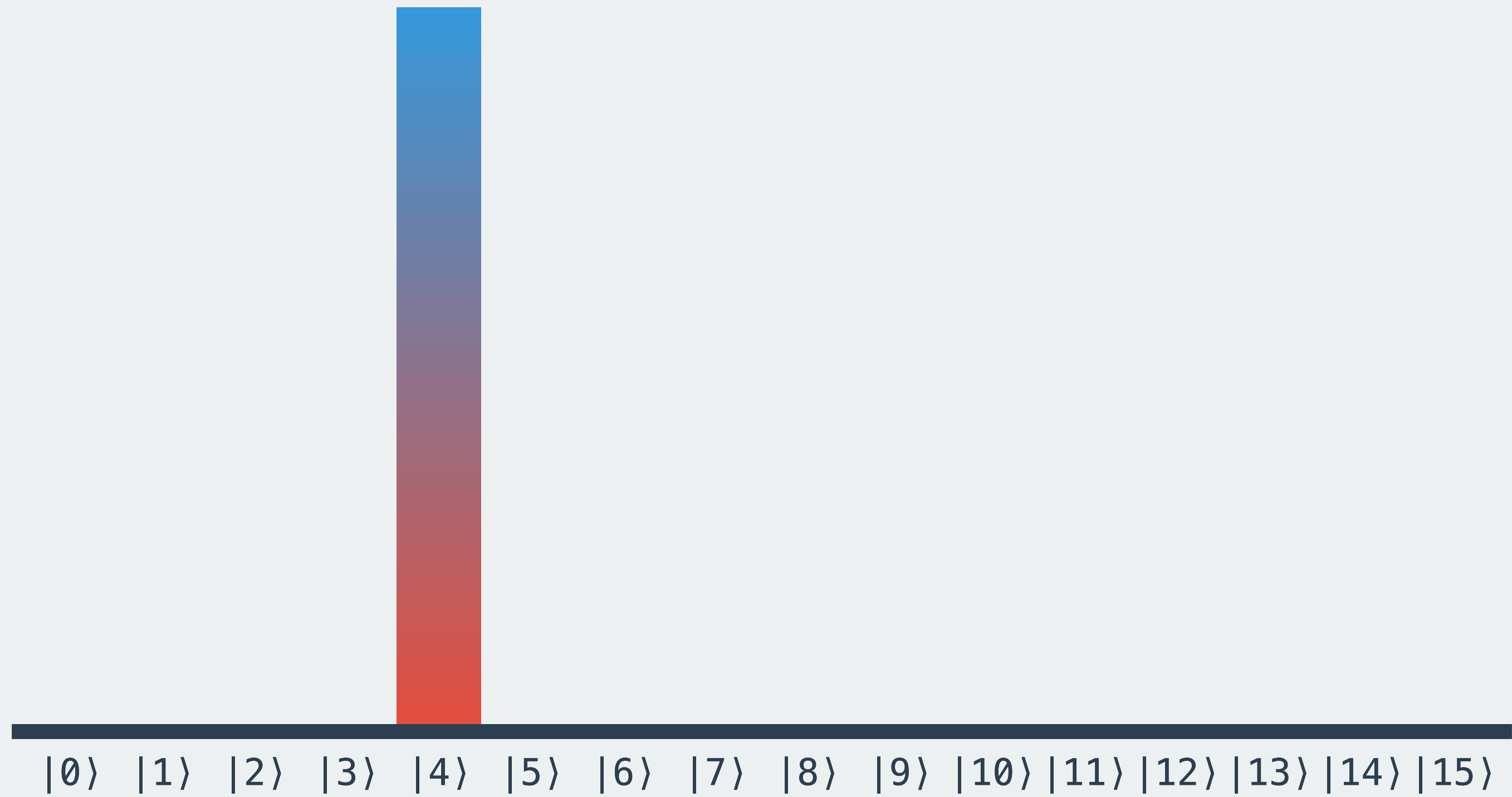
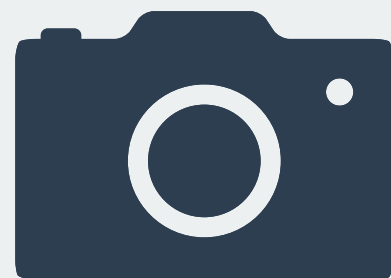
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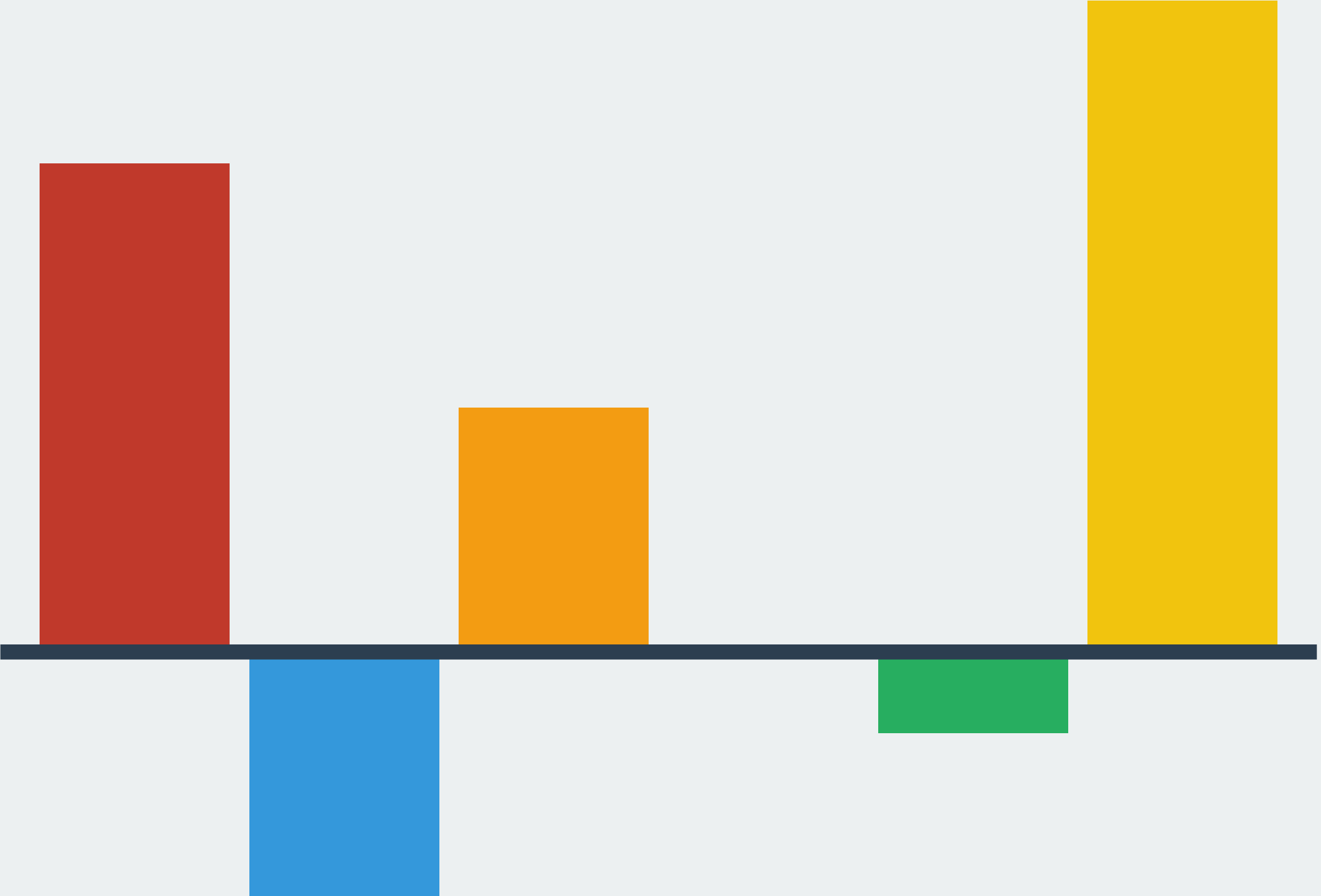


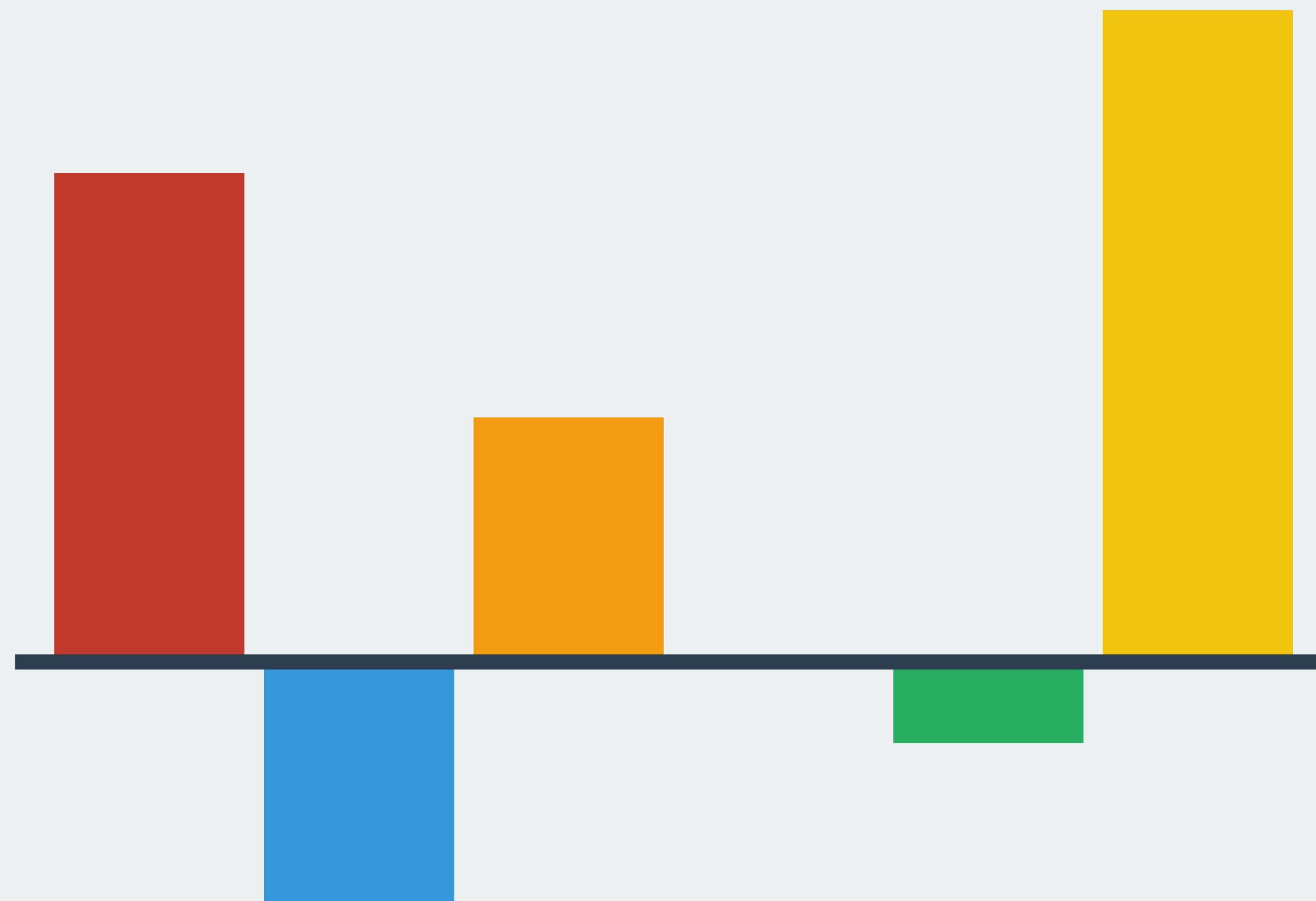


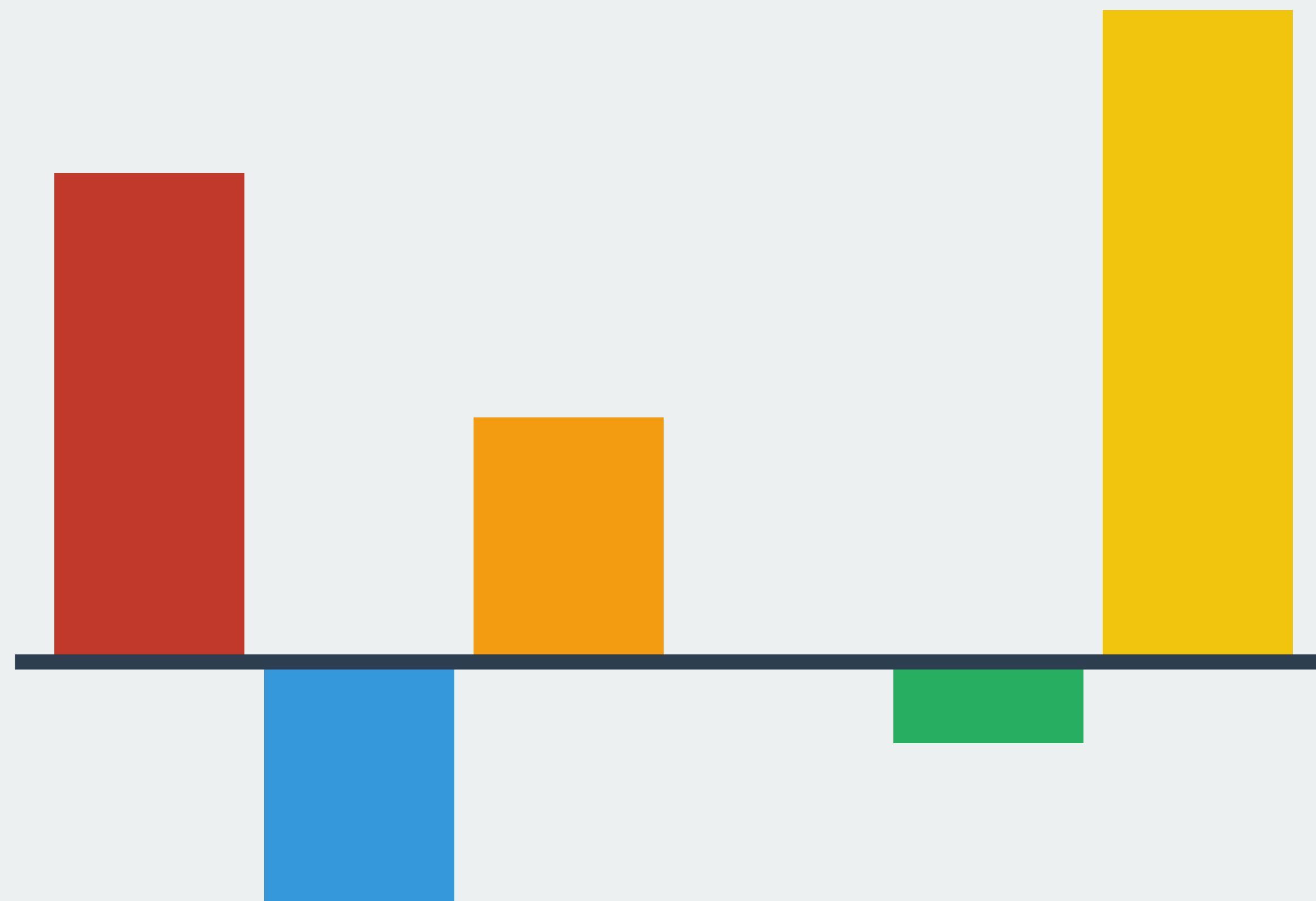


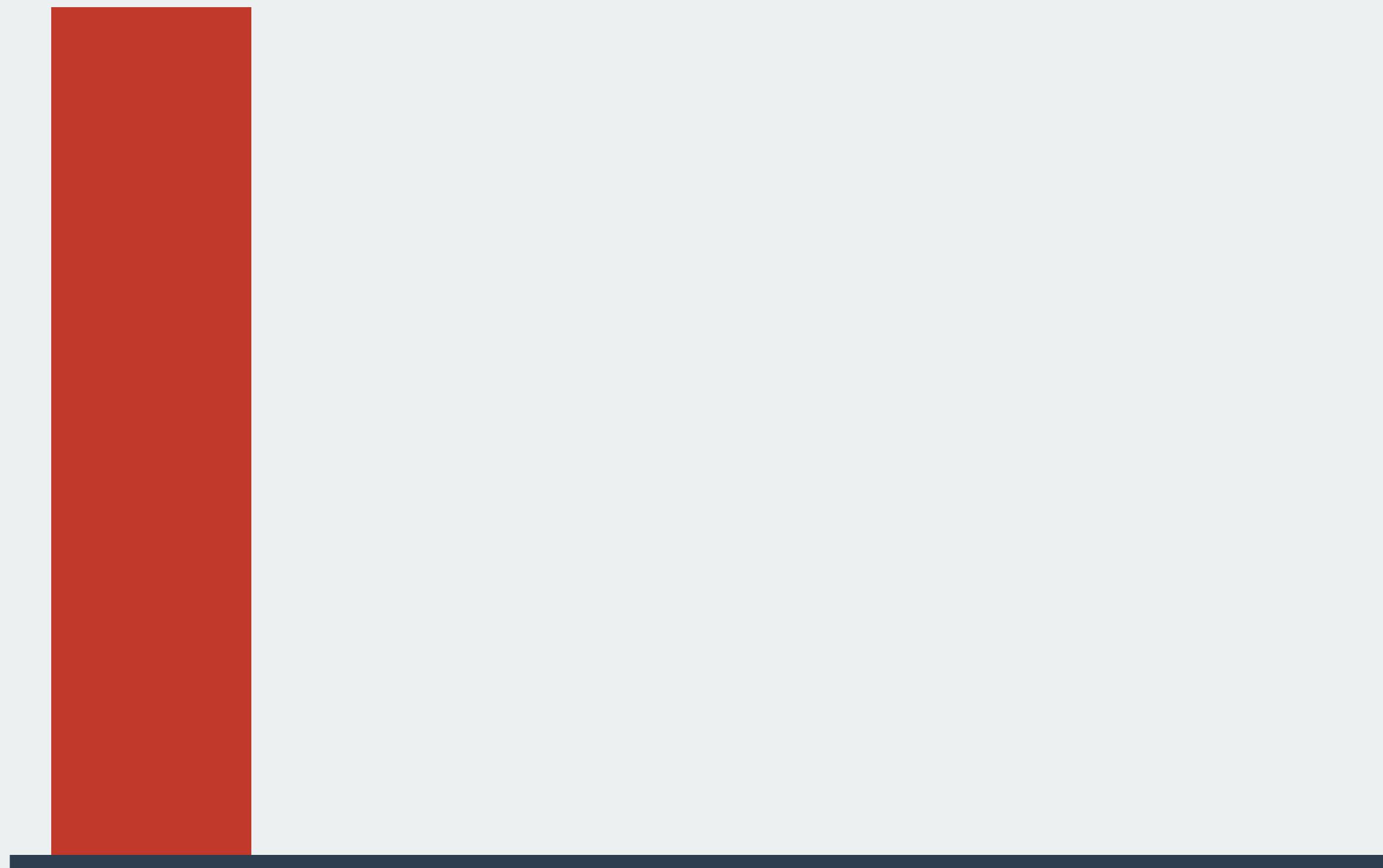
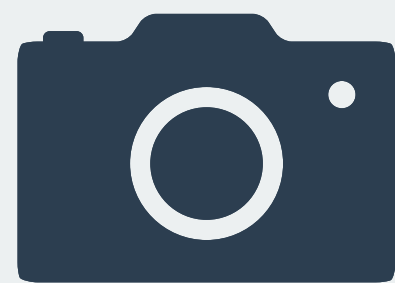


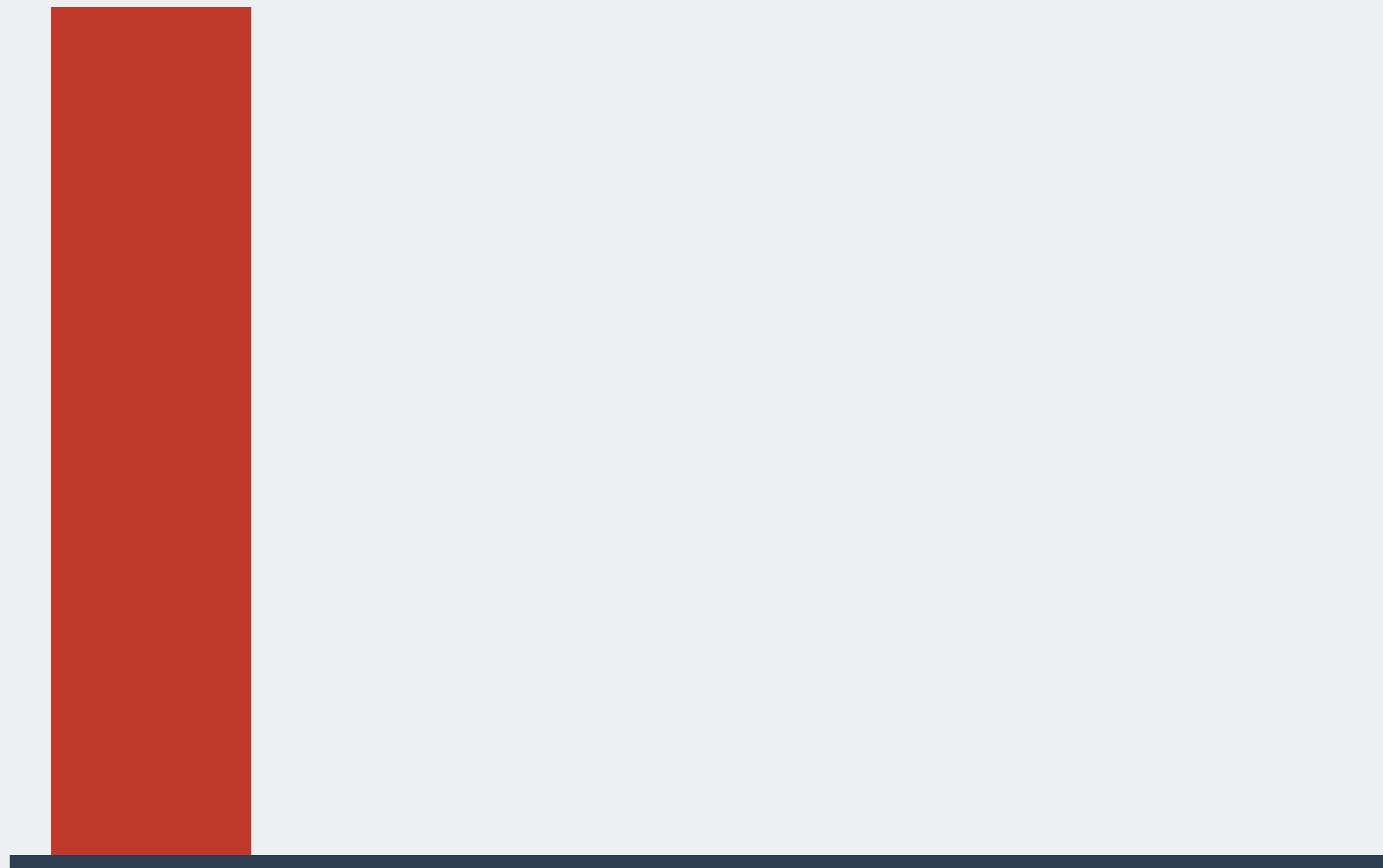
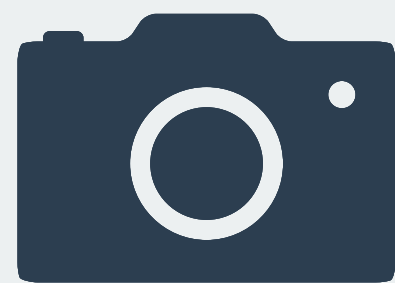




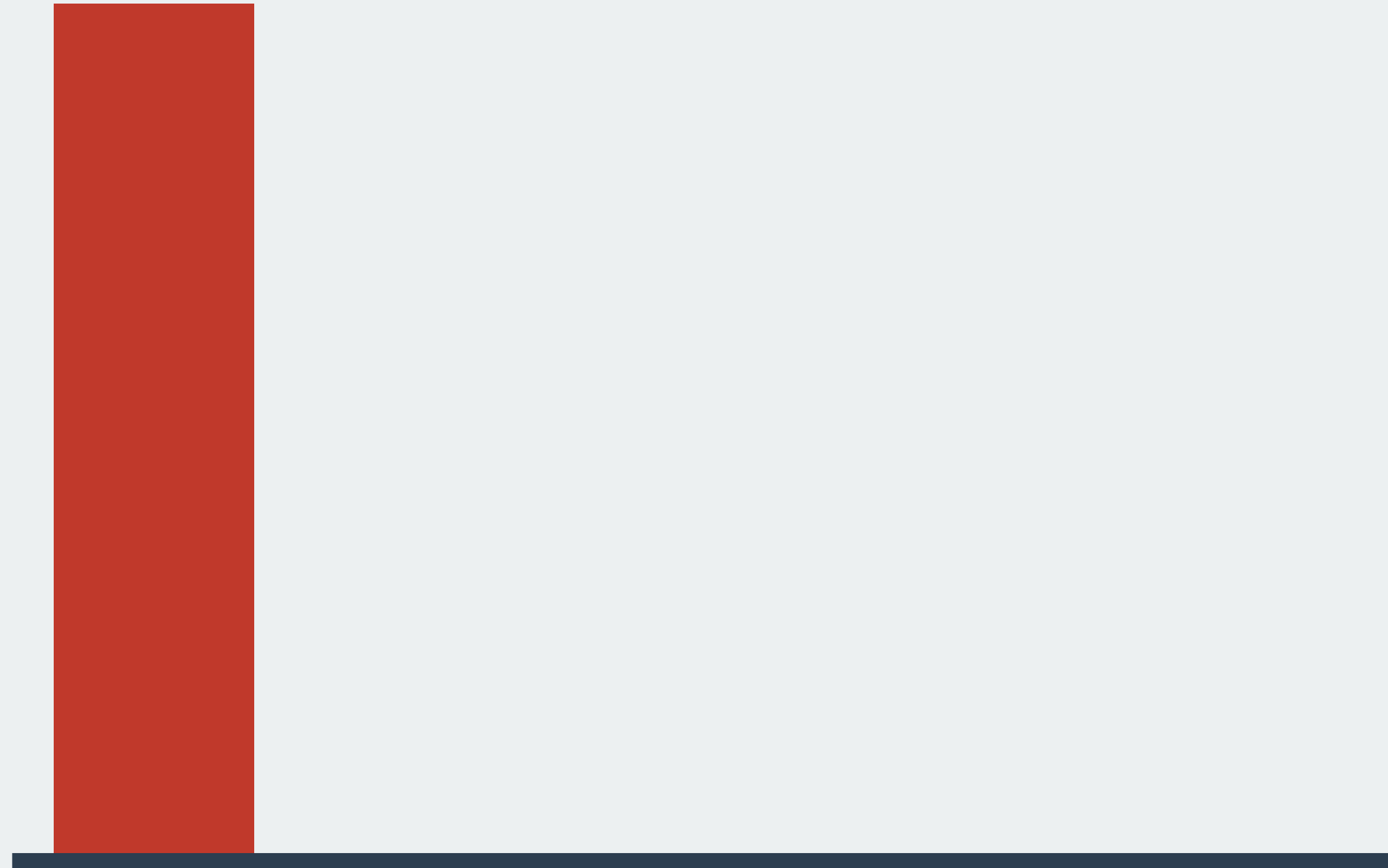
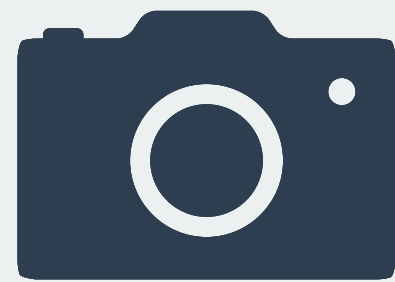




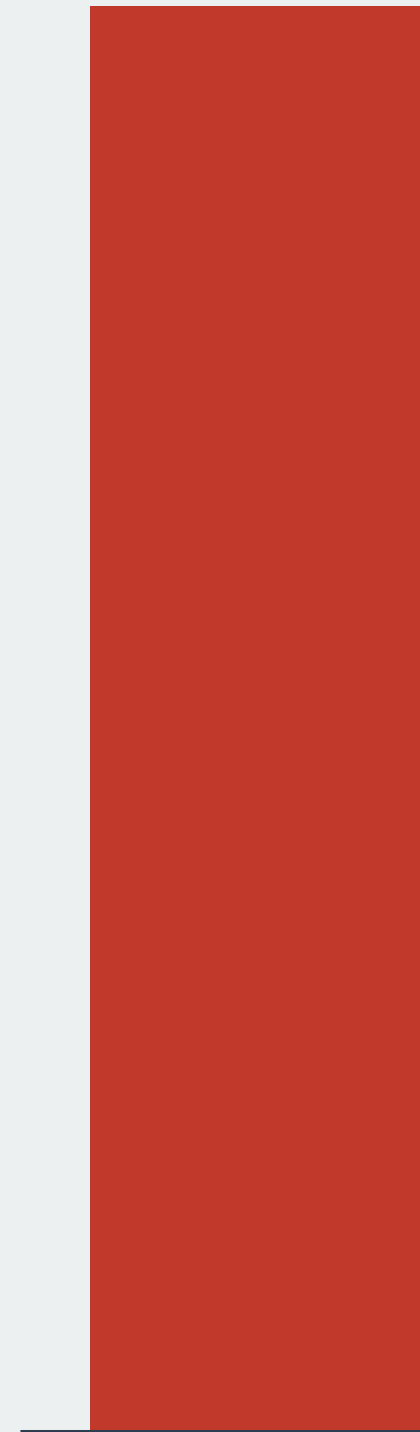
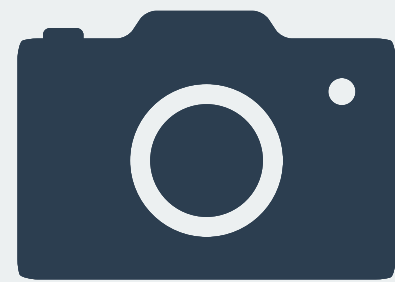




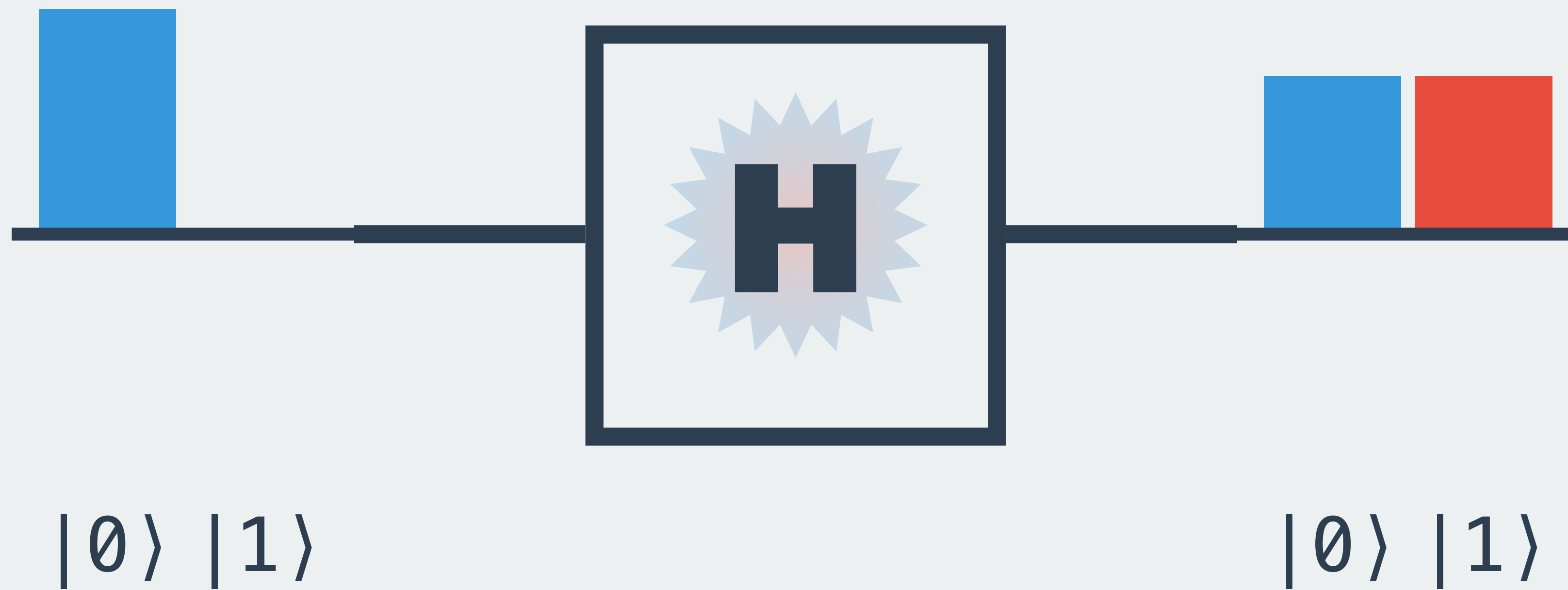
$$|\alpha_1|^2 + \dots + |\alpha_n|^2 = 1$$

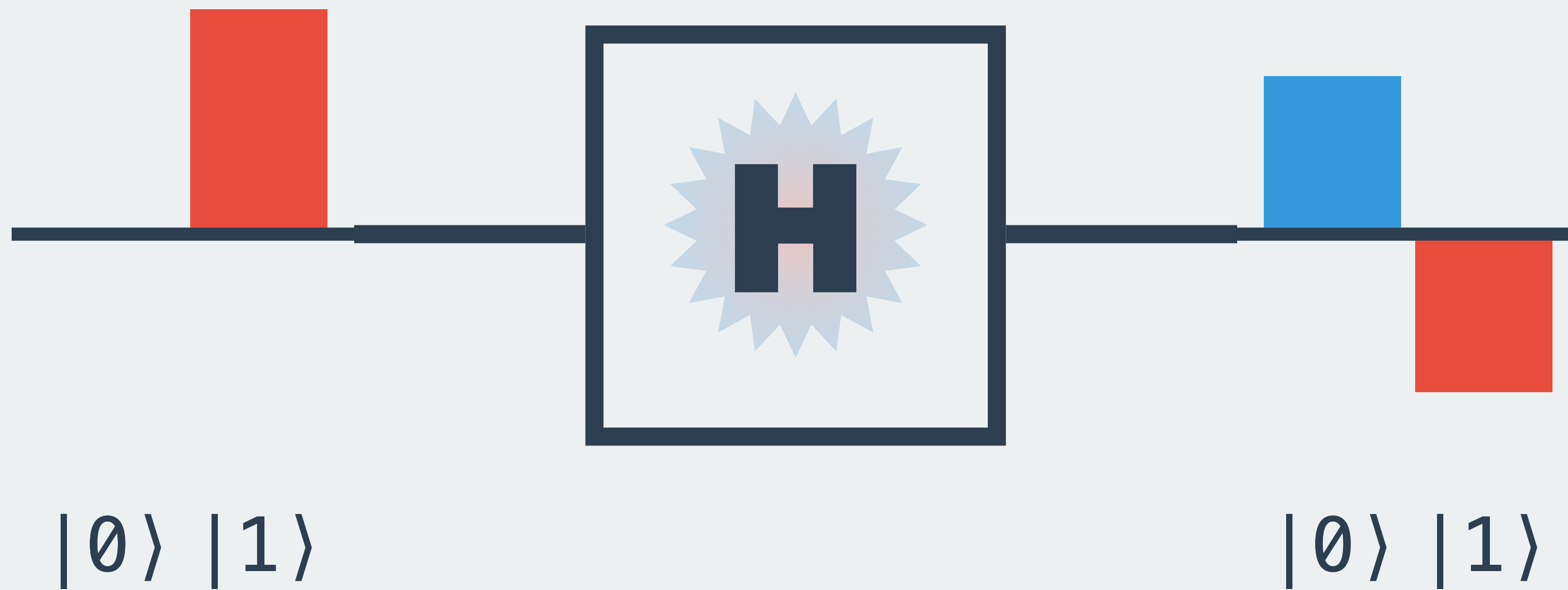


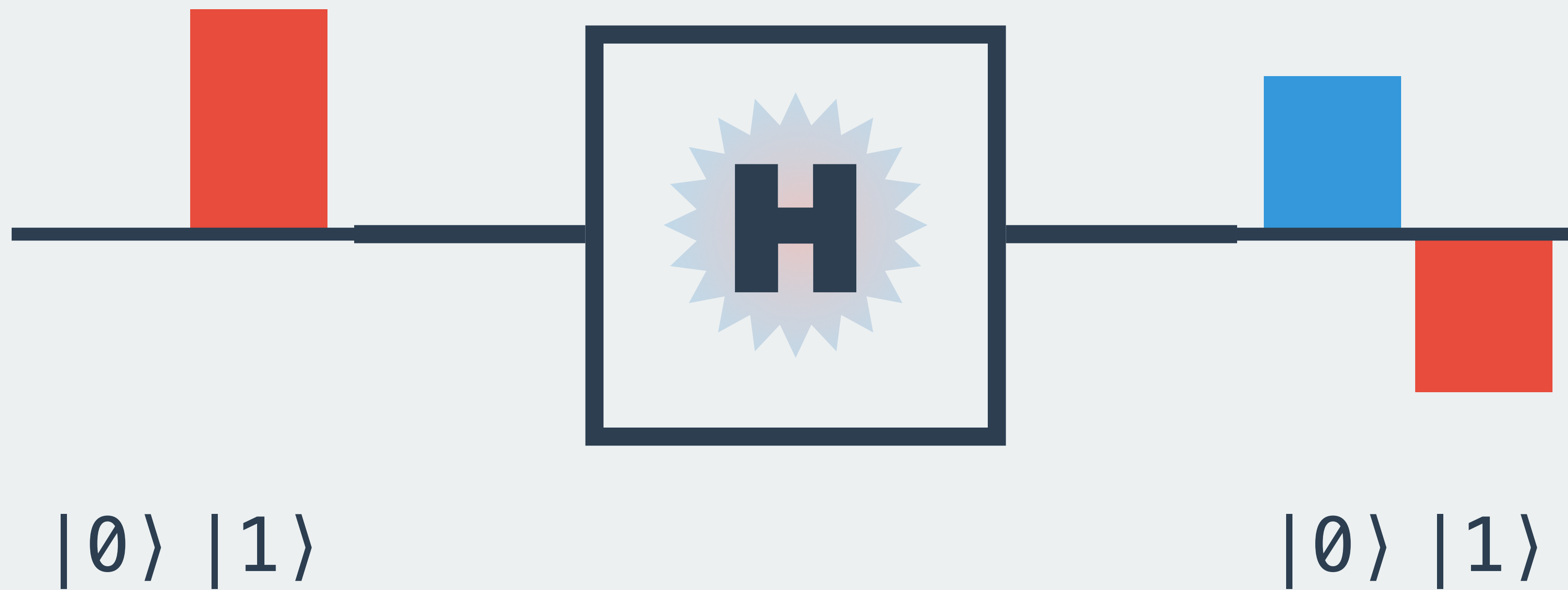
$$|\alpha_1|^2 + \dots + |\alpha_n|^2 = 1$$
$$P(x) = |\alpha_x|^2$$

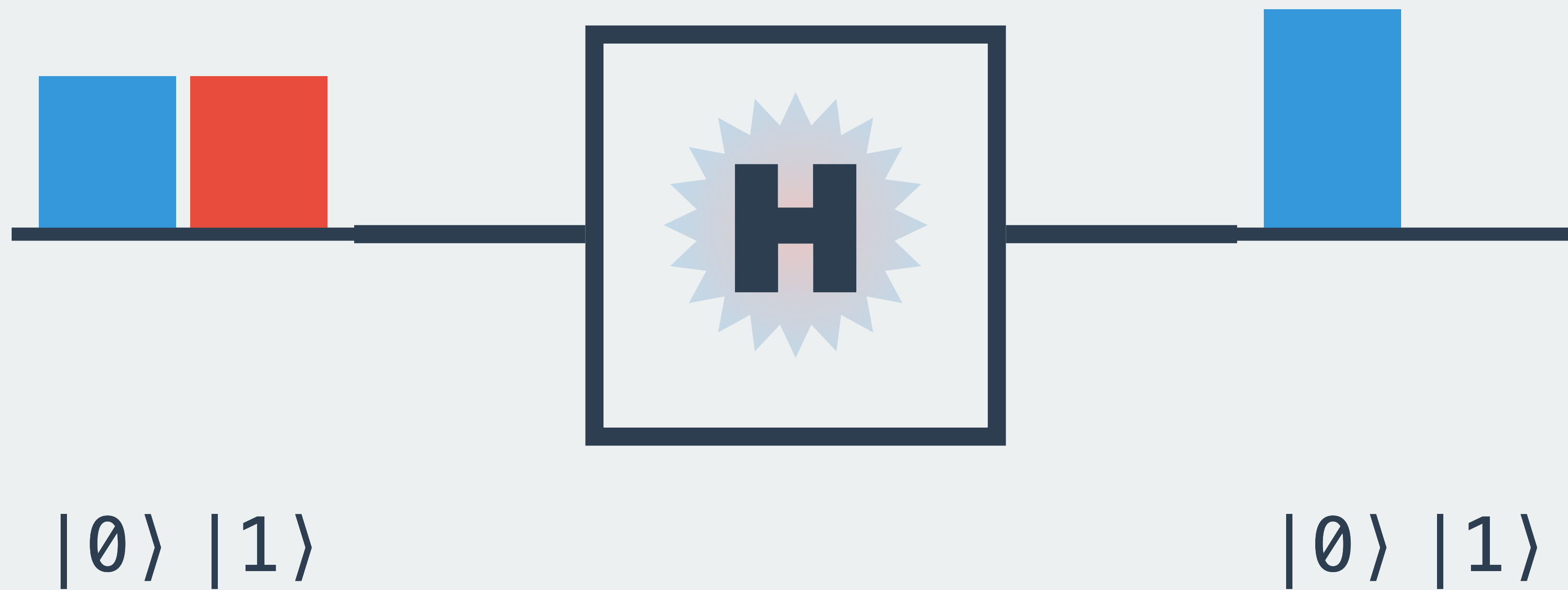


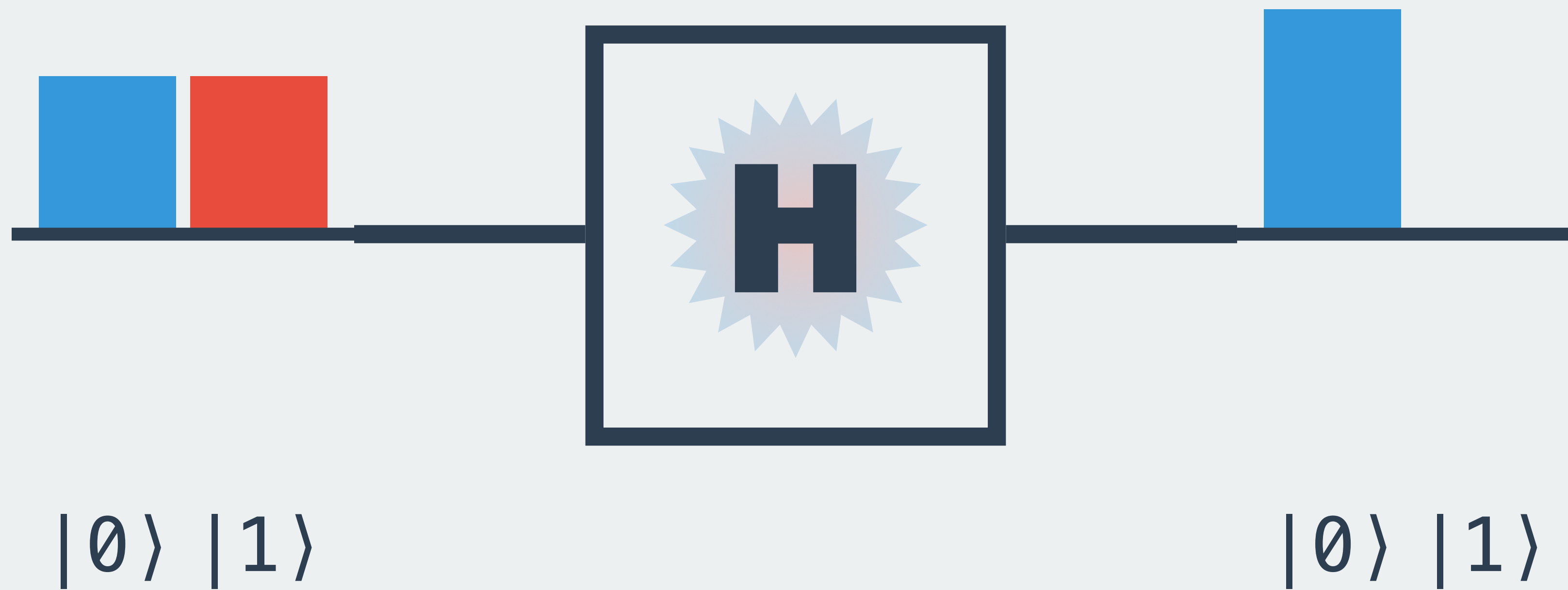


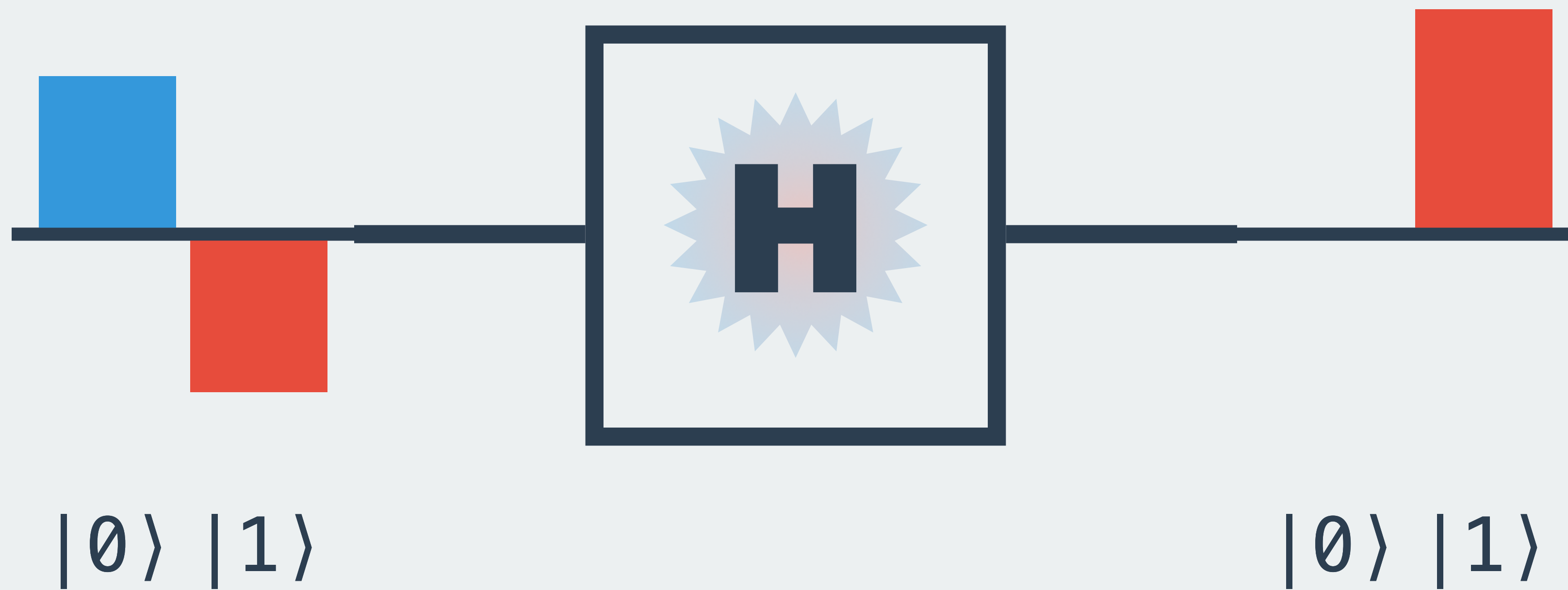












A detailed view of a quantum computing laboratory setup. The background is a complex arrangement of scientific equipment on a perforated metal table. It includes various optical components like lenses, mirrors, and beam splitters, along with fiber optic cables and electronic control units. The scene is dimly lit, with some equipment glowing with a blue light. The overall atmosphere is technical and high-tech.

KVANTNO RAČUNALNIŠTVO

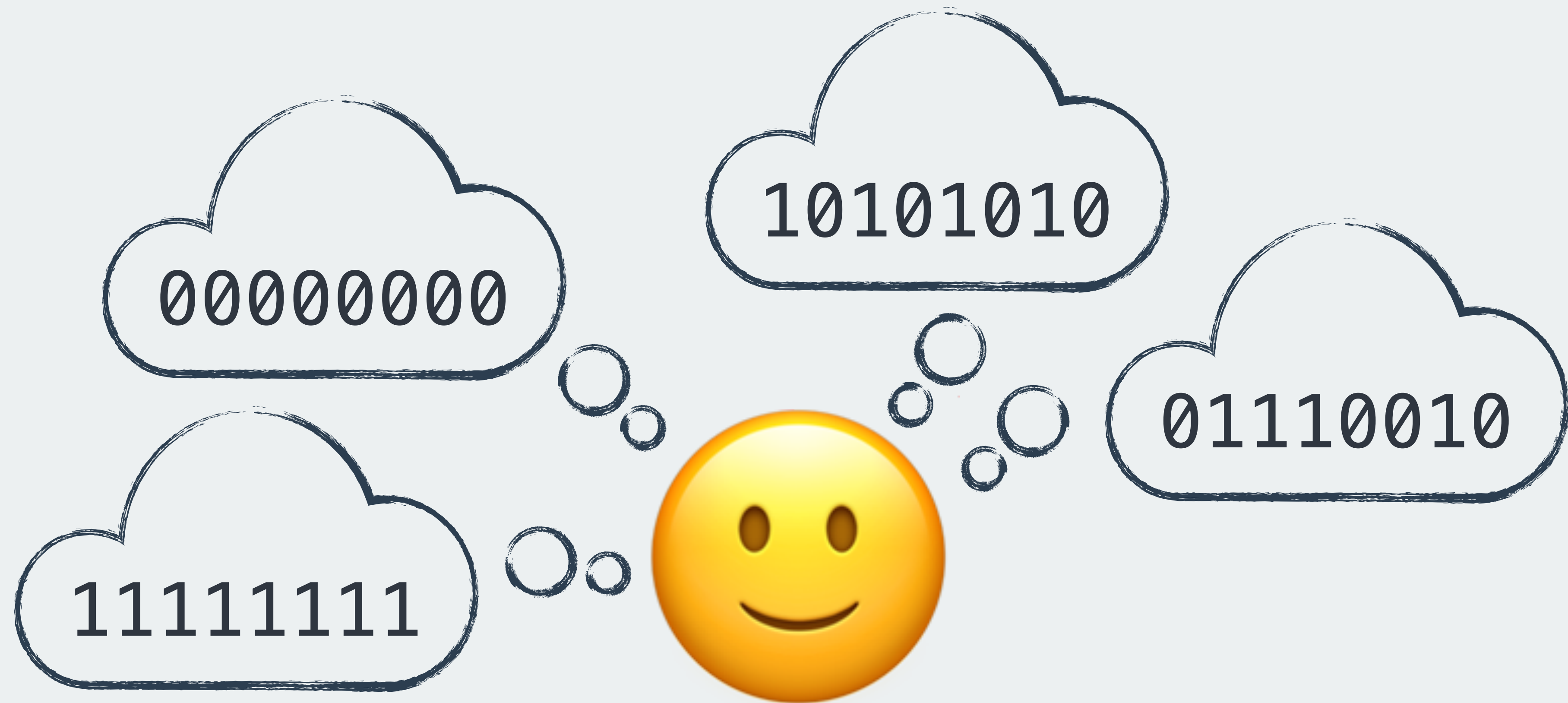
DEUTSCHEV ALGORITEM



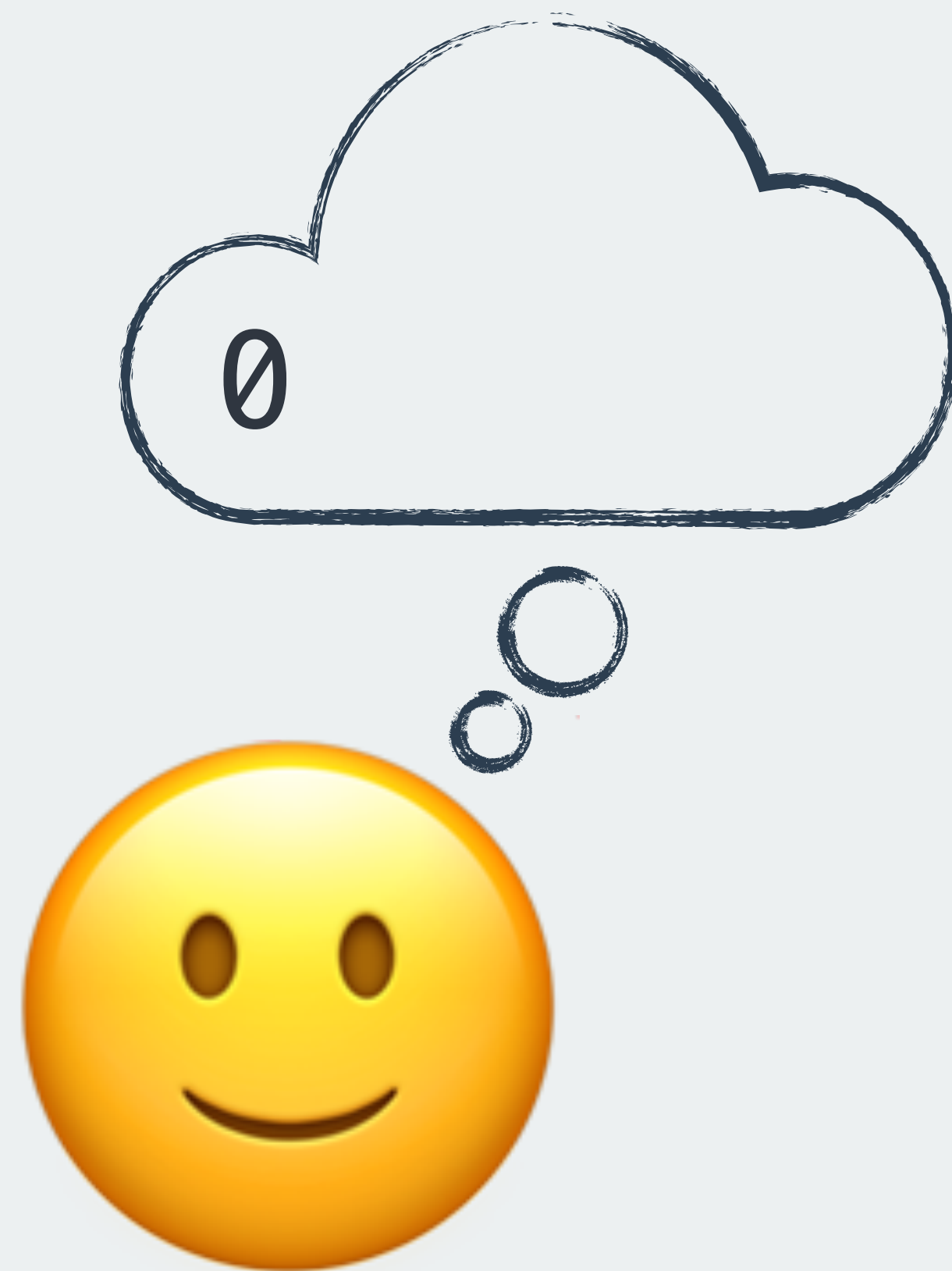
Richard Jozsa
1953-

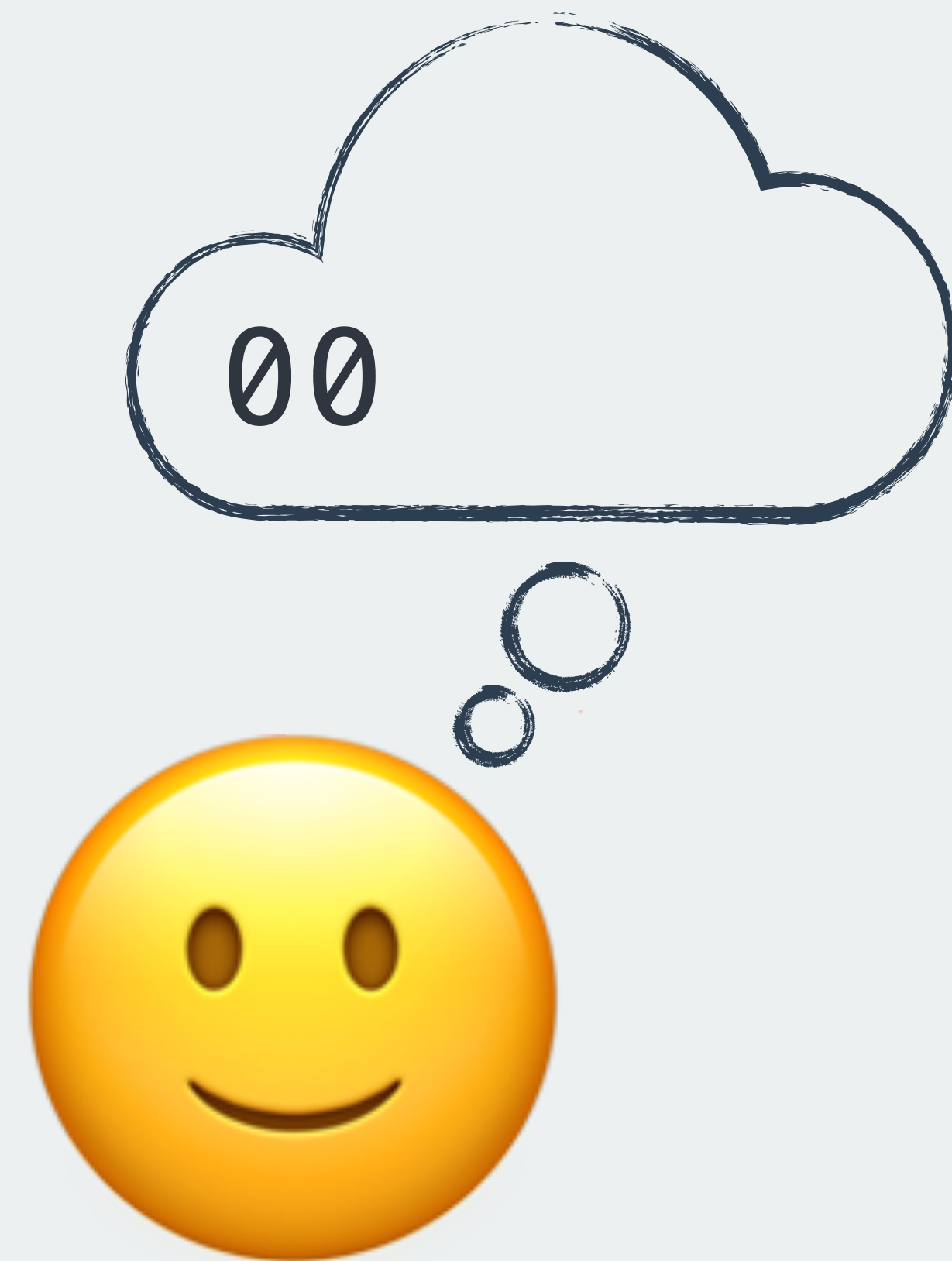


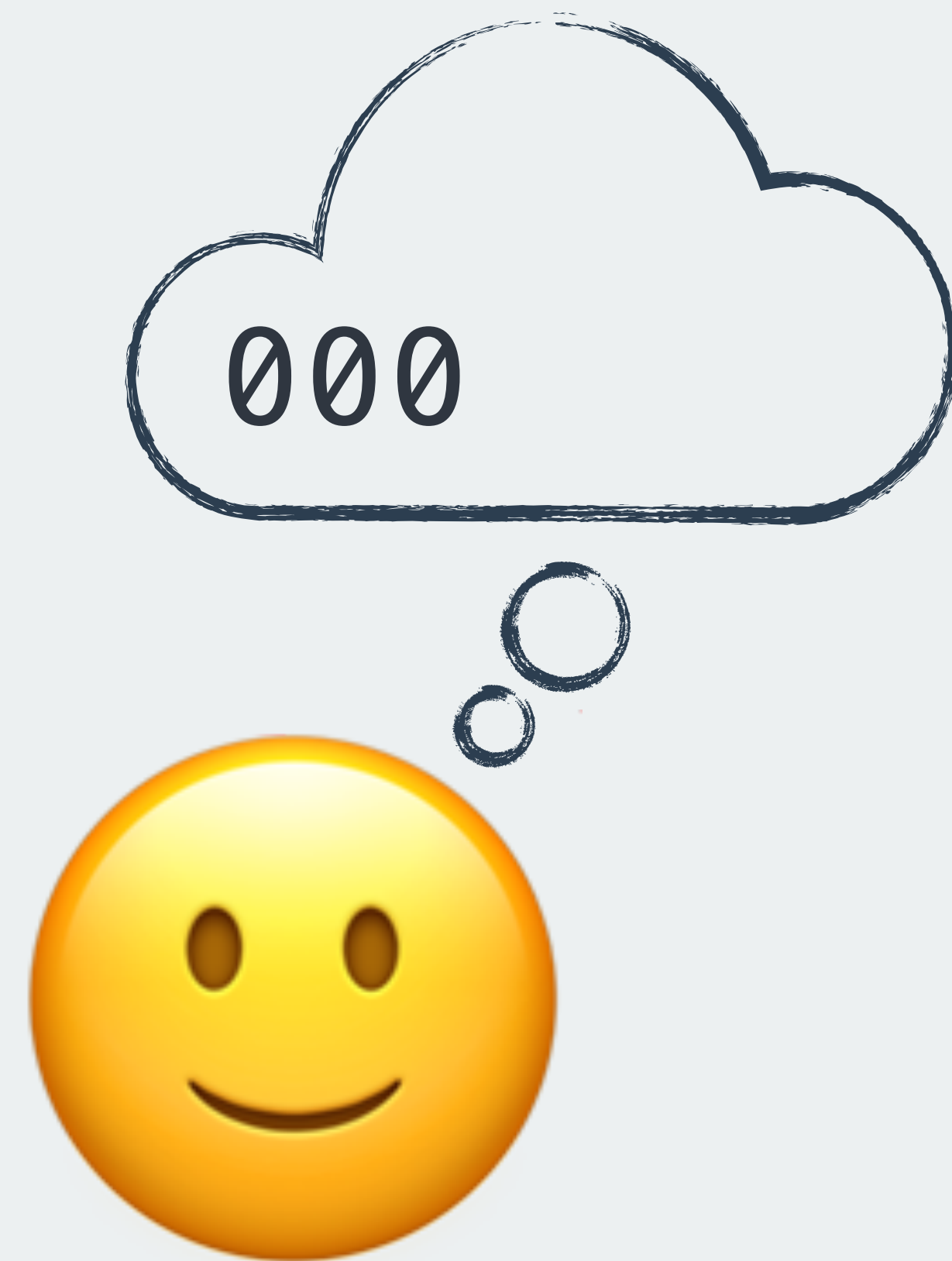
David Deutsch
1953-

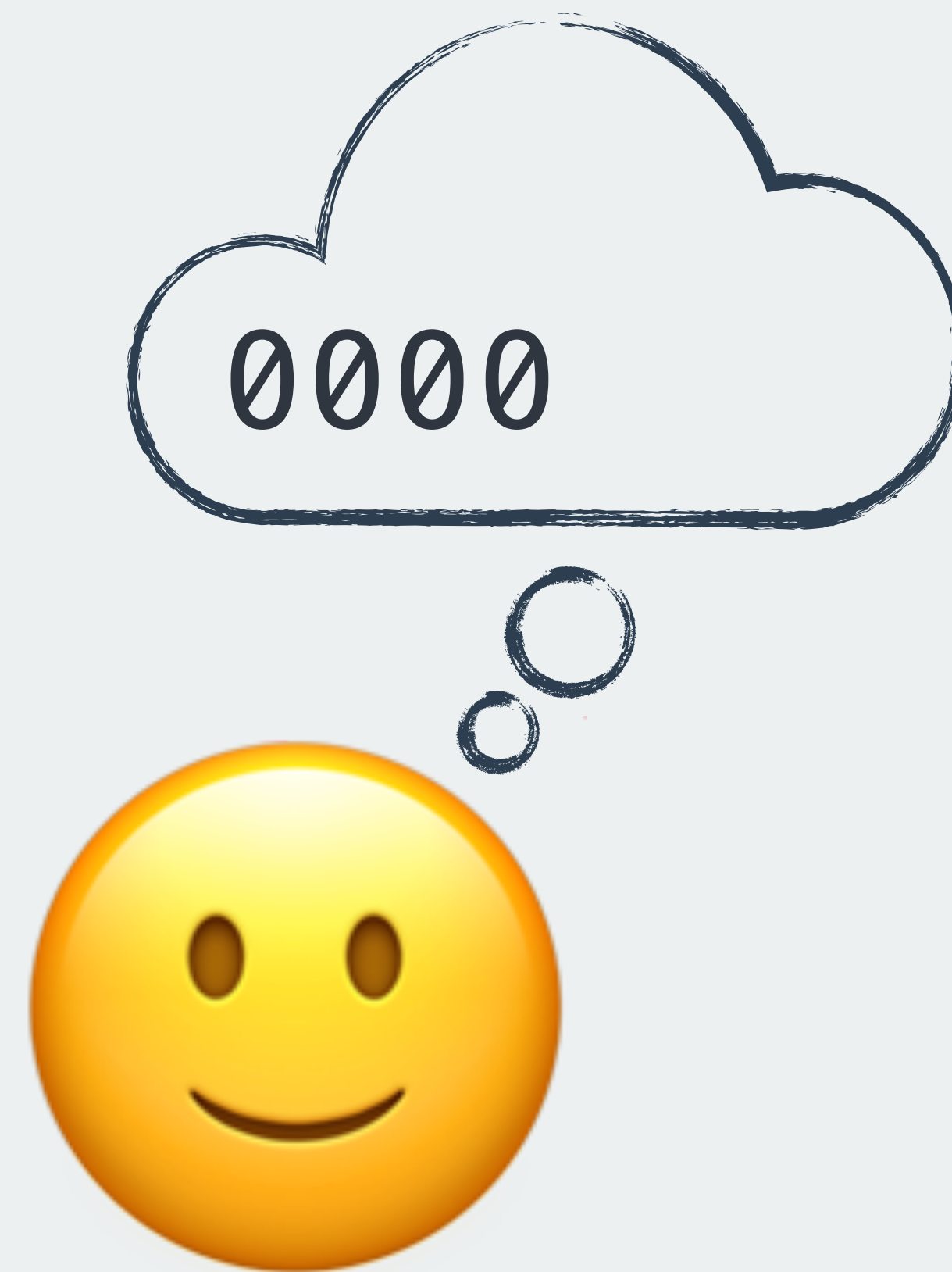




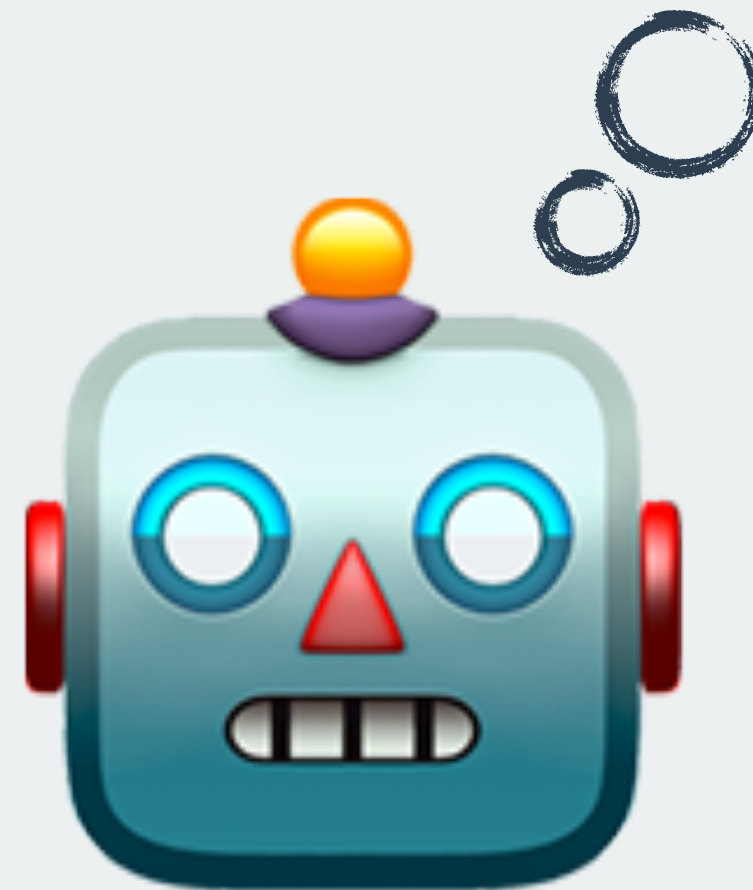




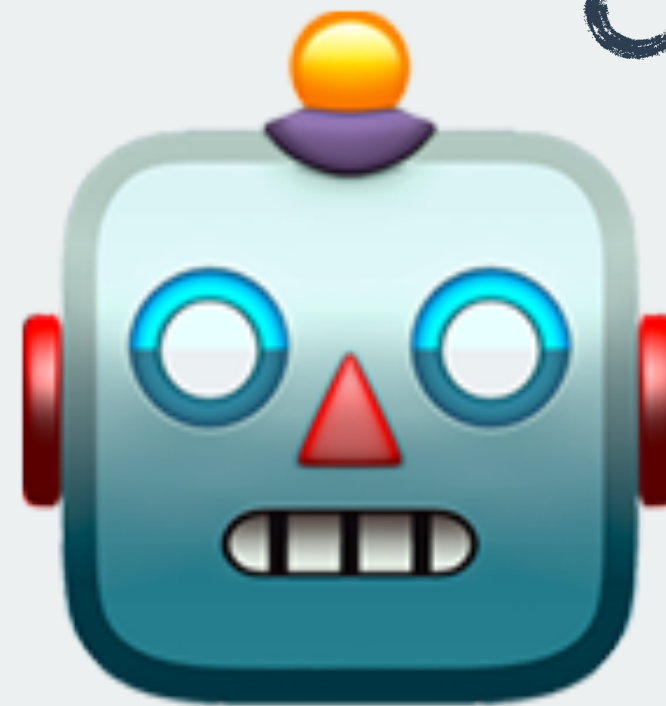


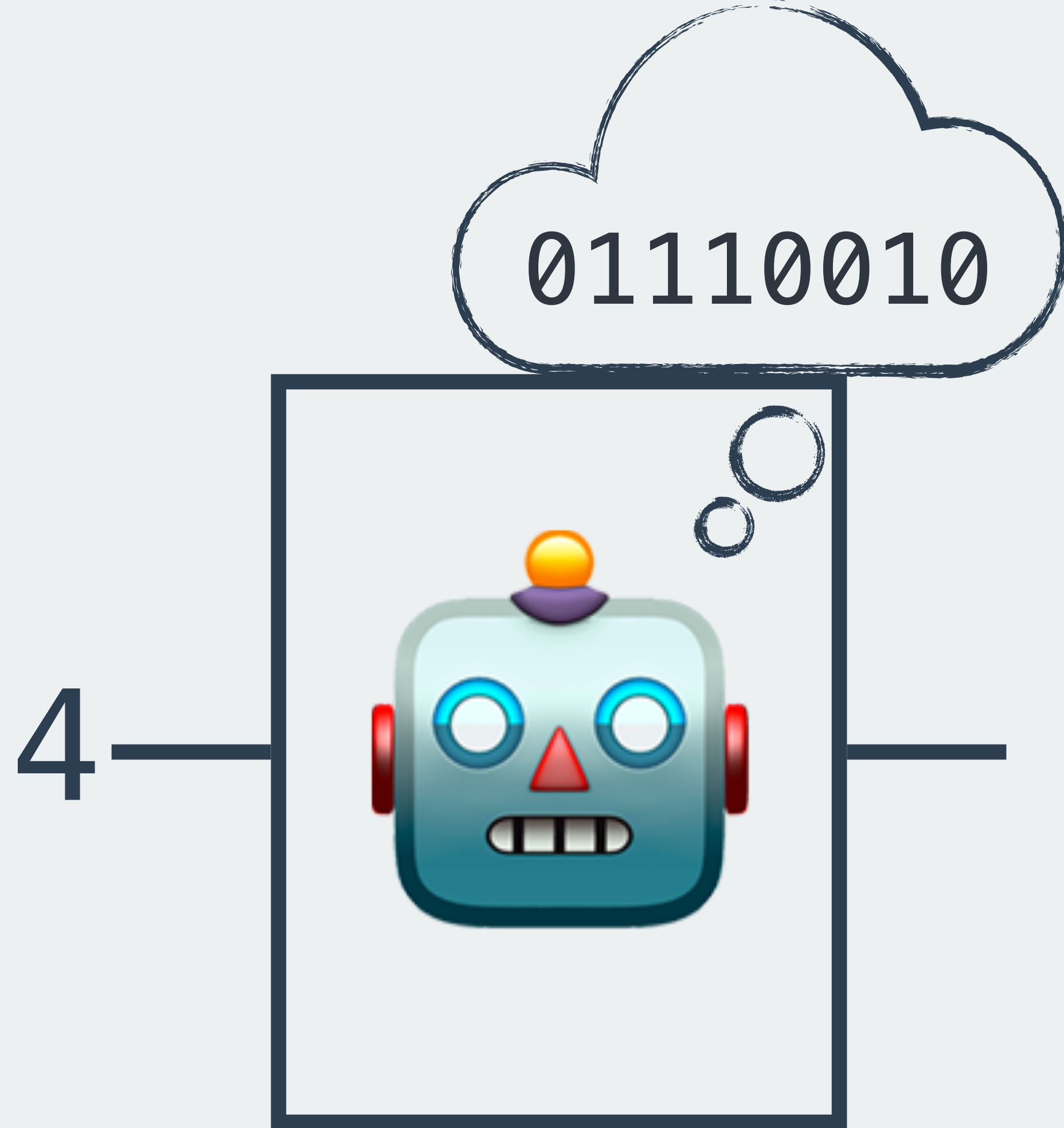


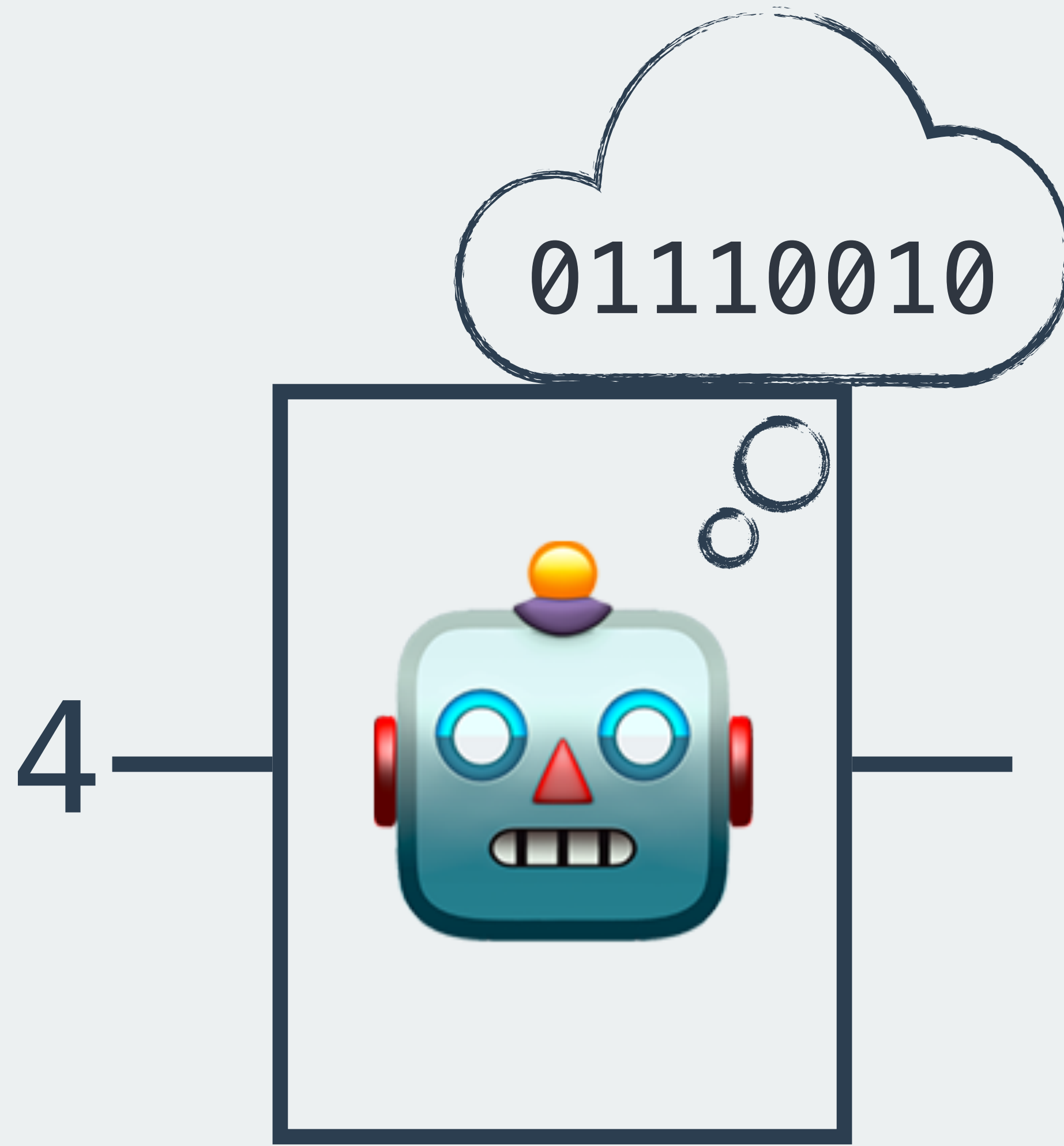
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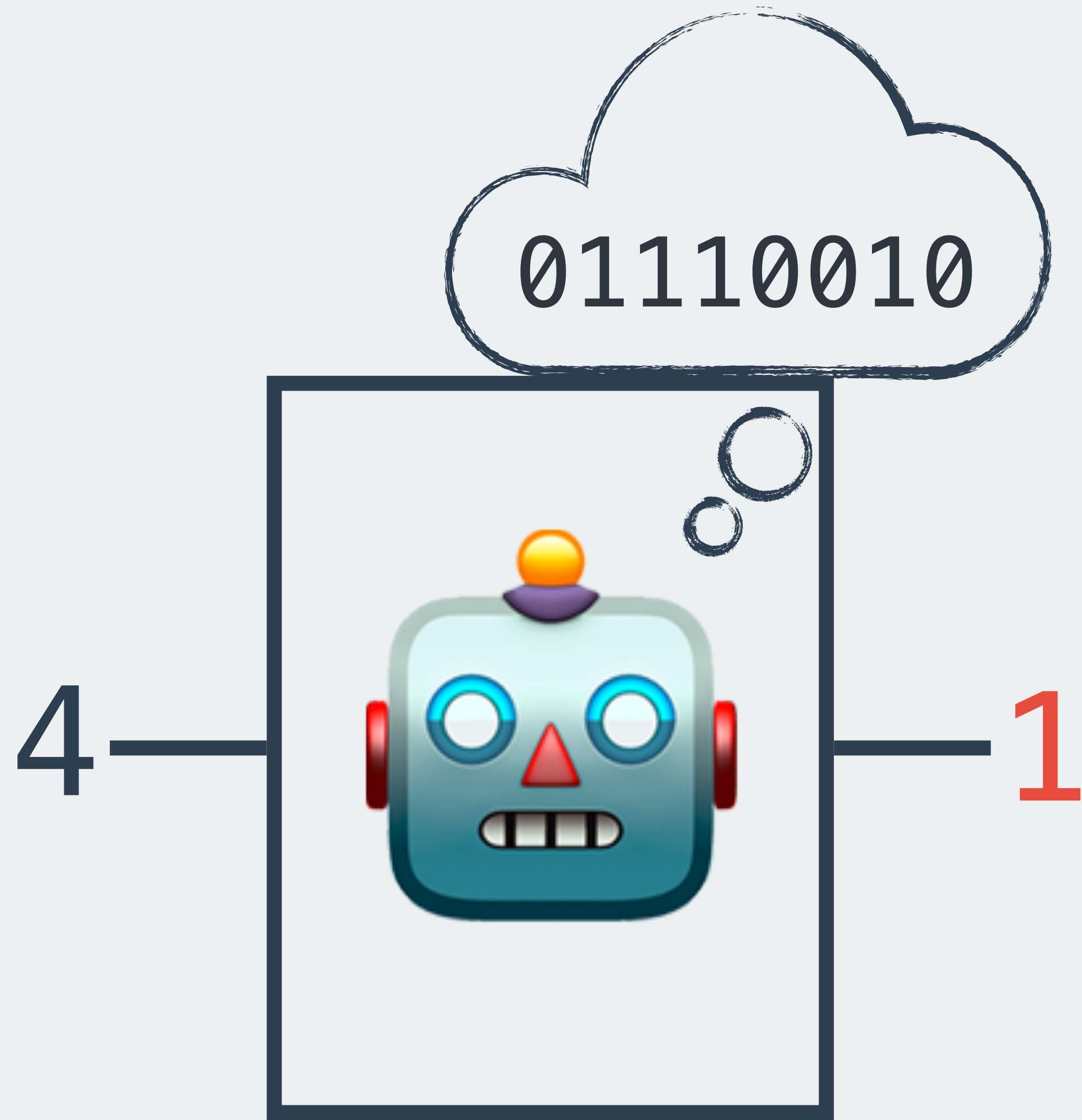


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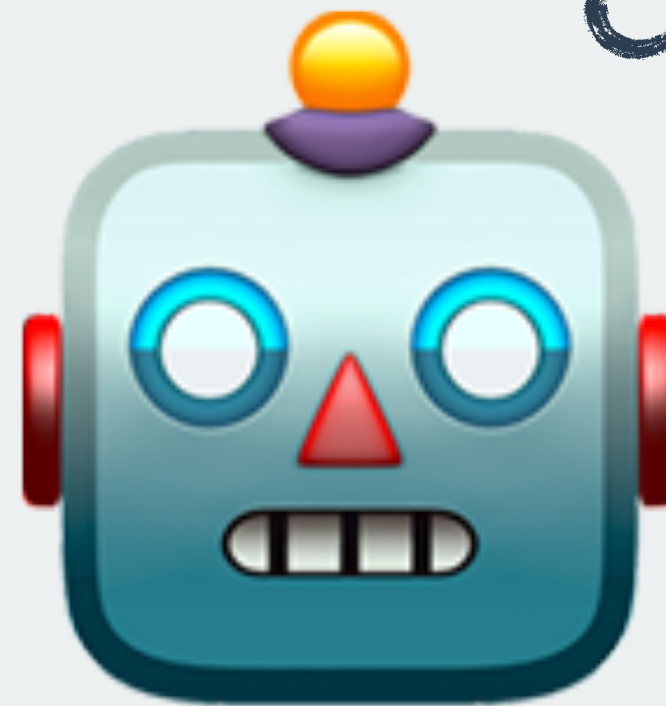


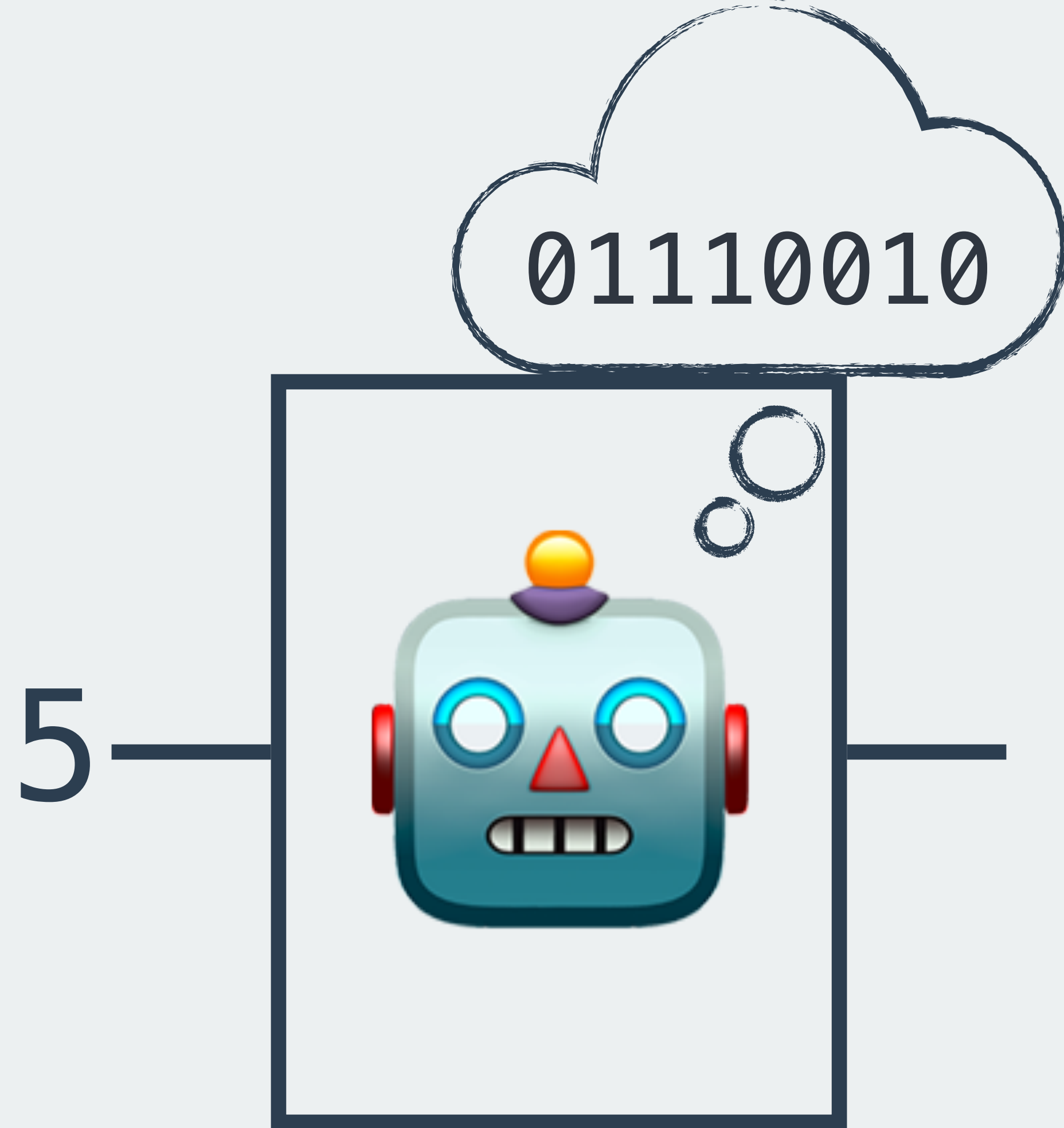


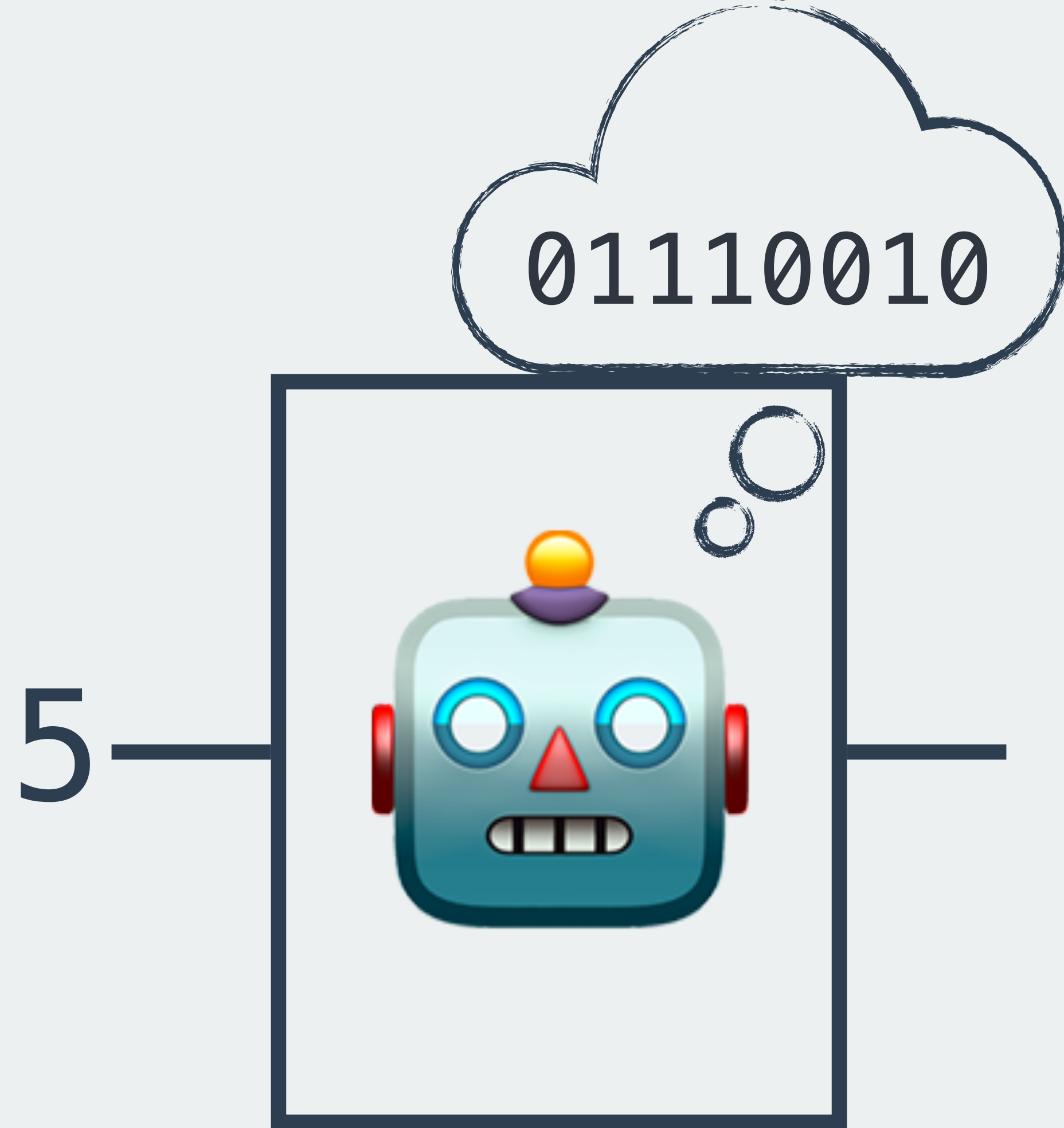


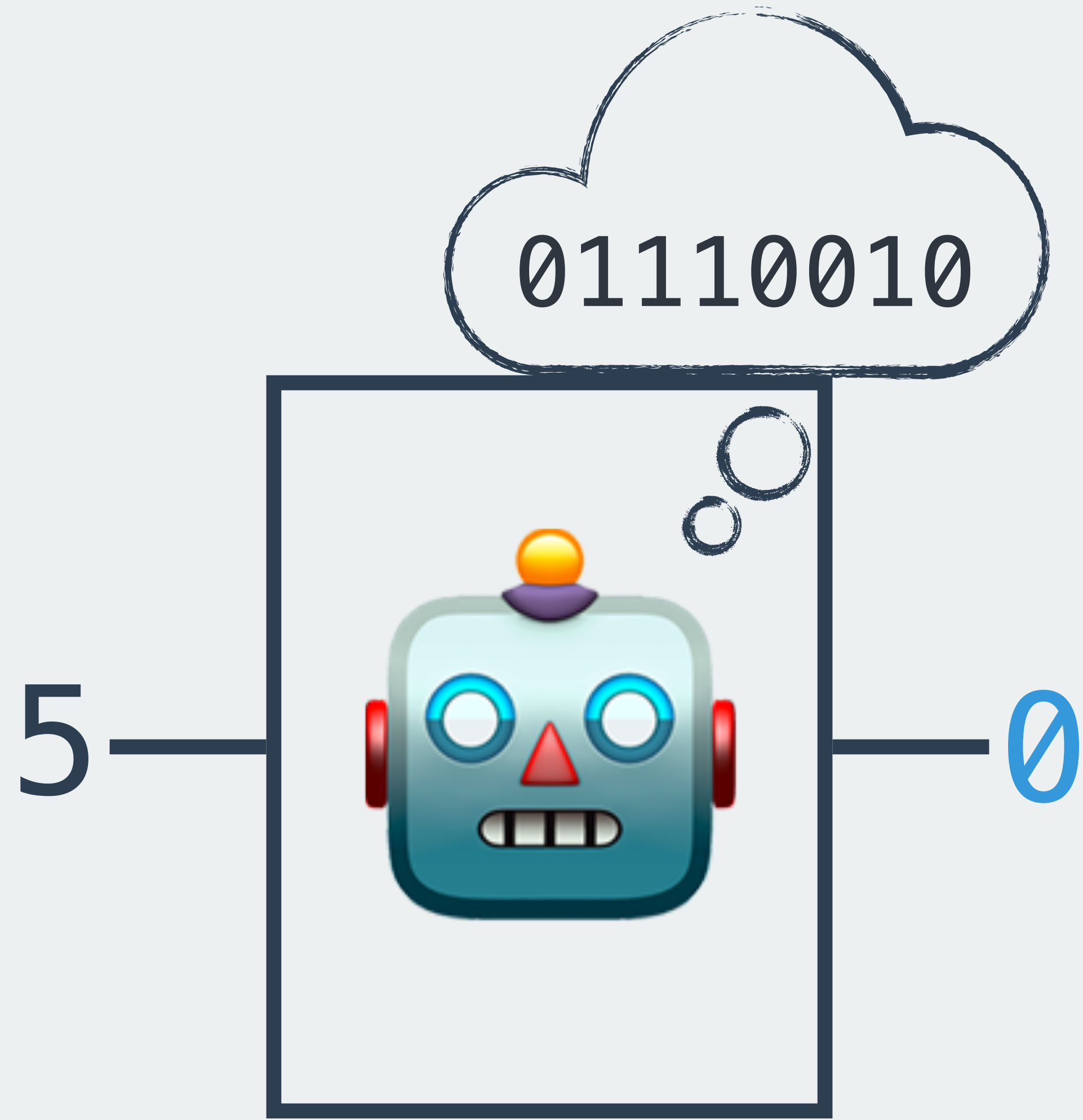


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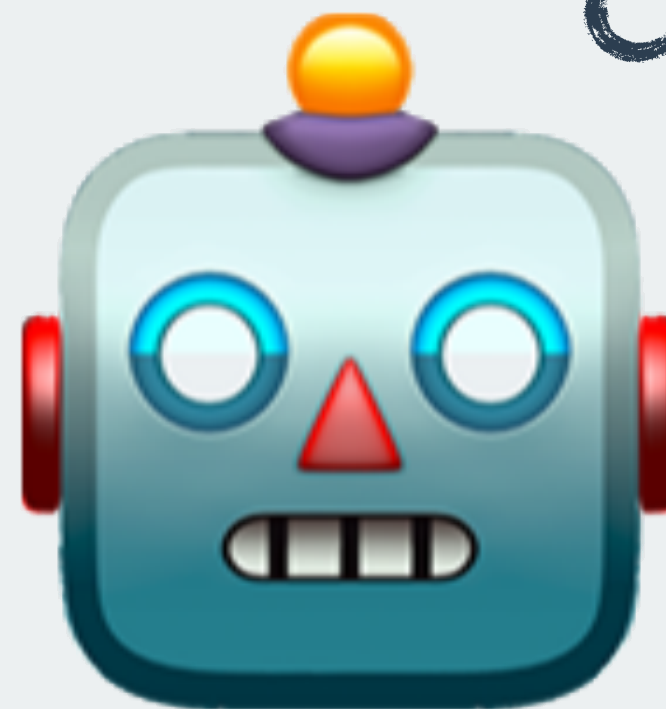




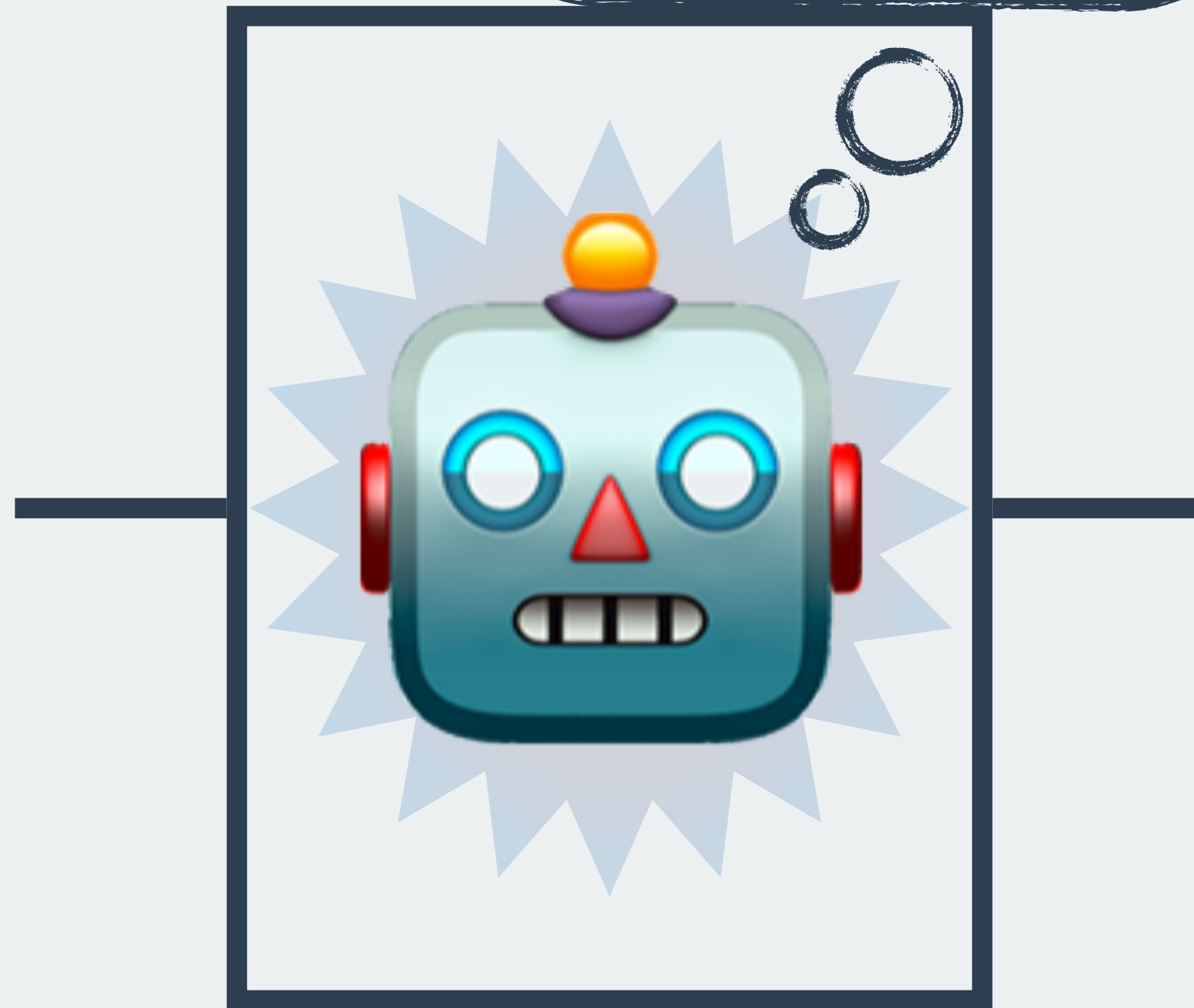


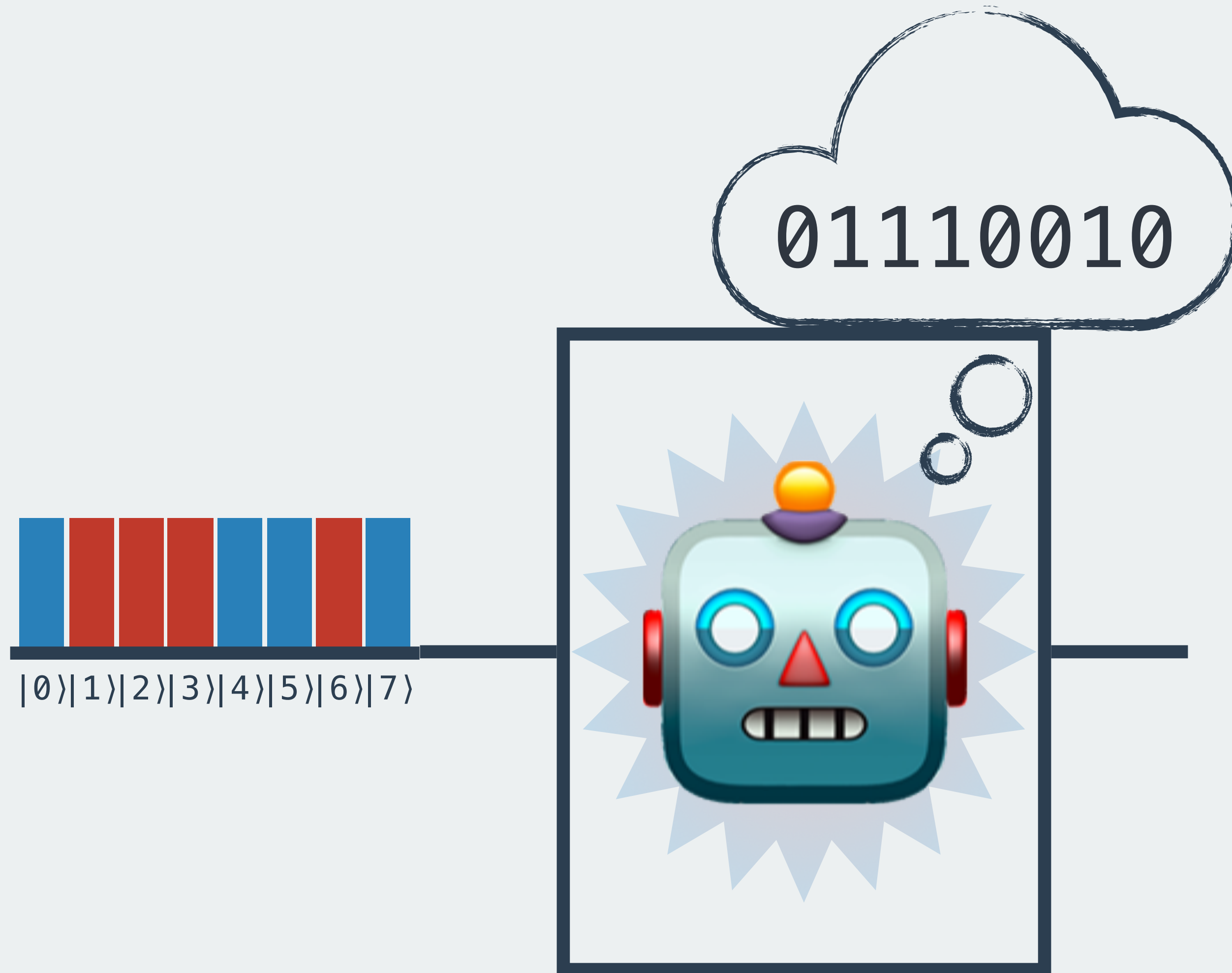


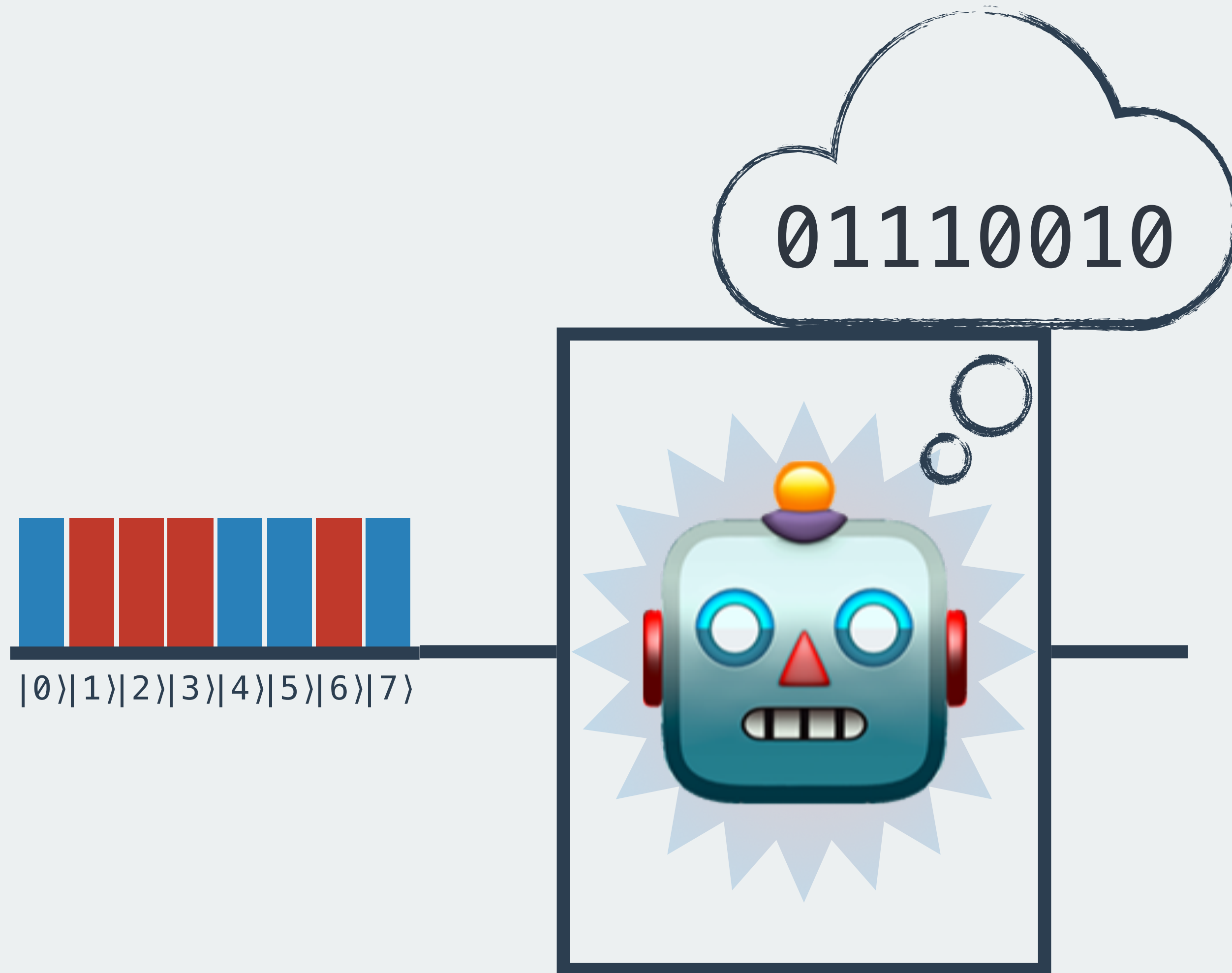
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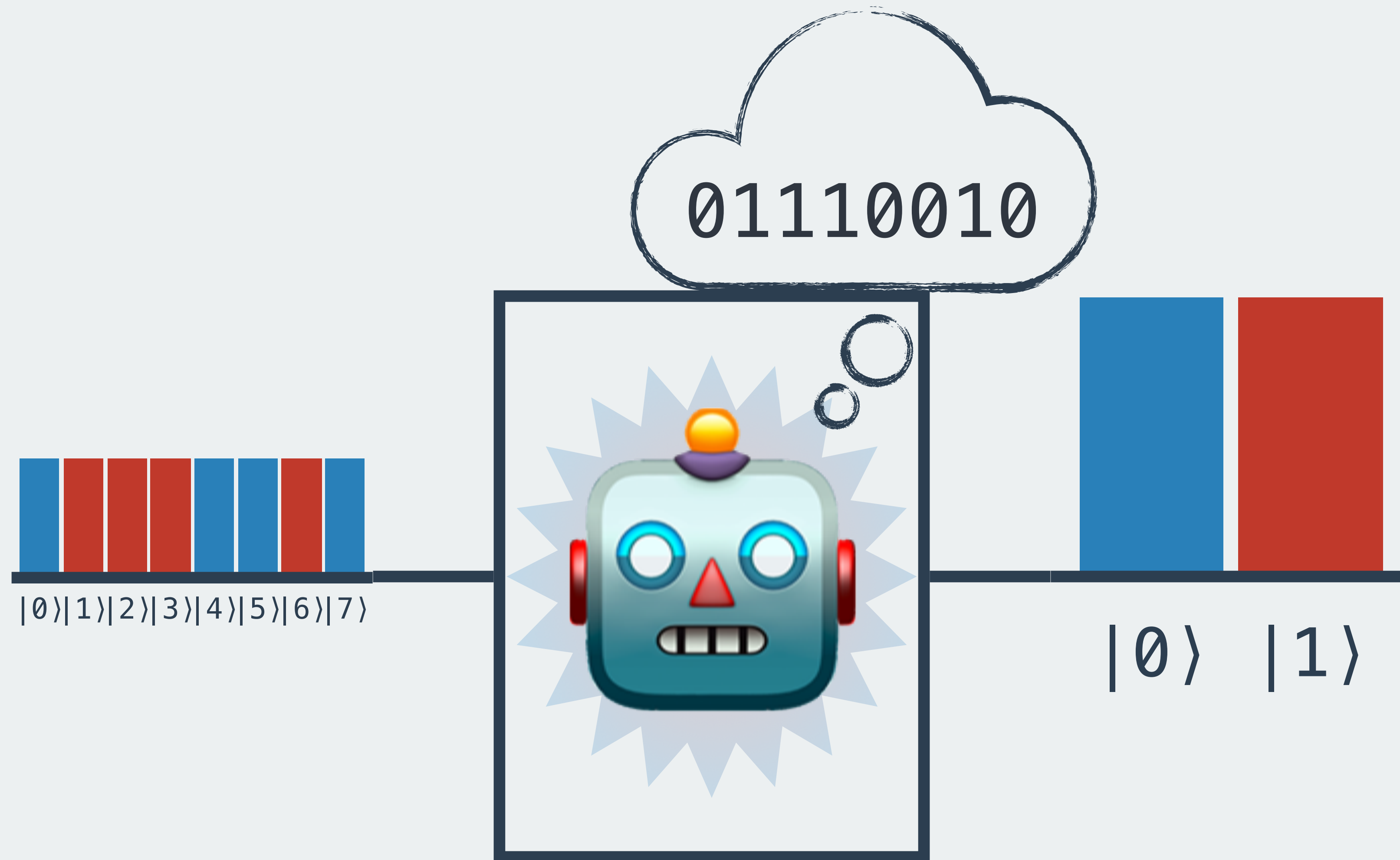


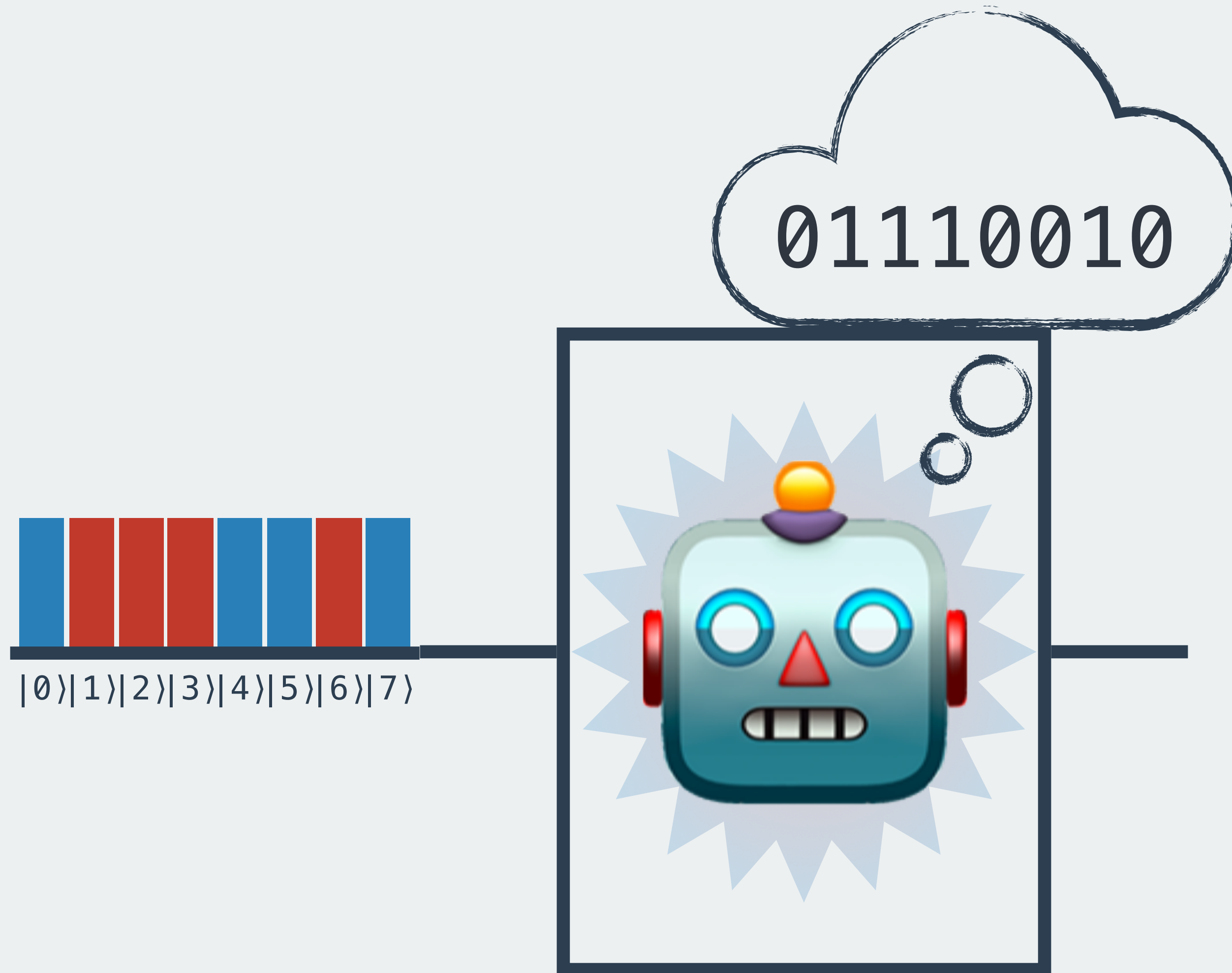
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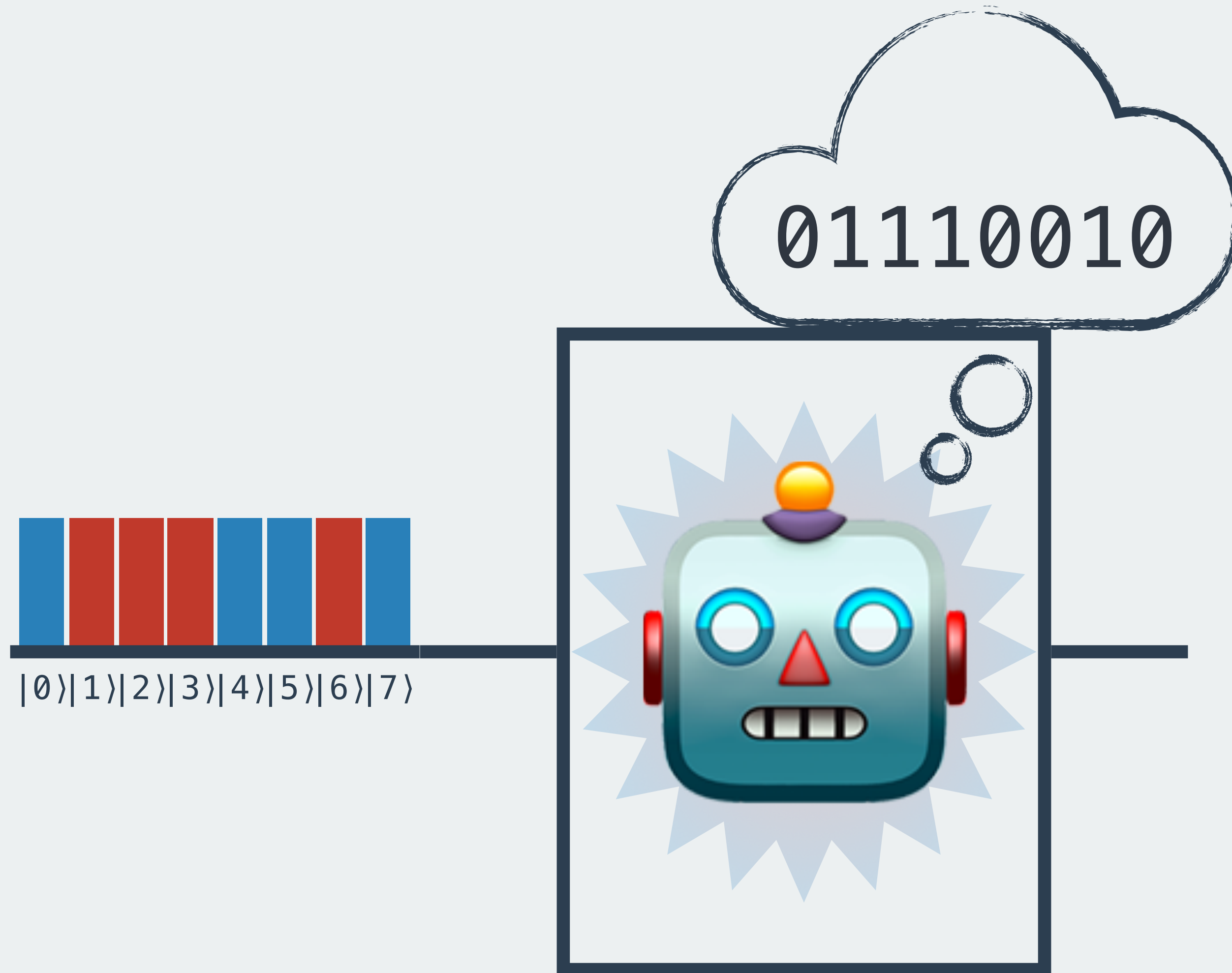


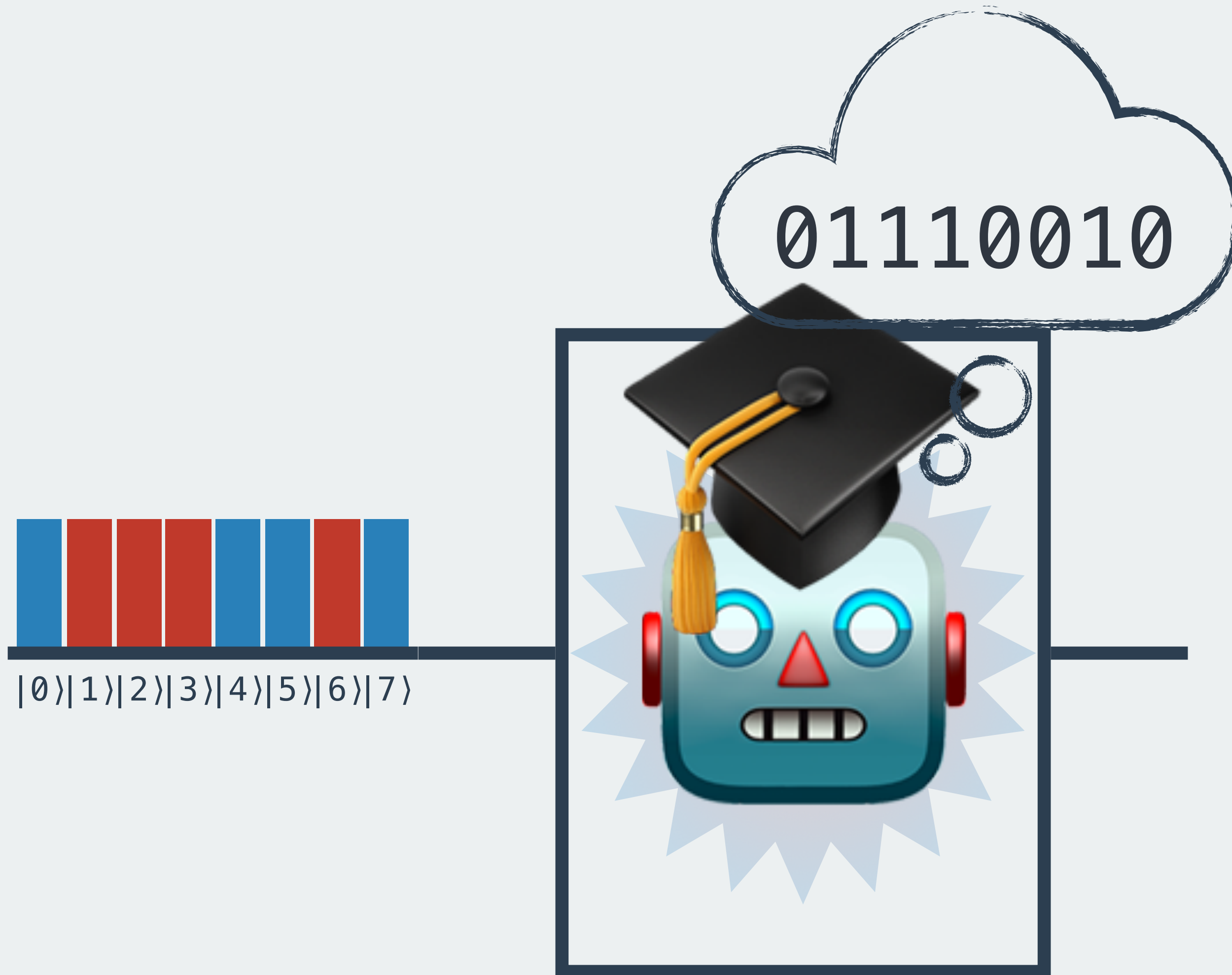


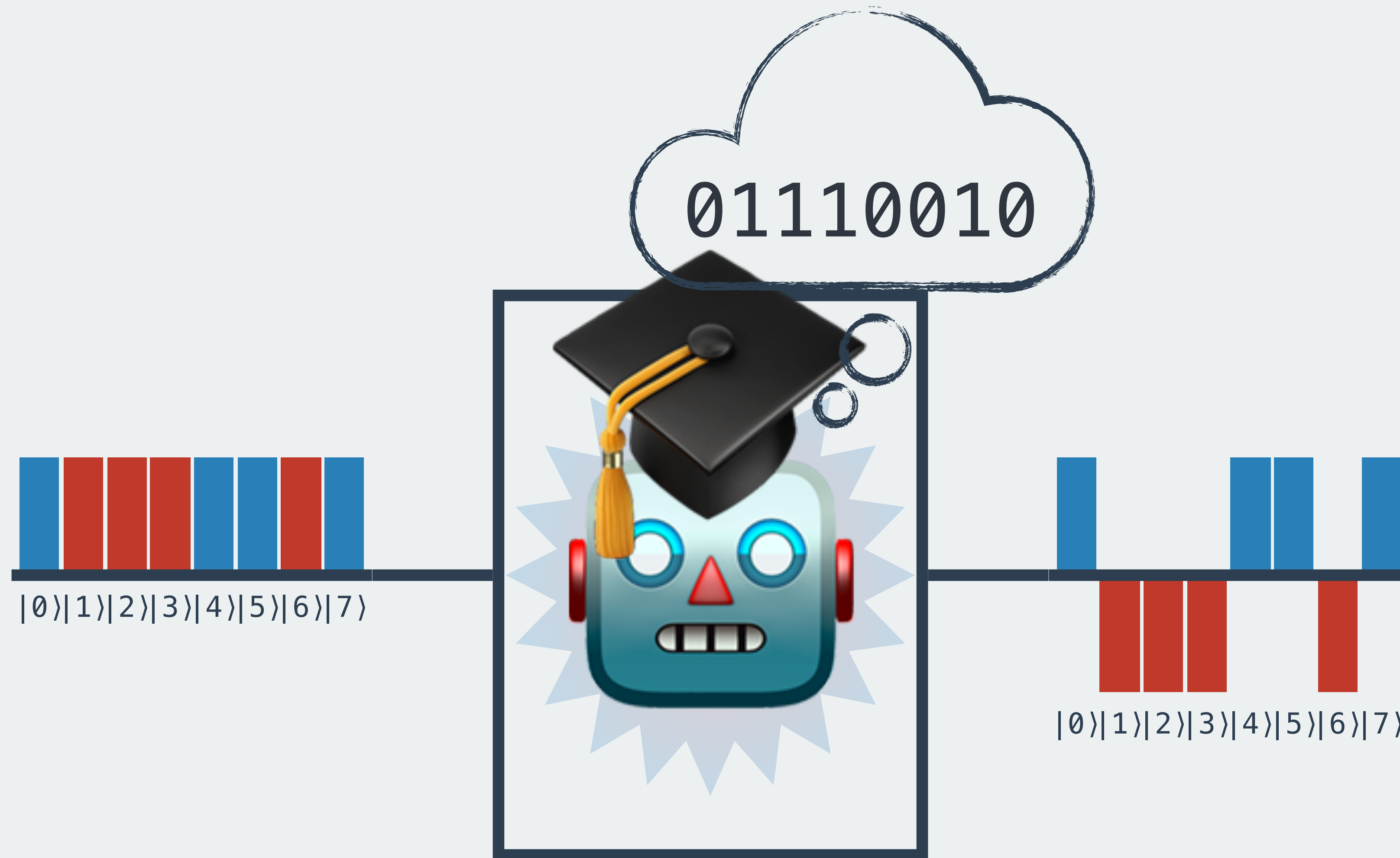


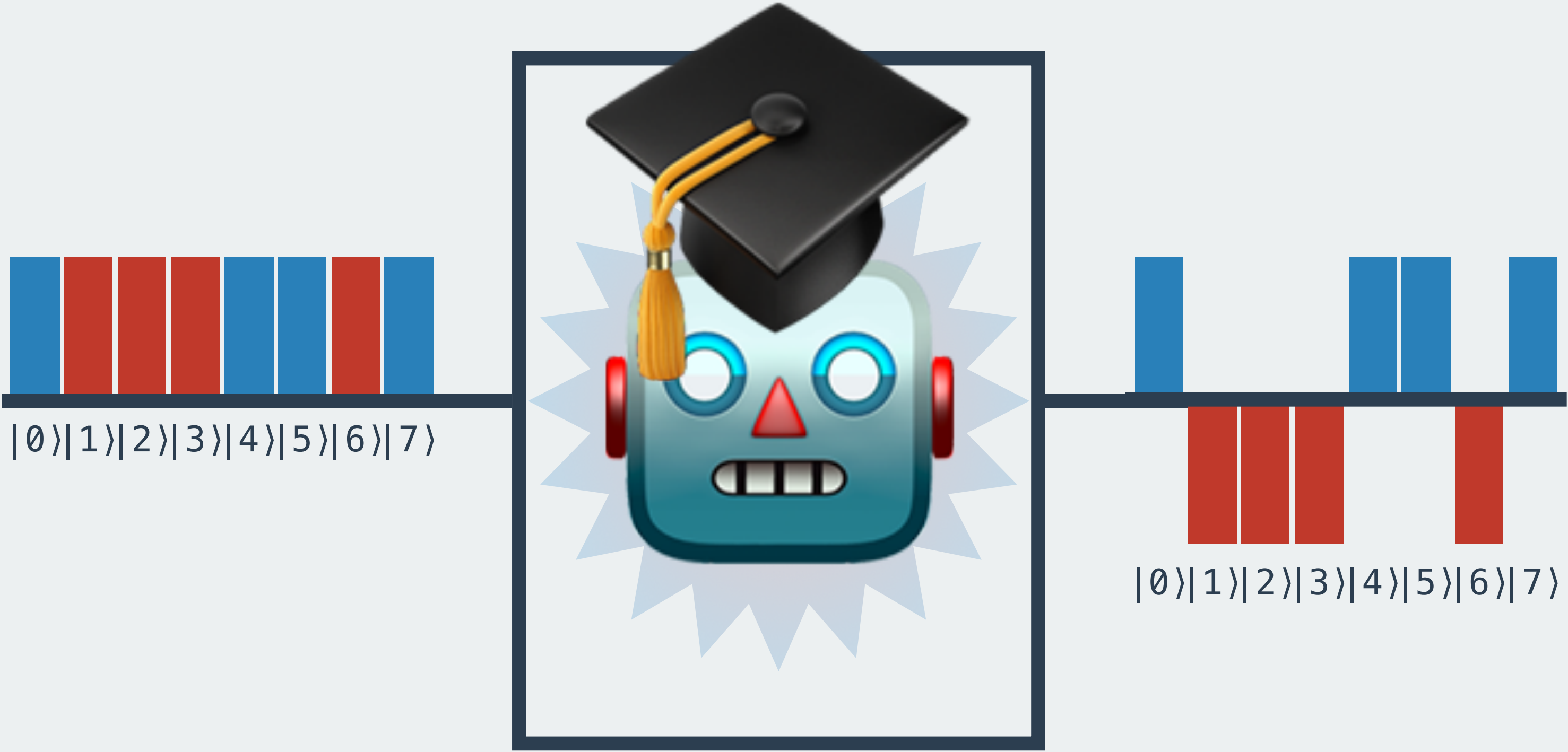


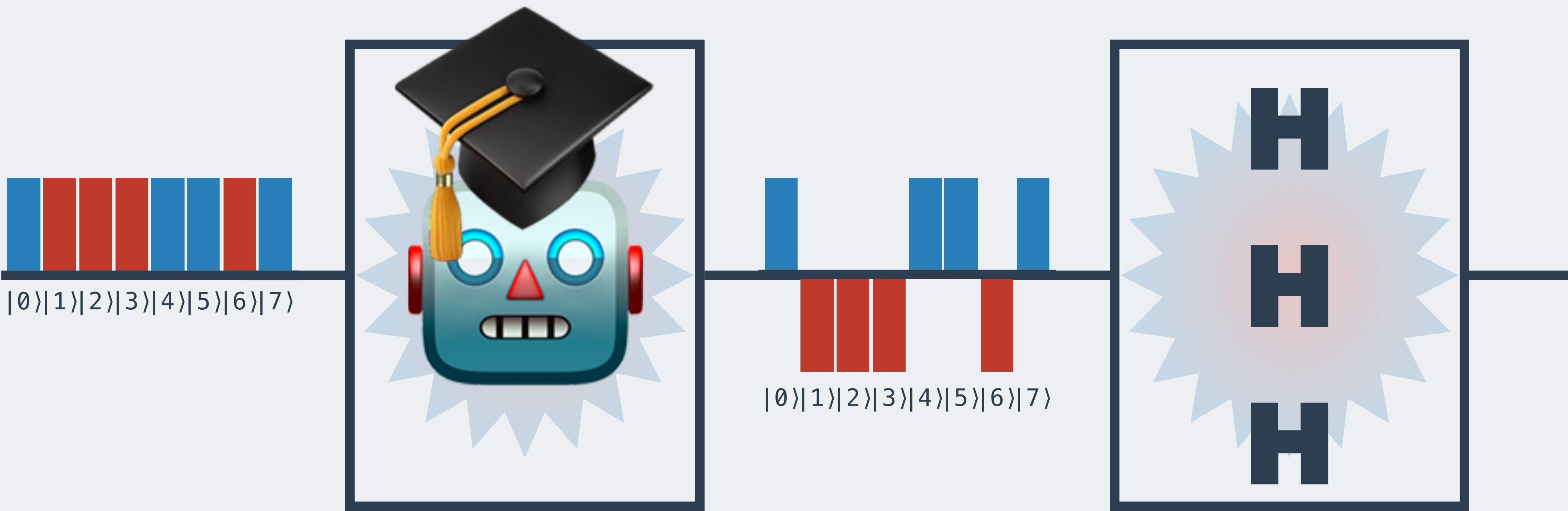




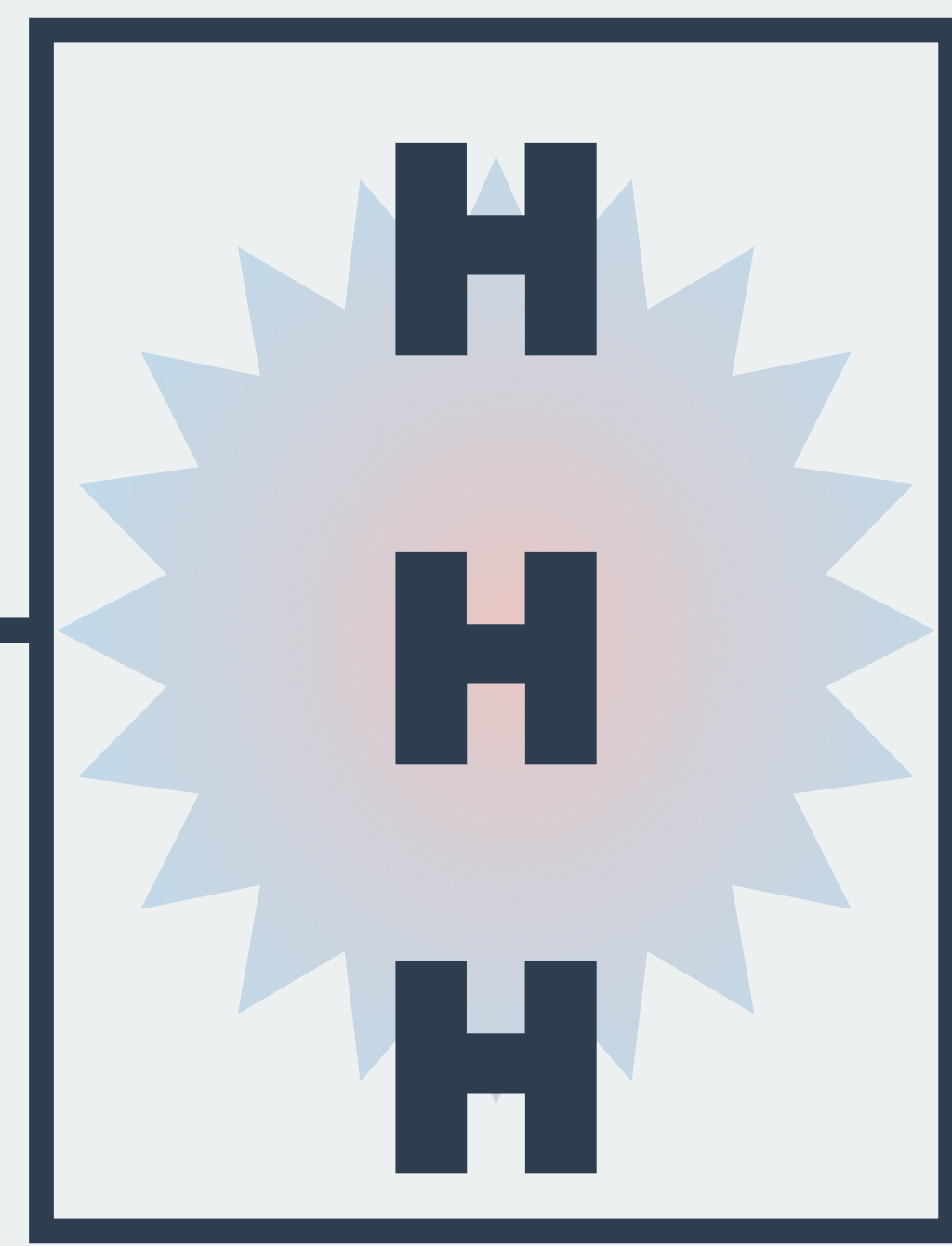
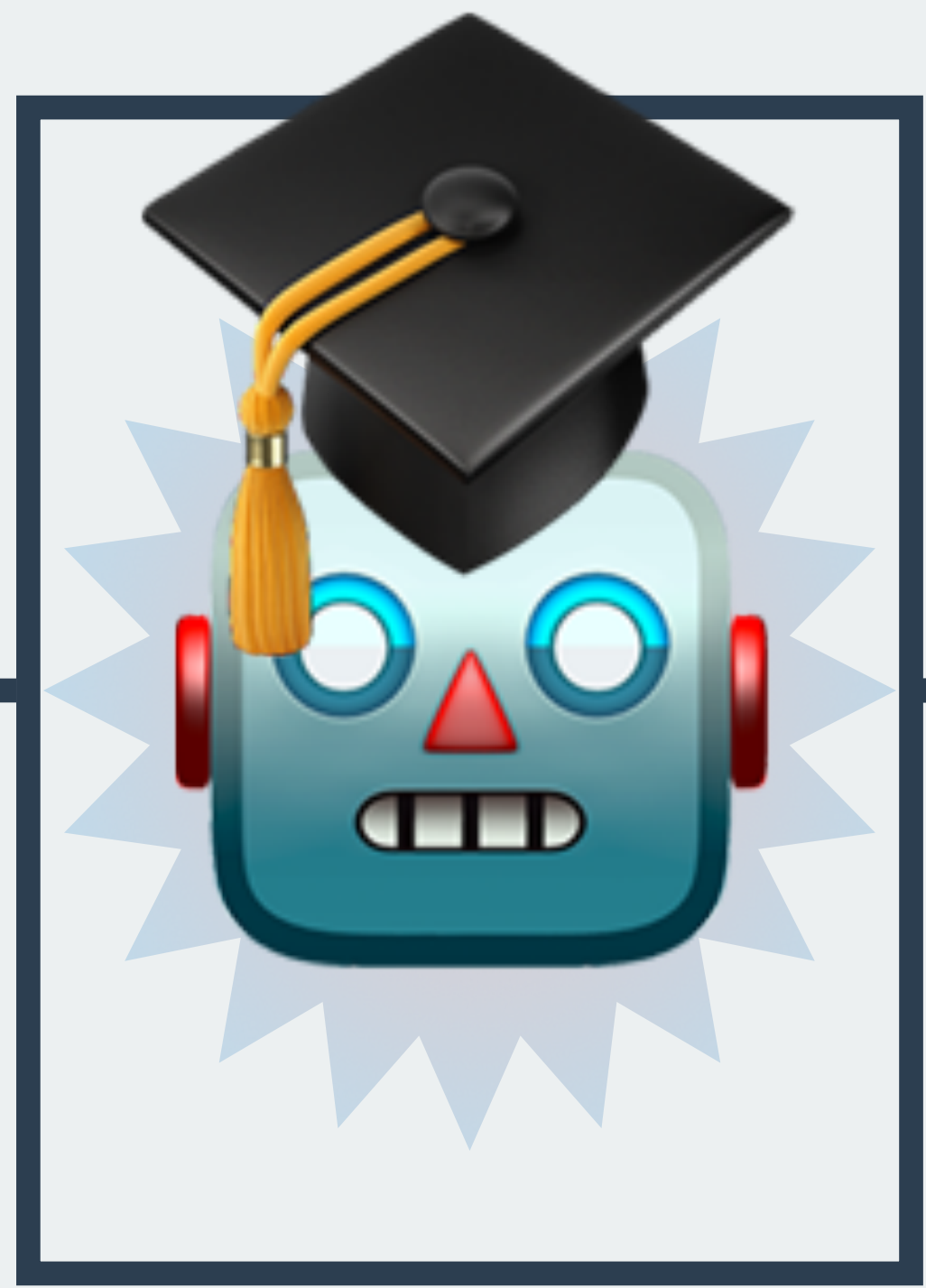


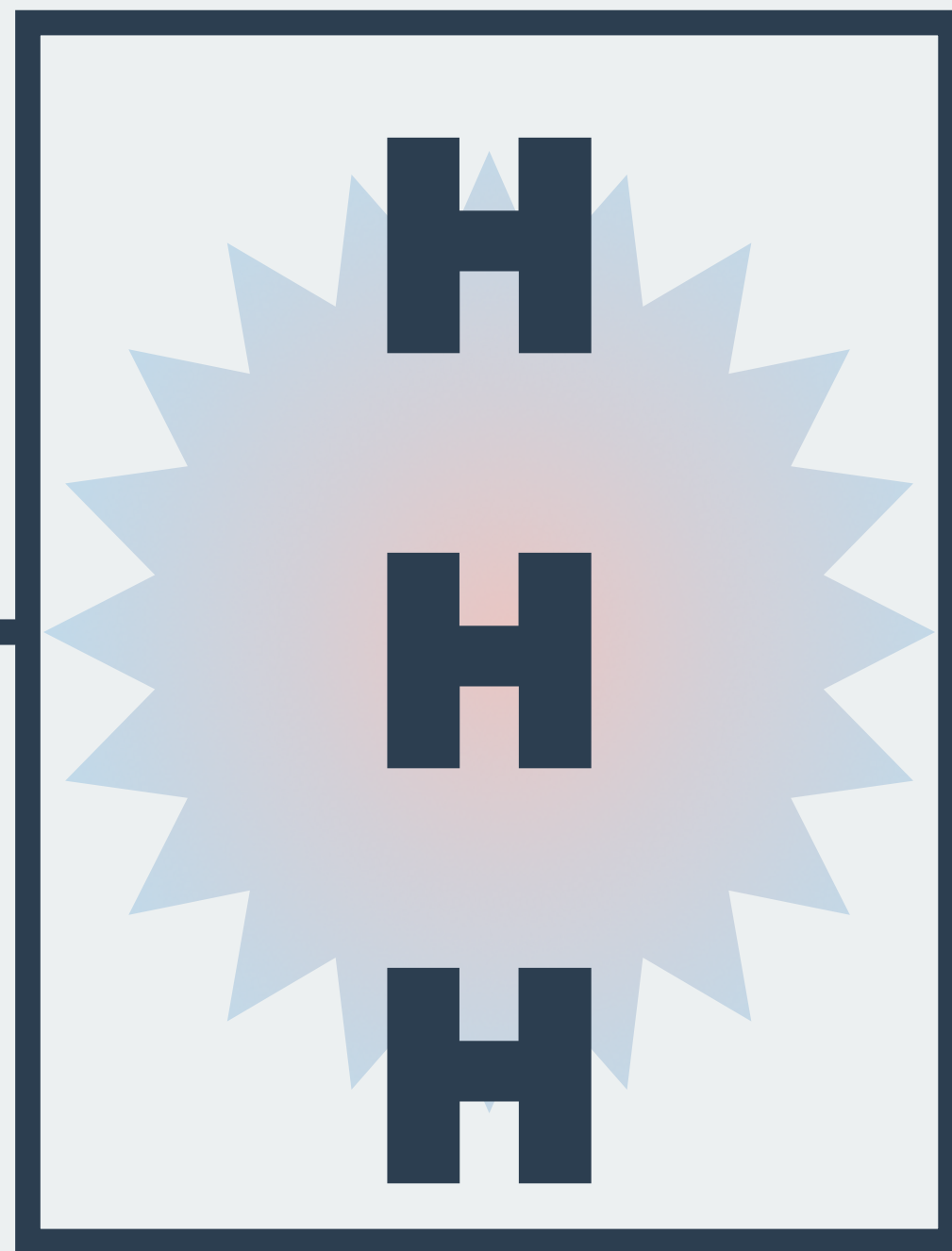
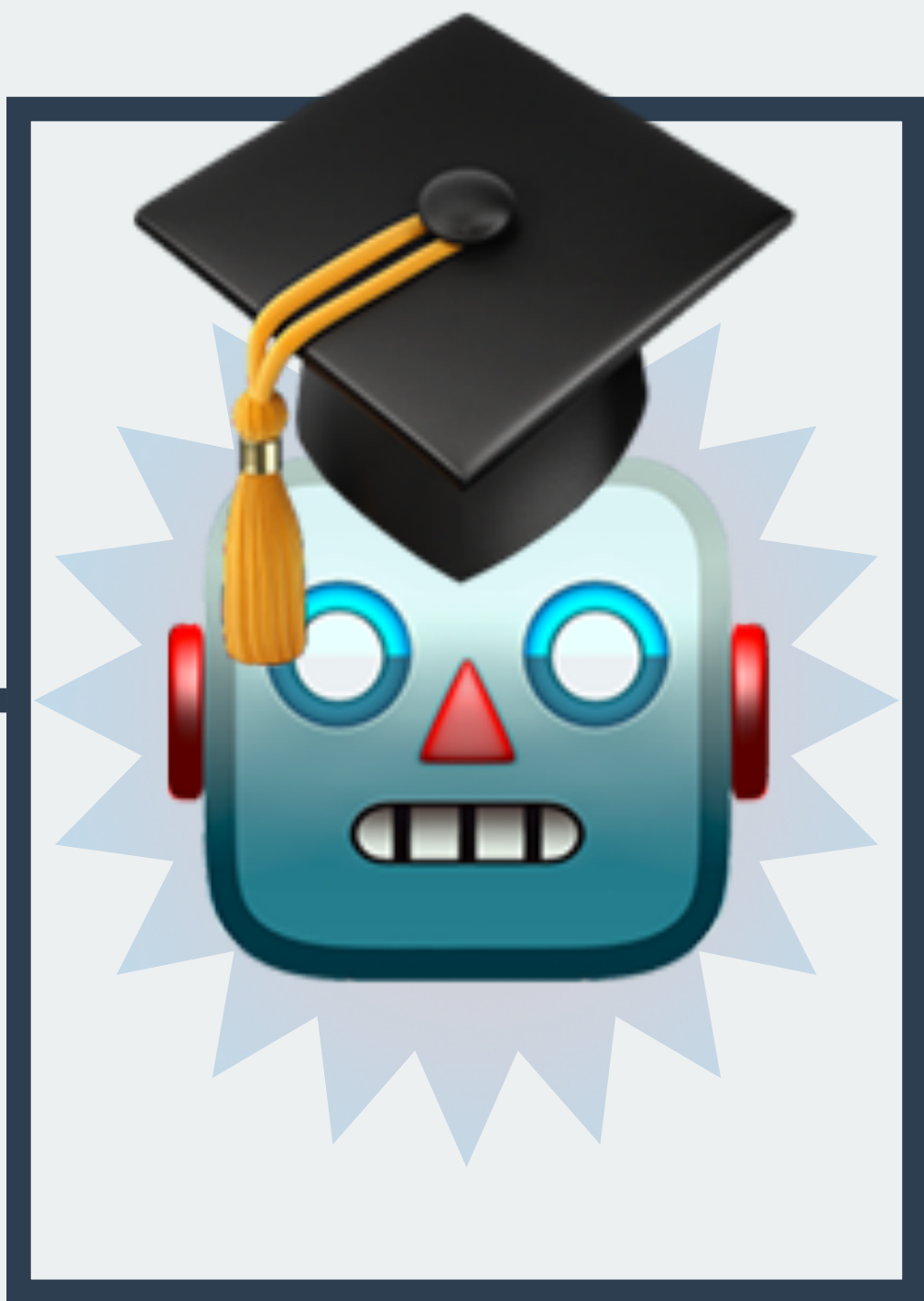


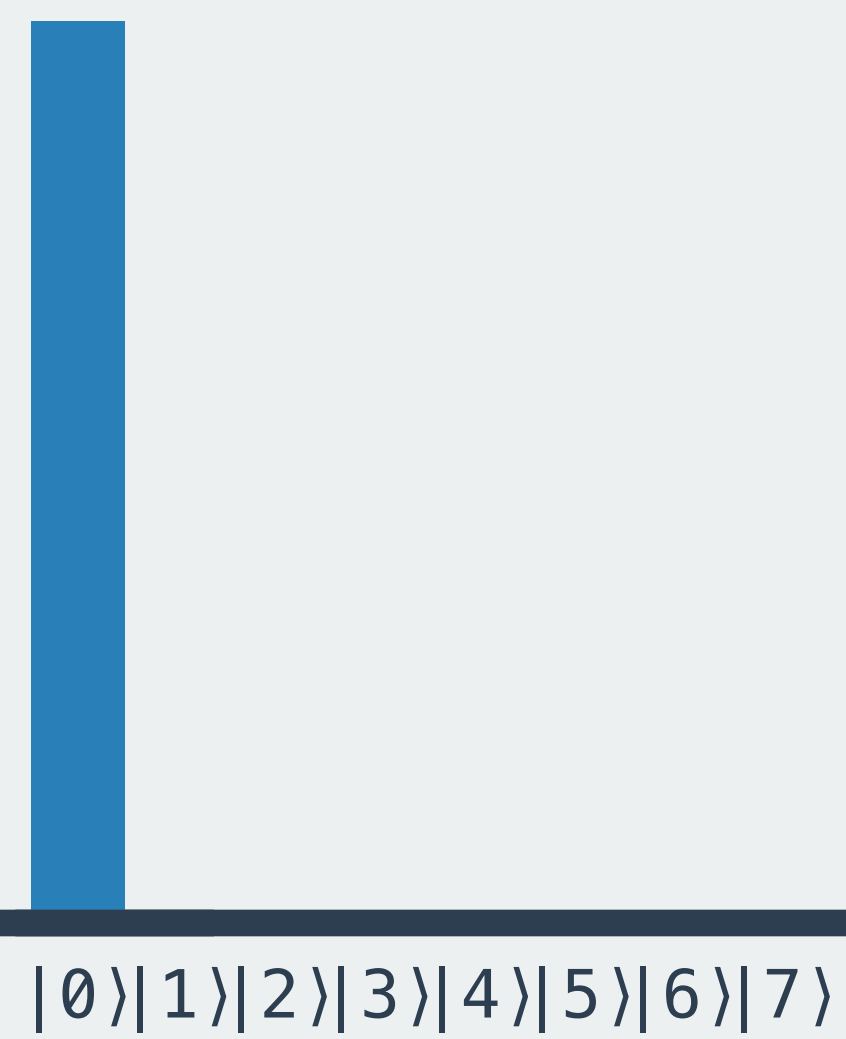
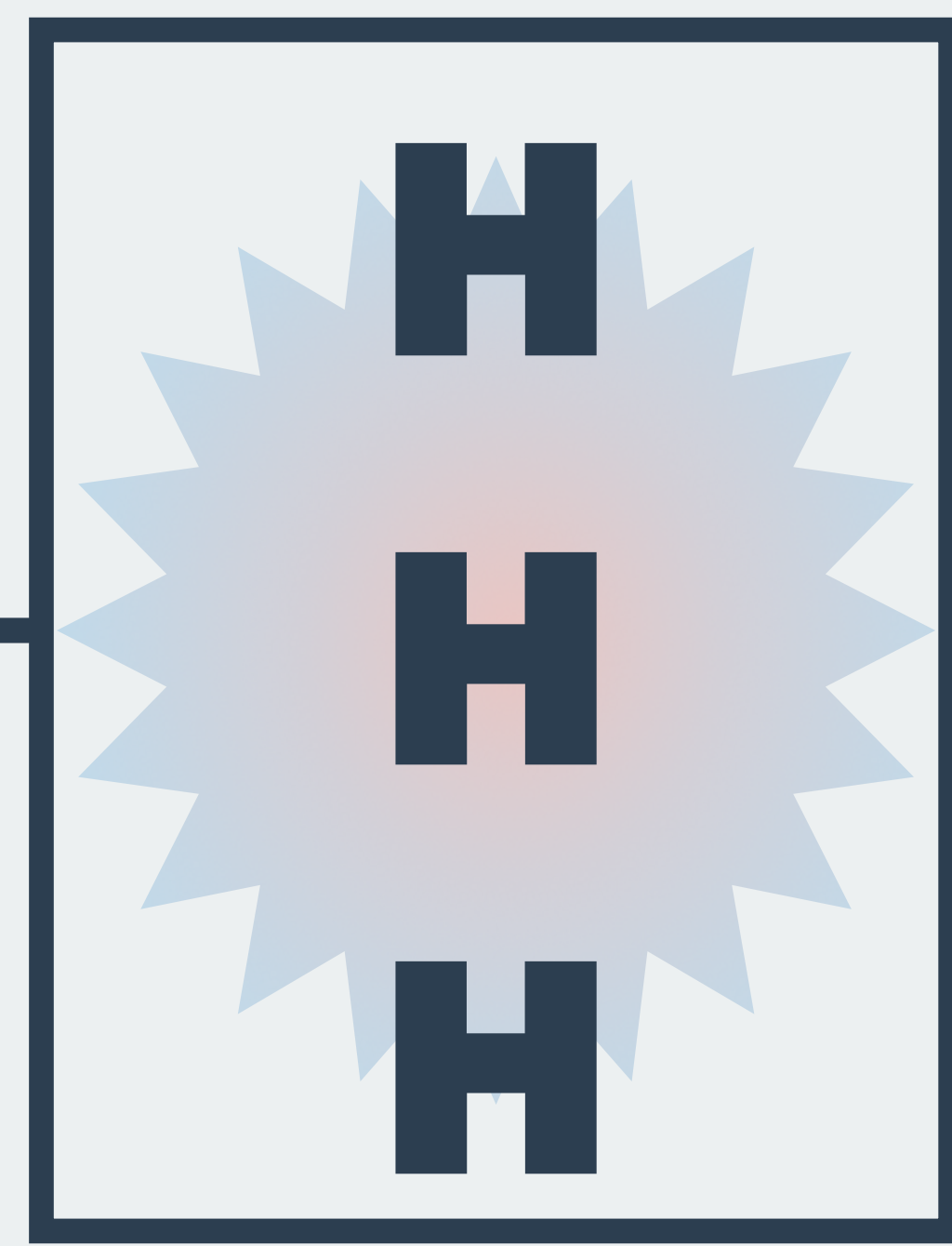
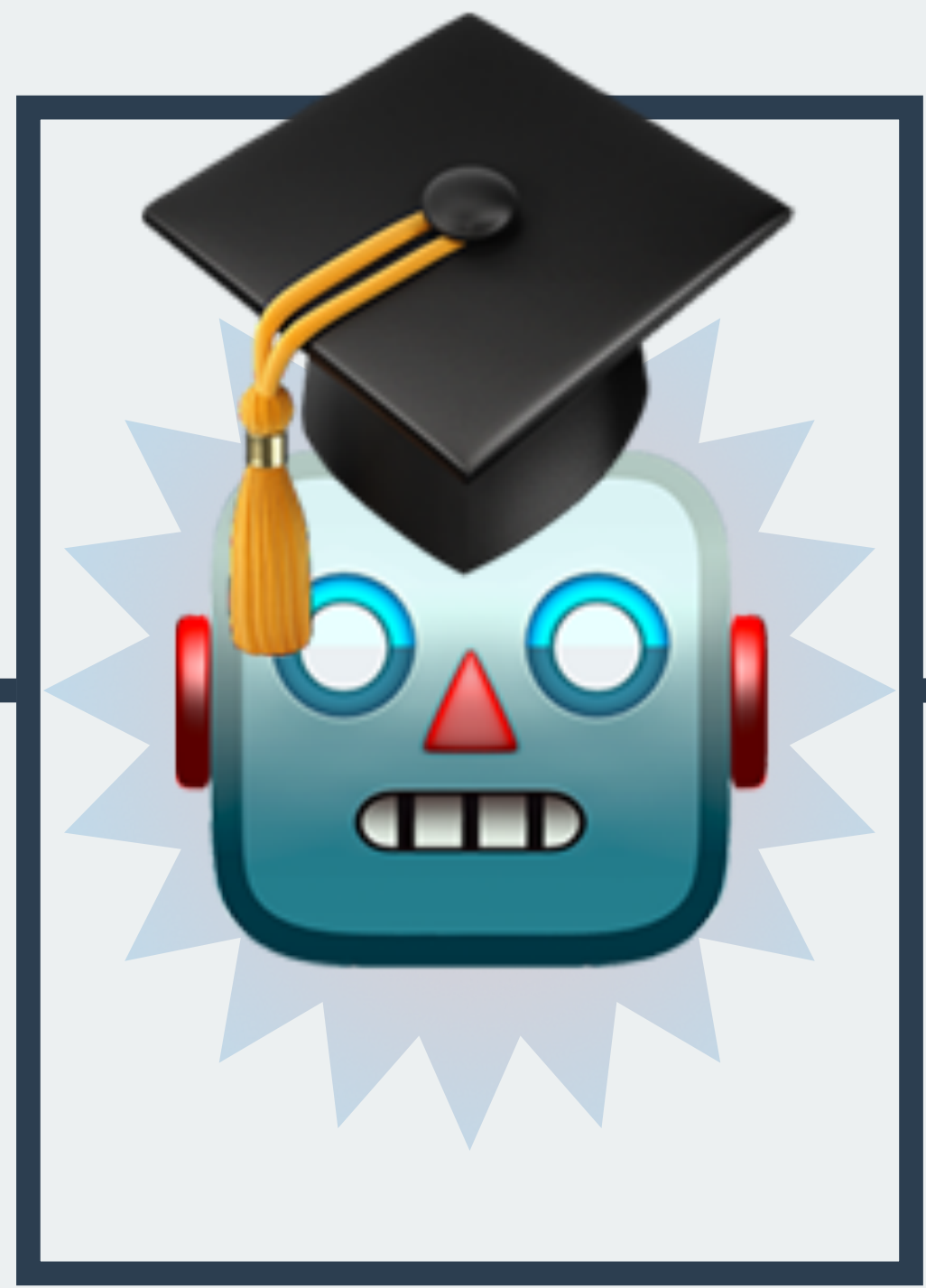




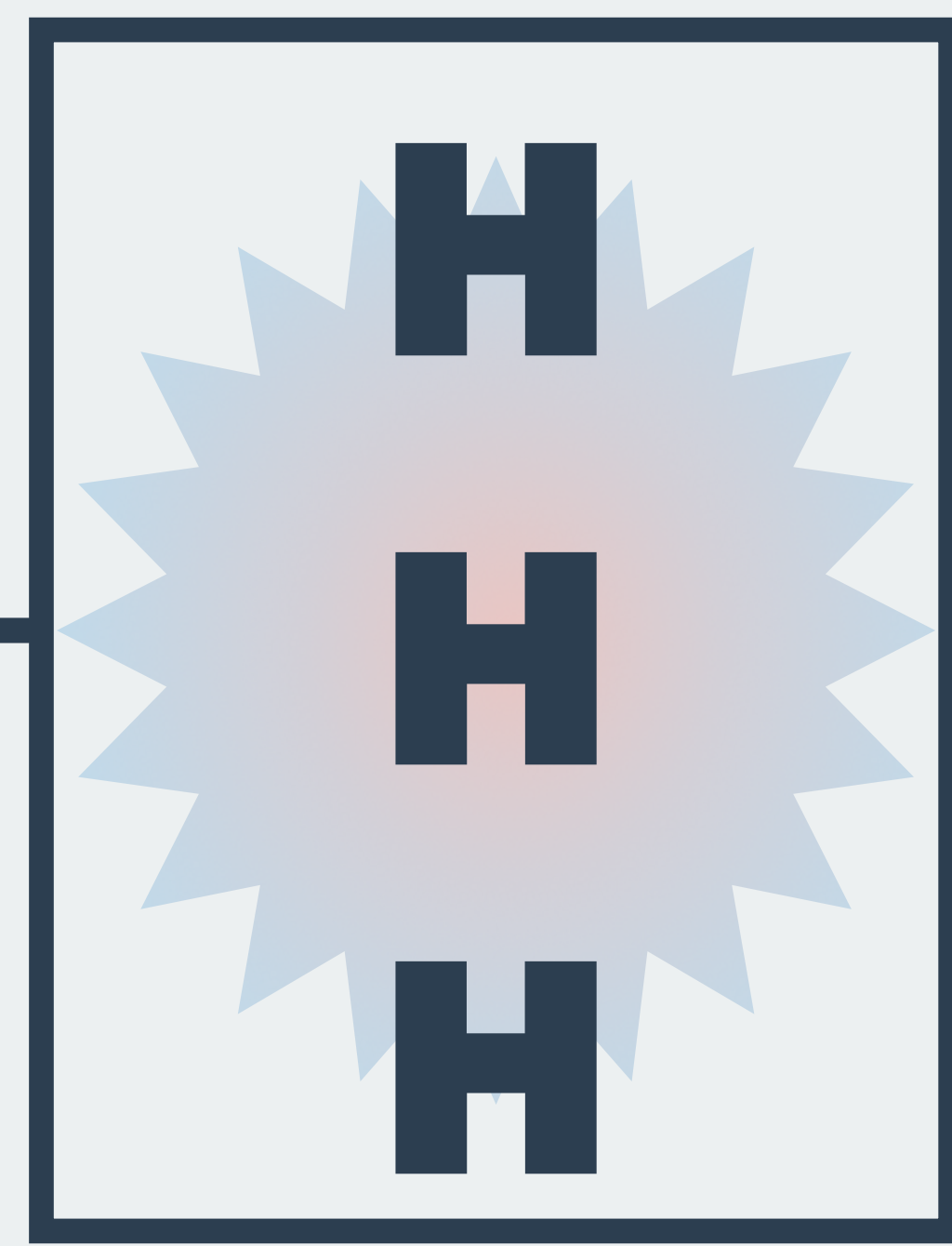
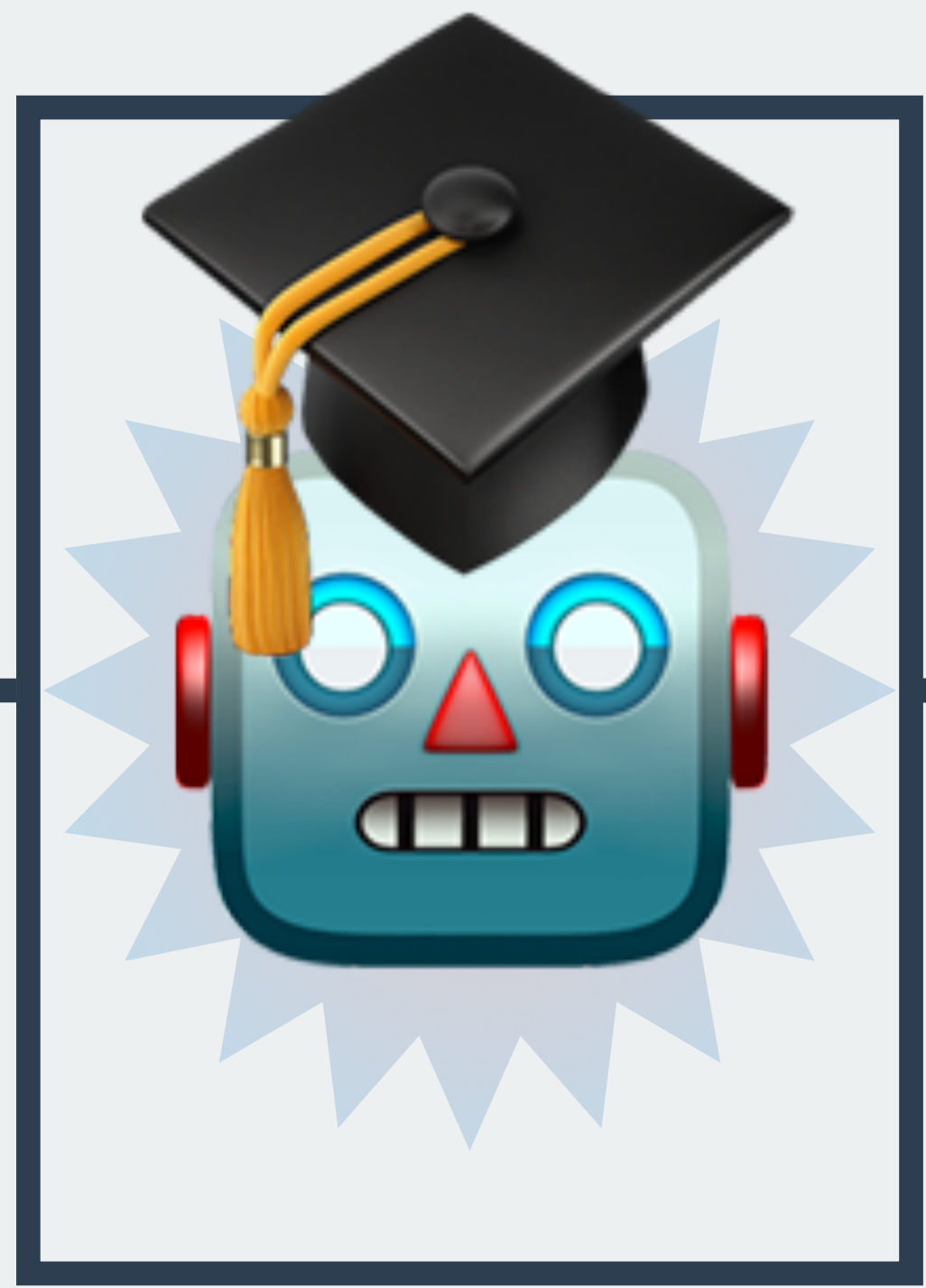
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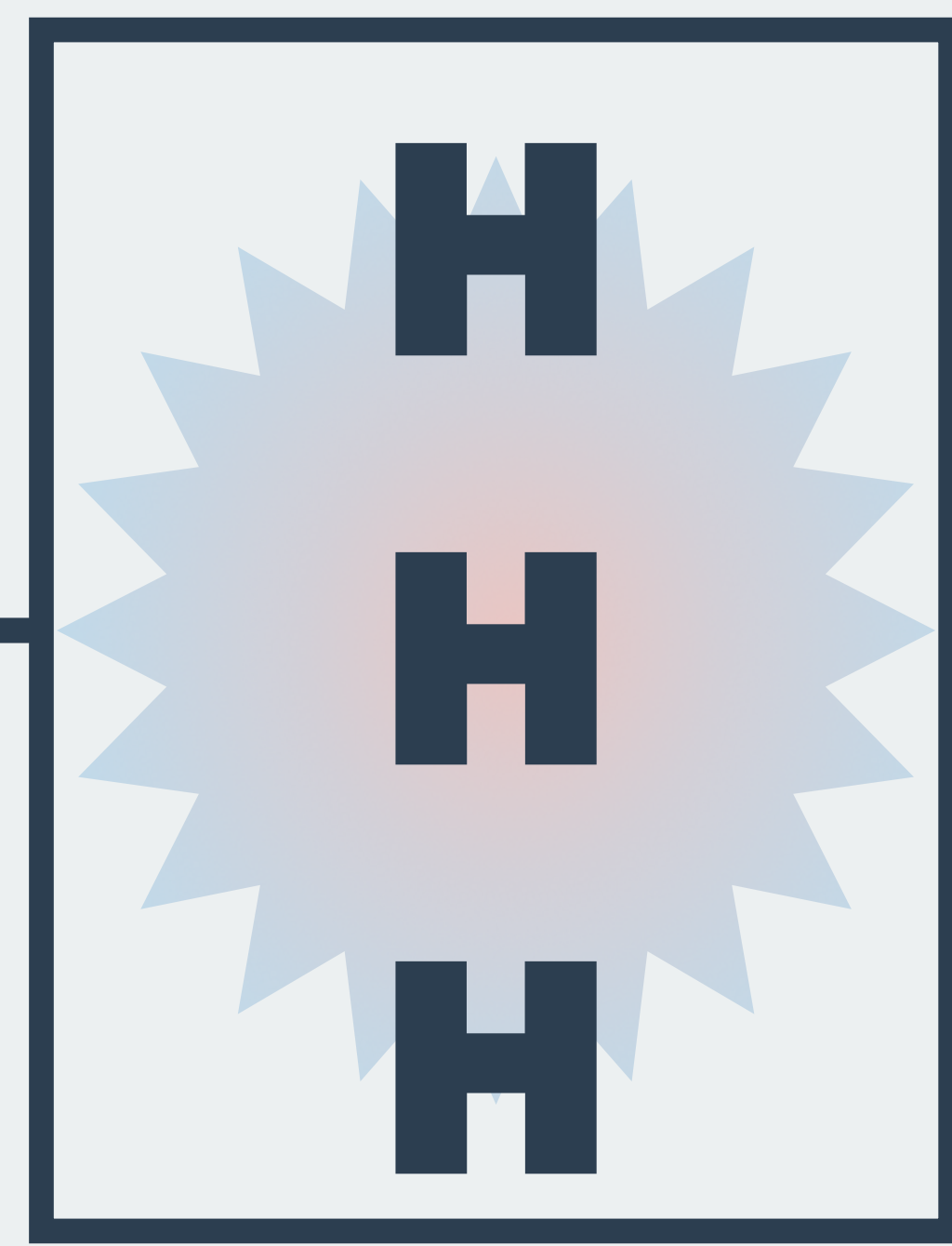
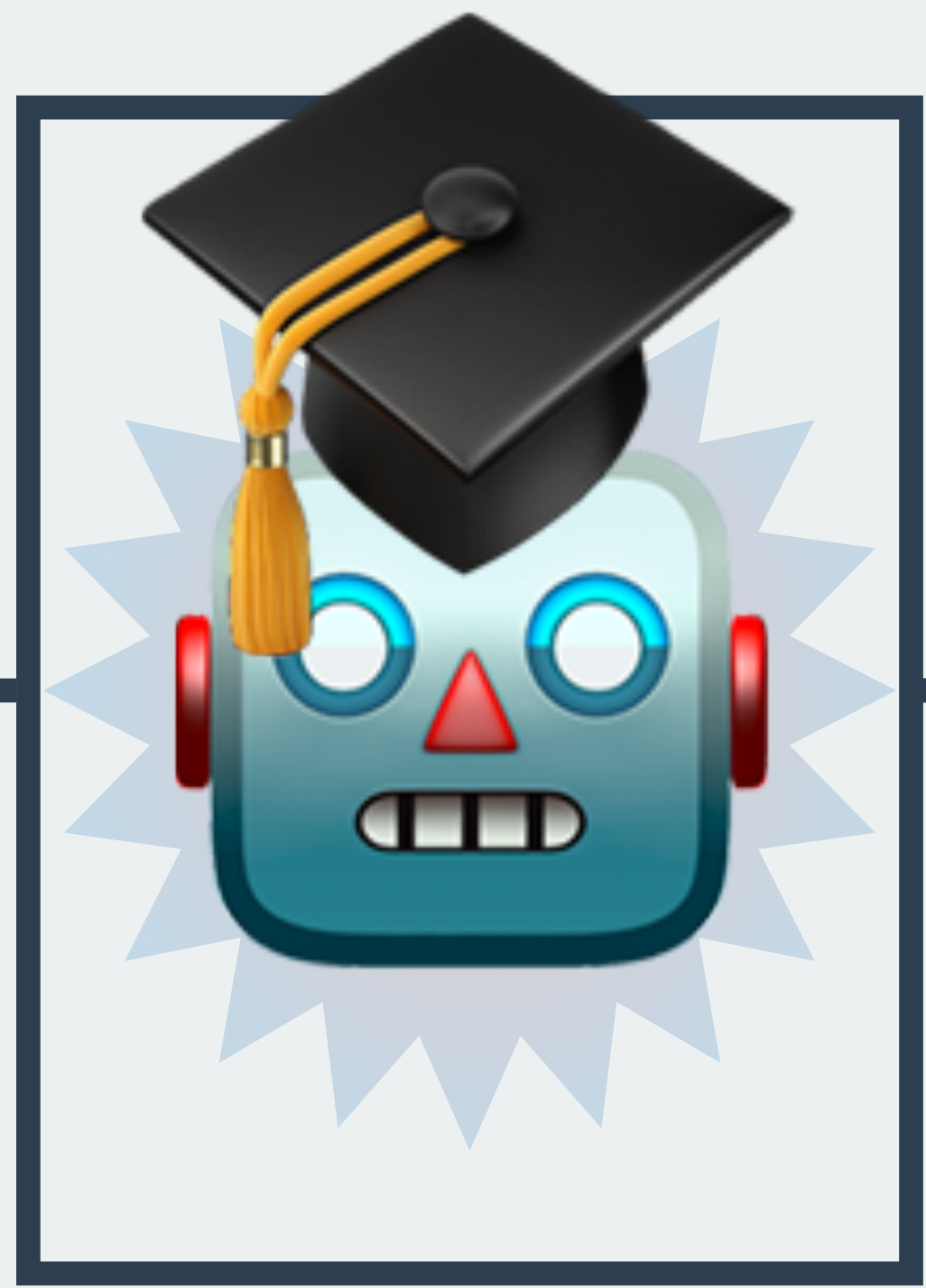


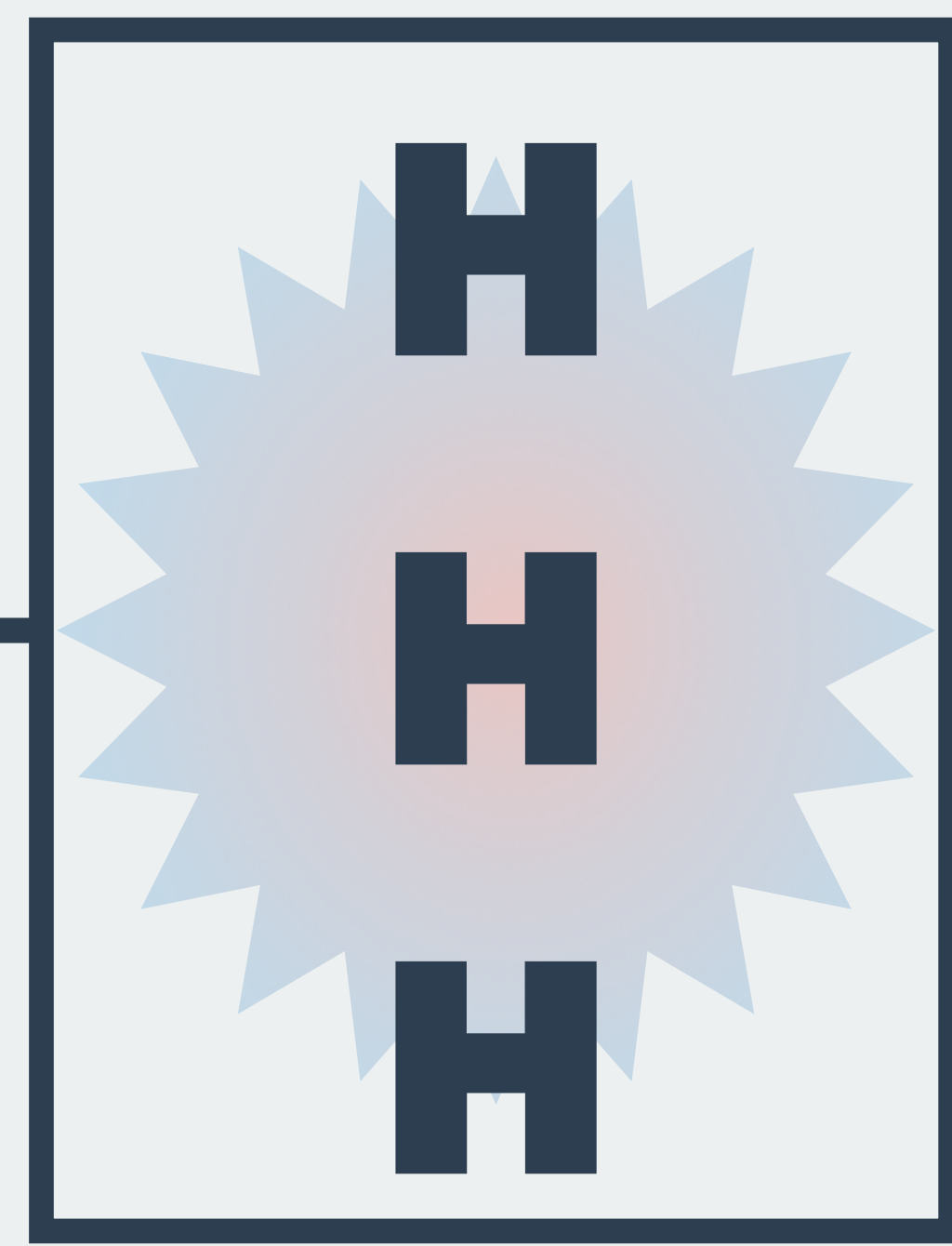
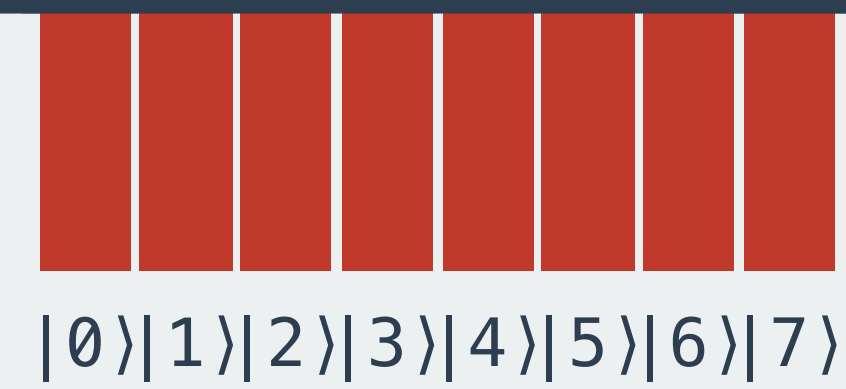
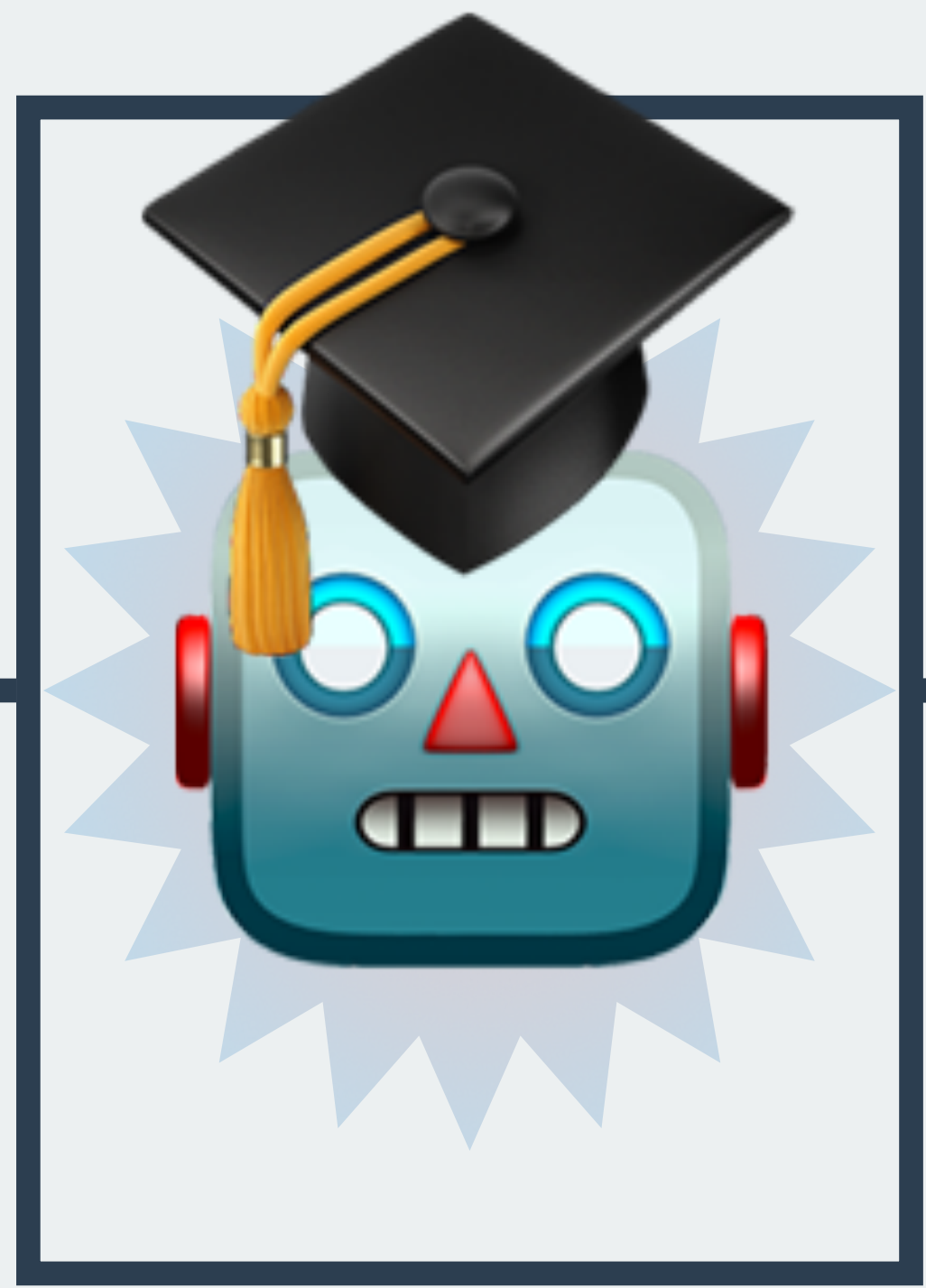


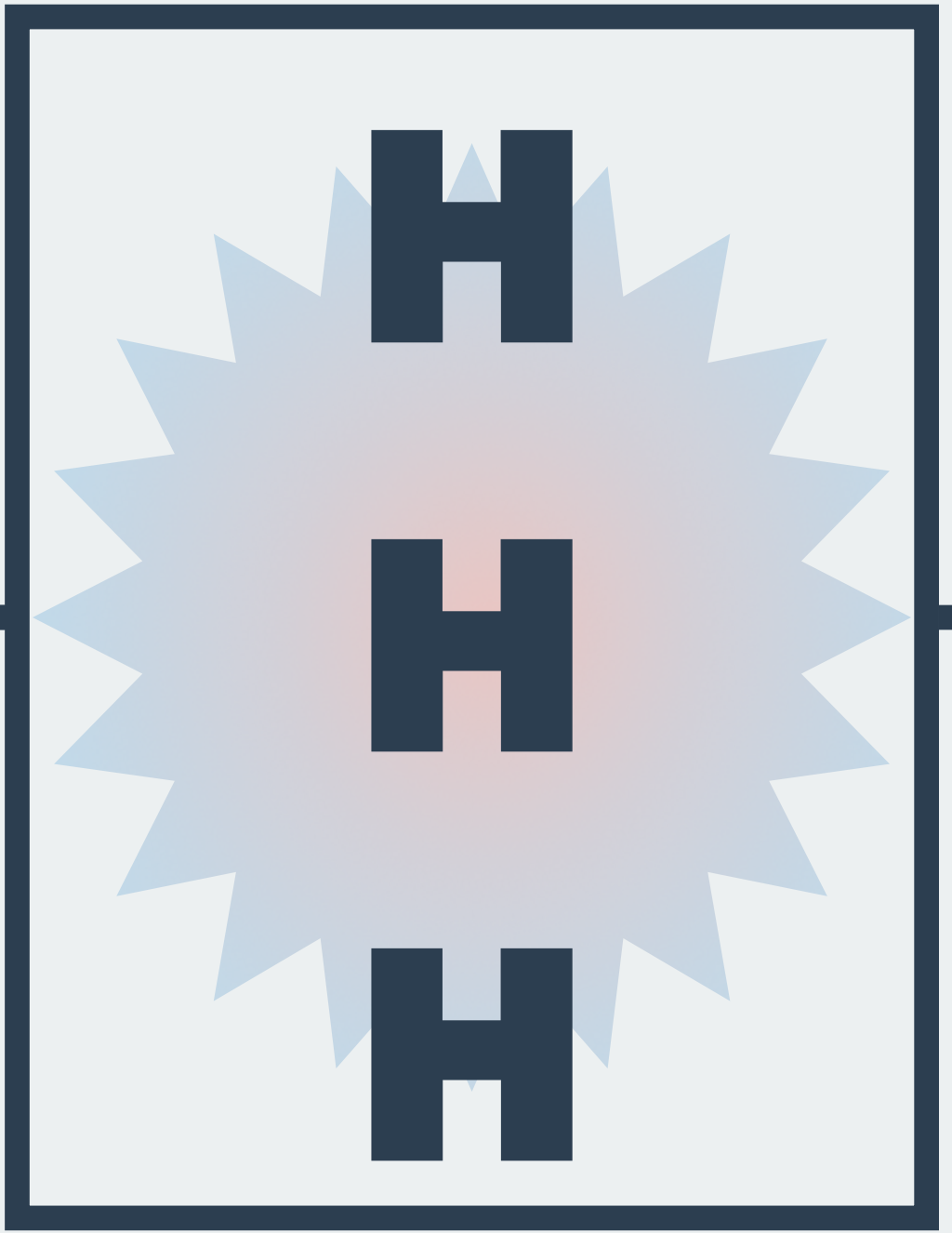
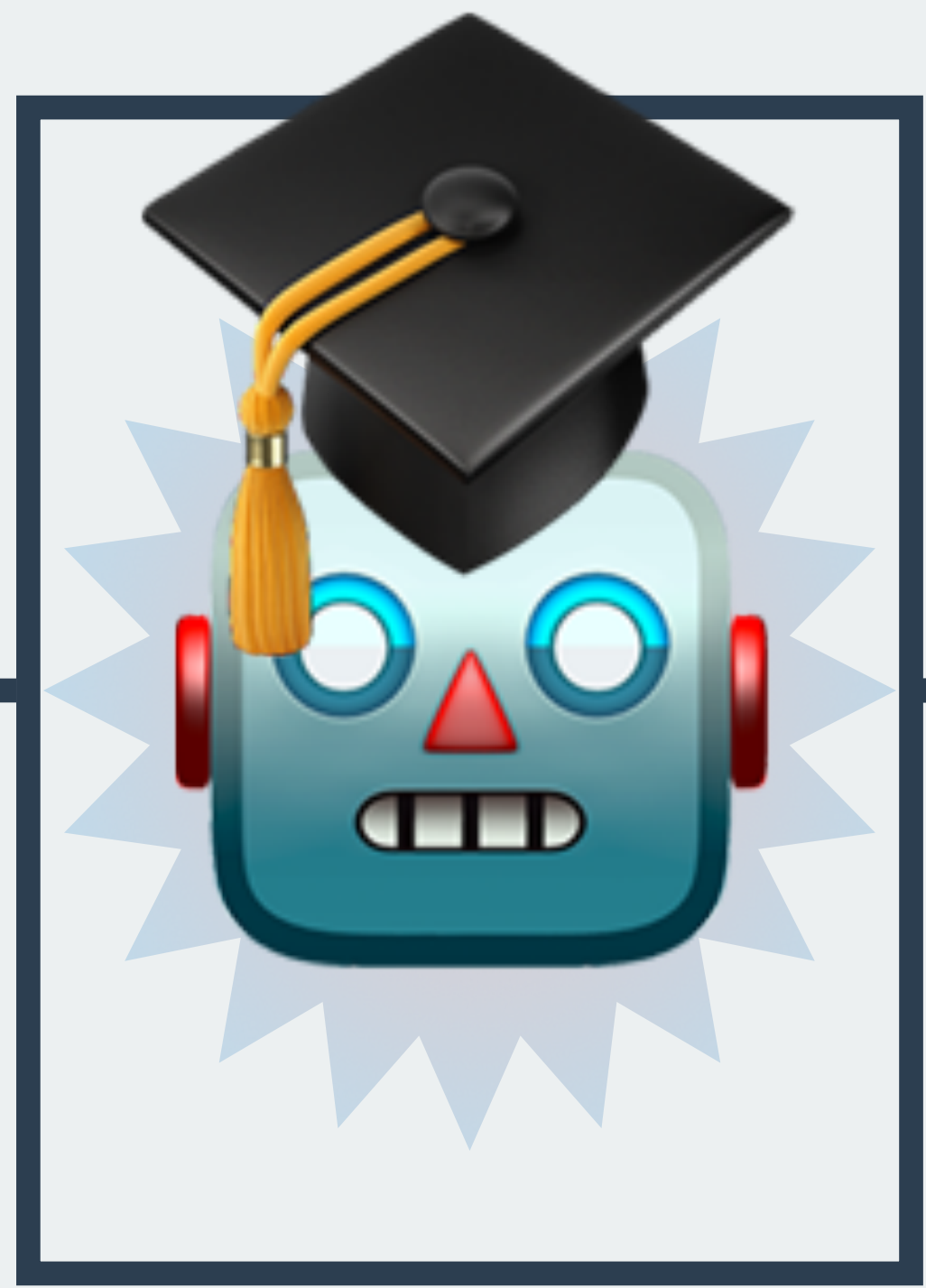


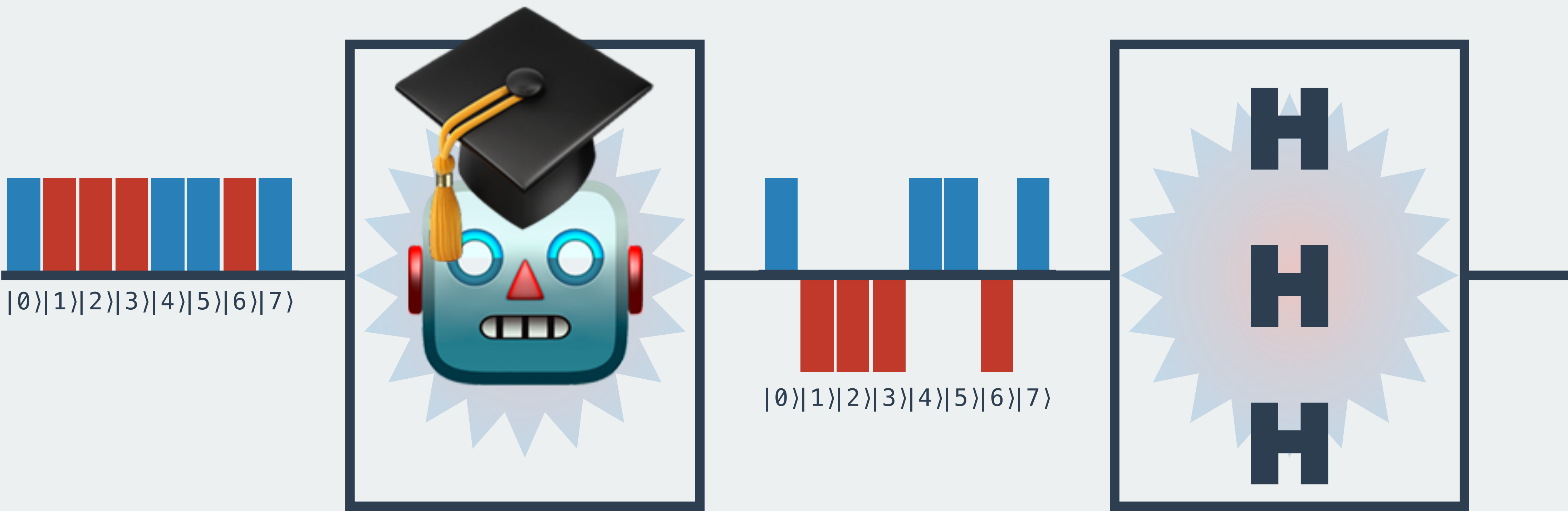
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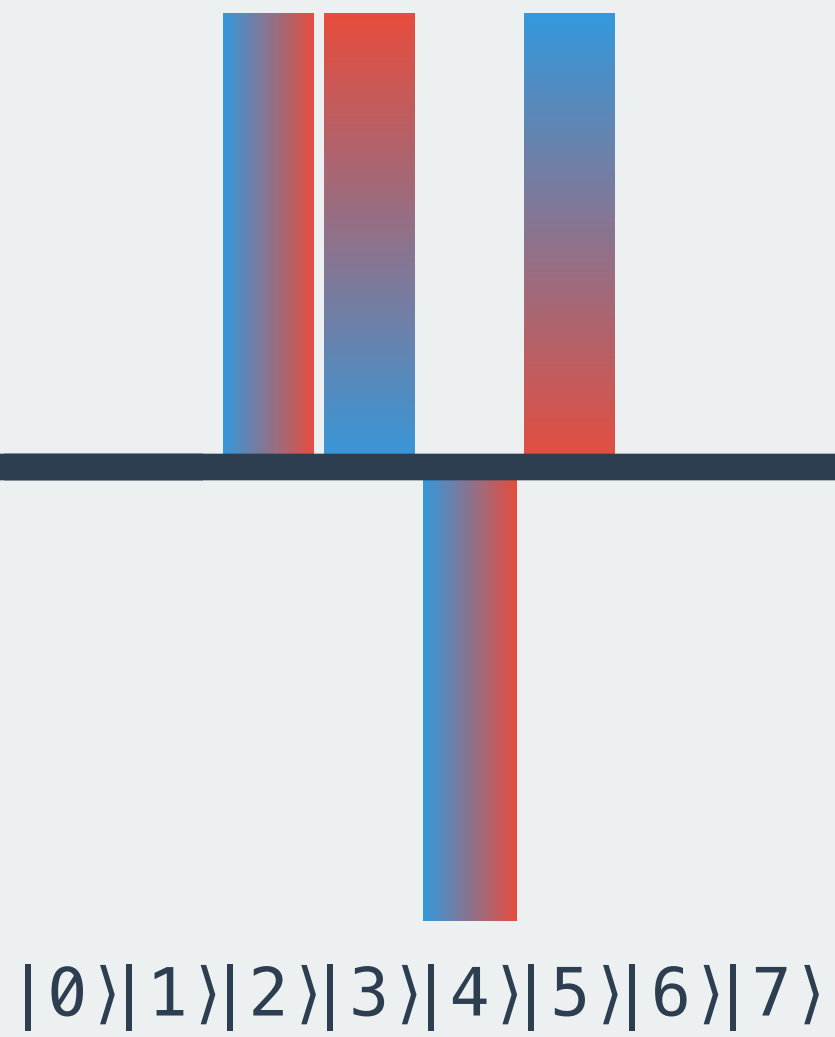
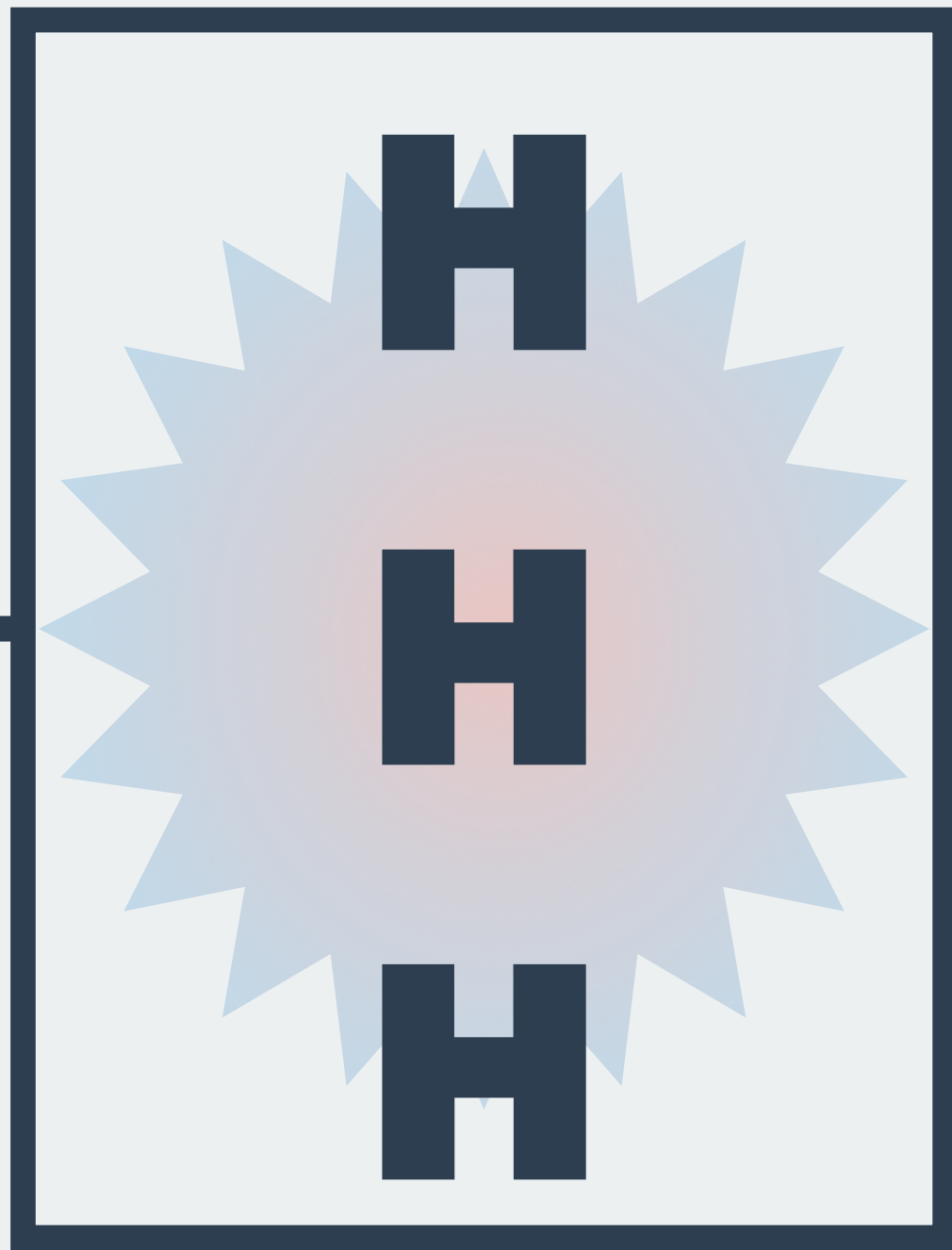
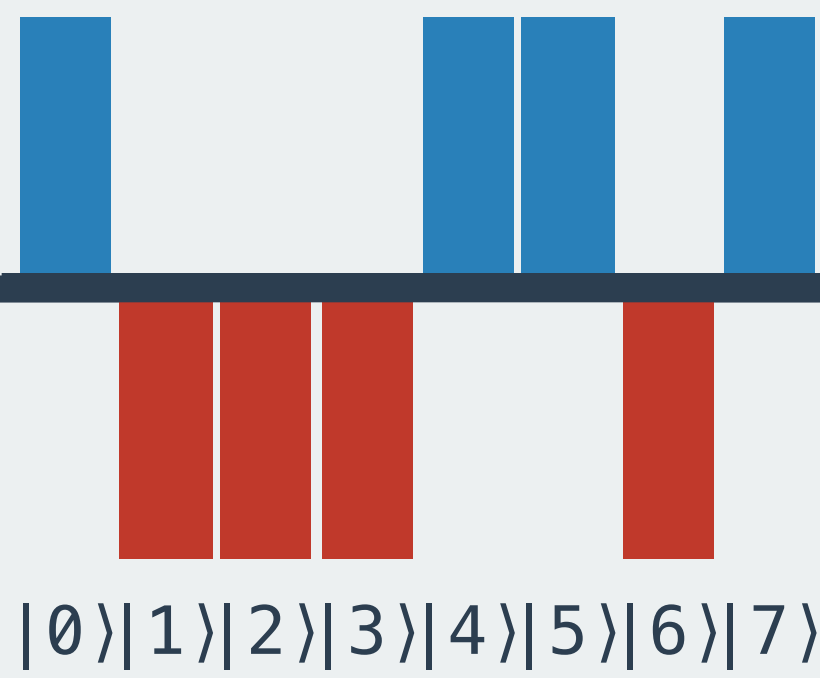
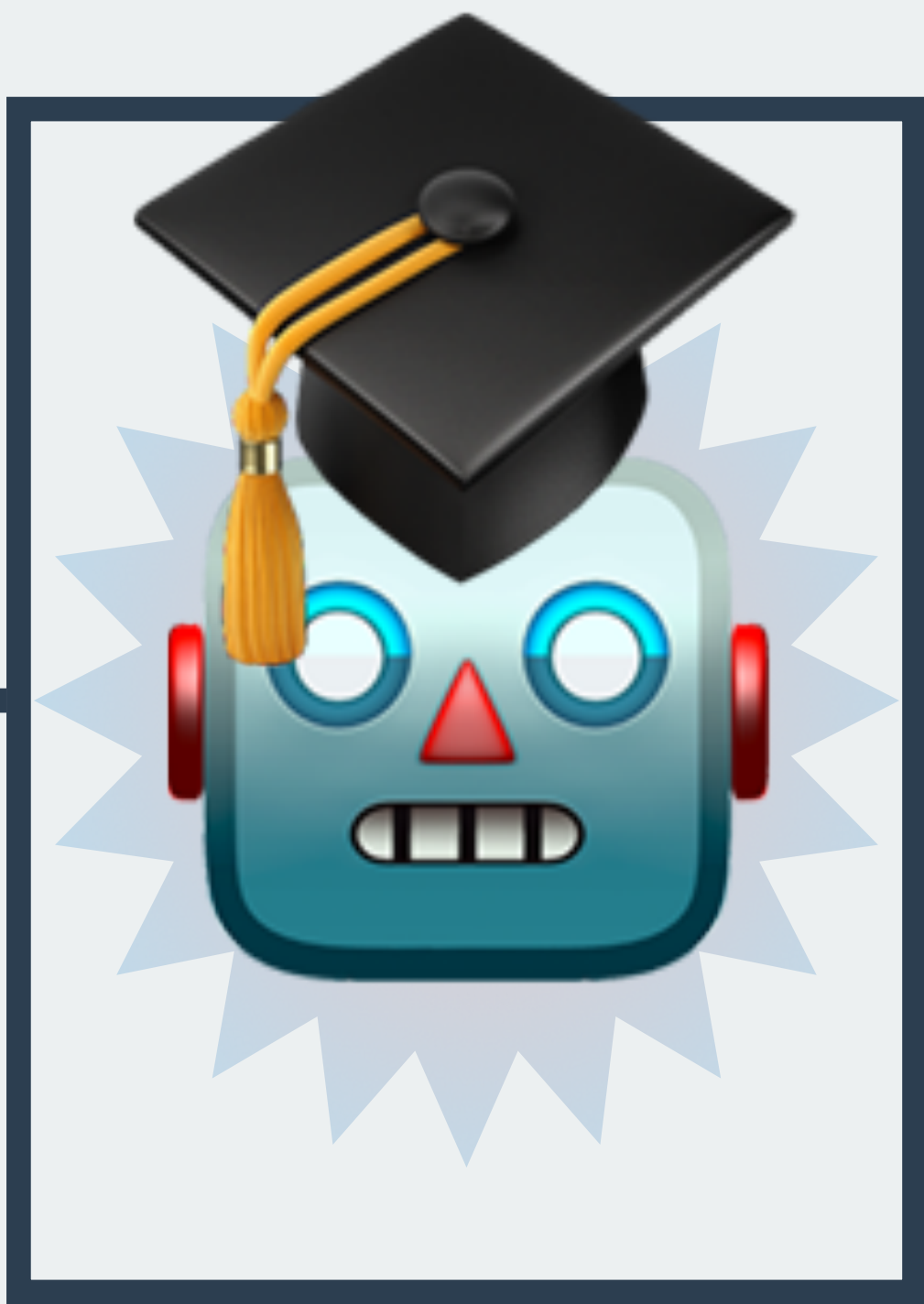
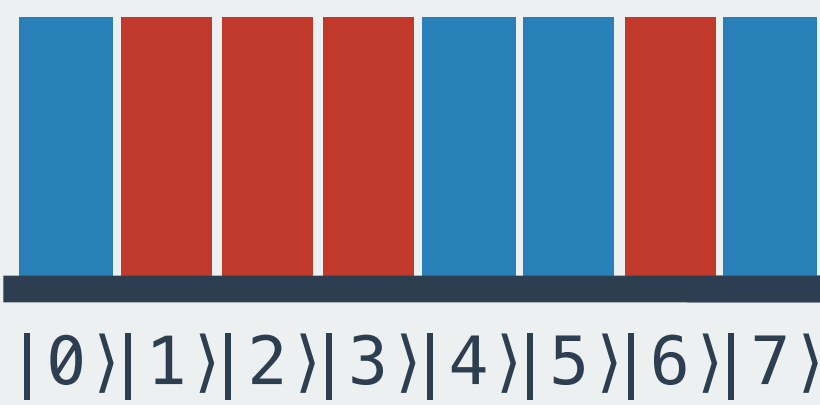










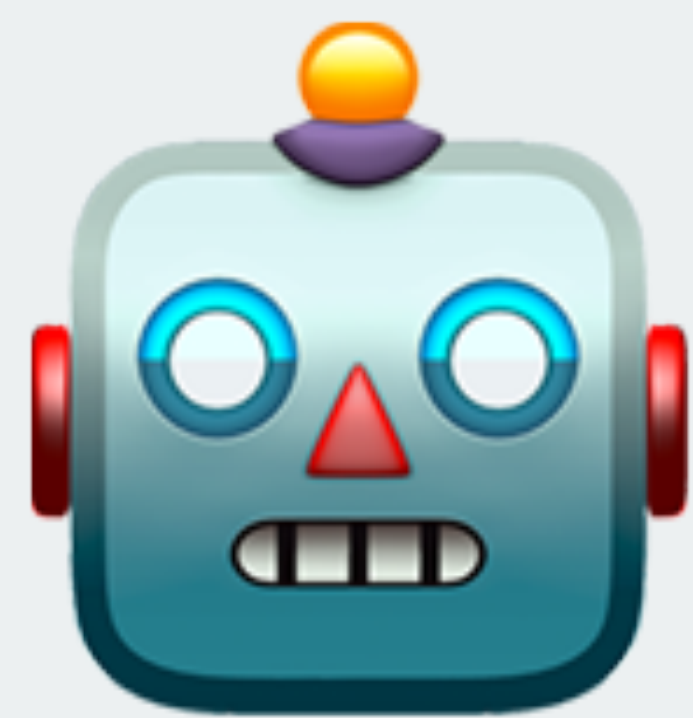


GROVERJEV

ALGORITEM

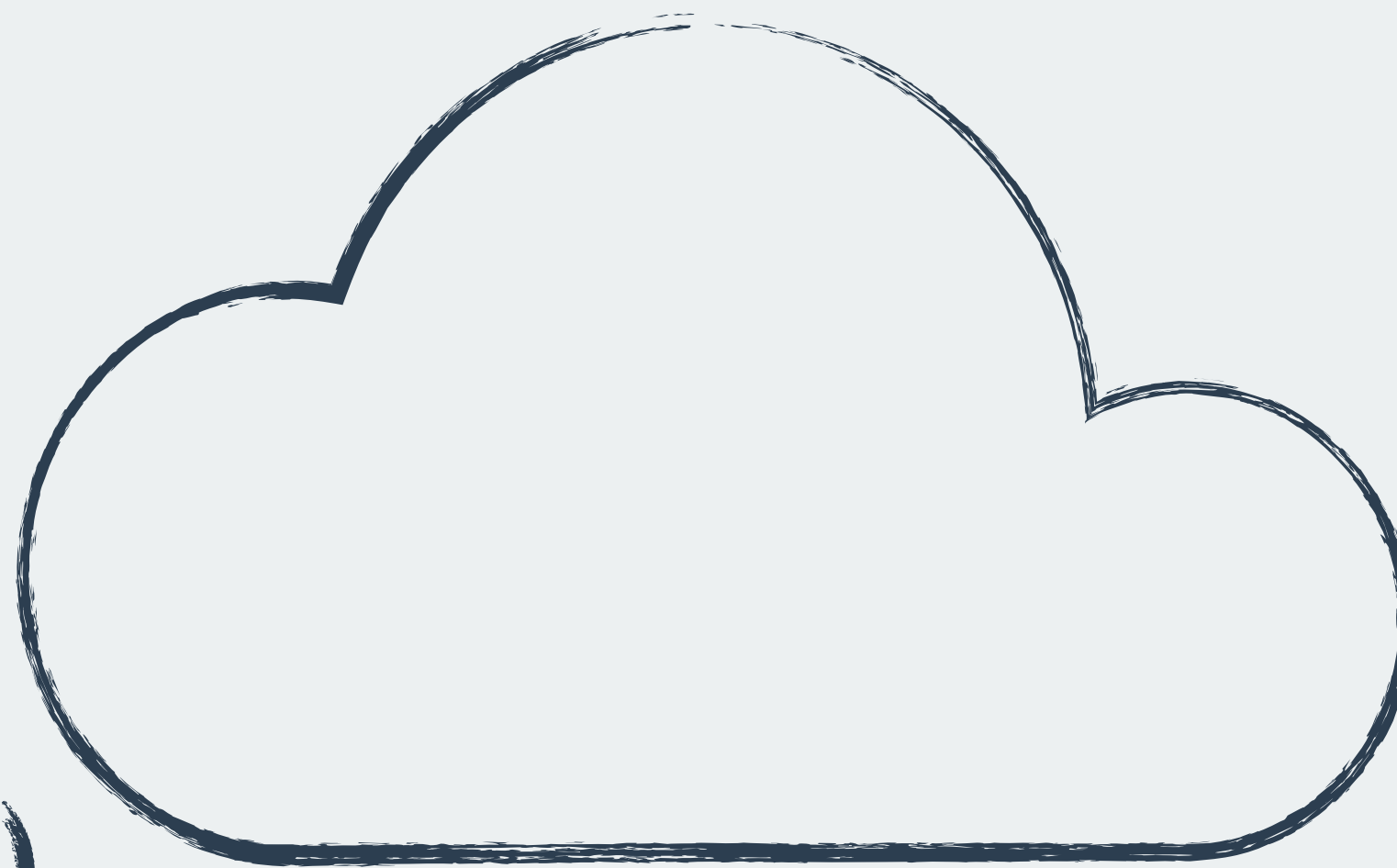
Lov Grover, 1961-





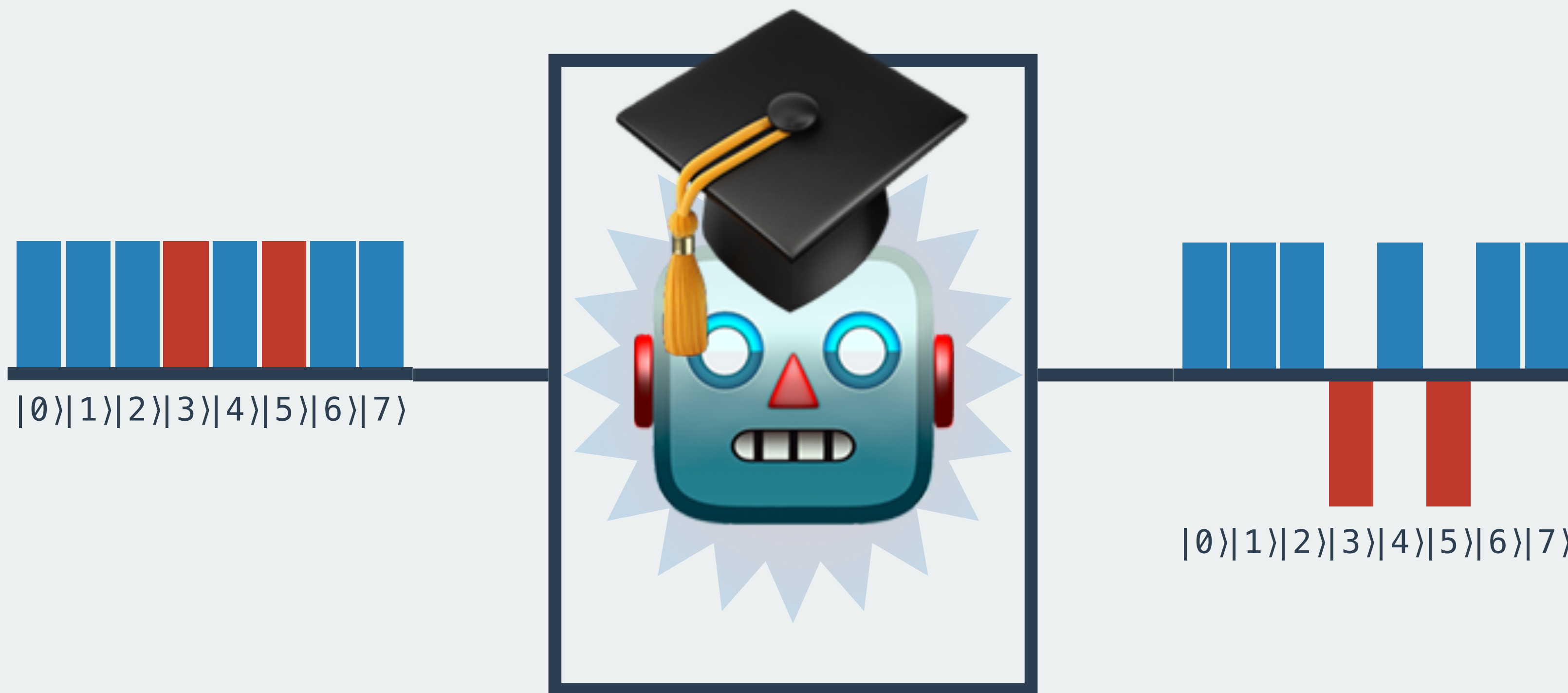
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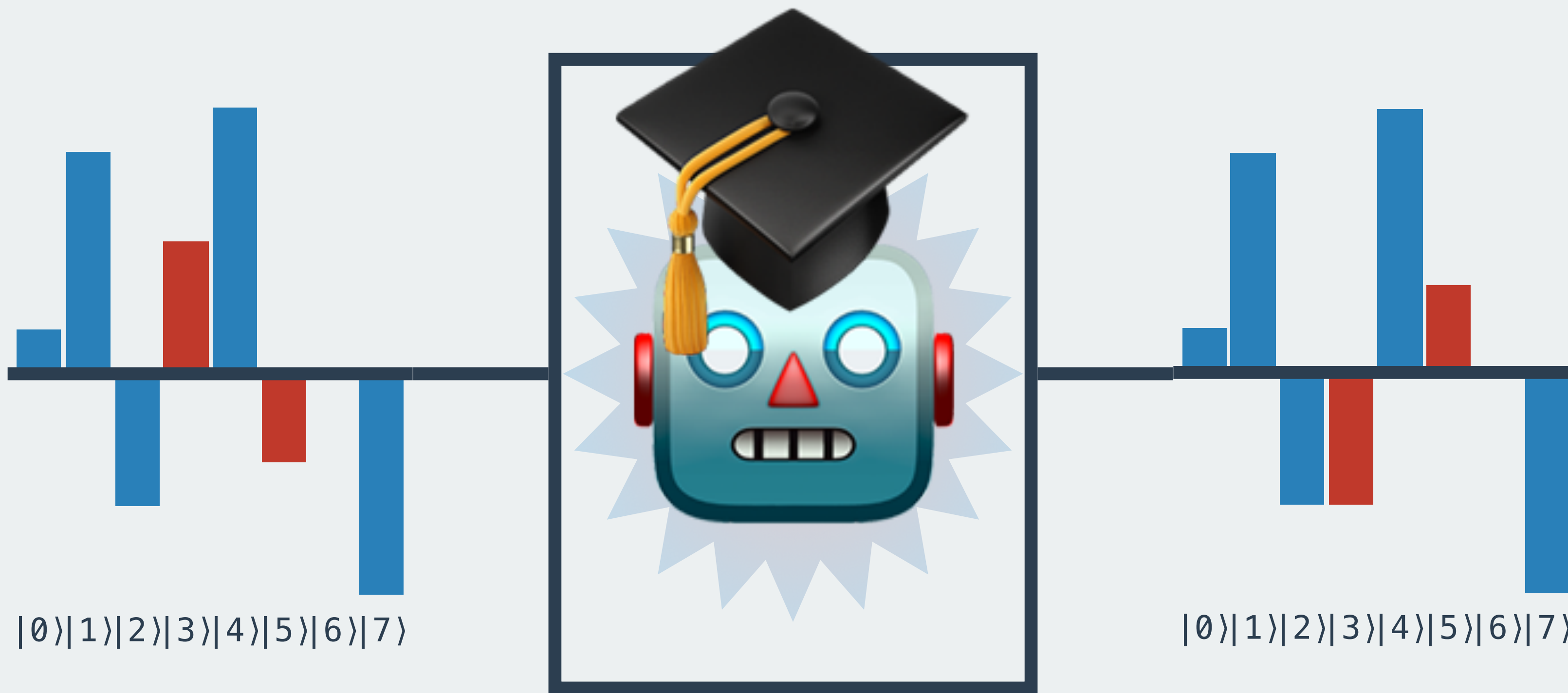


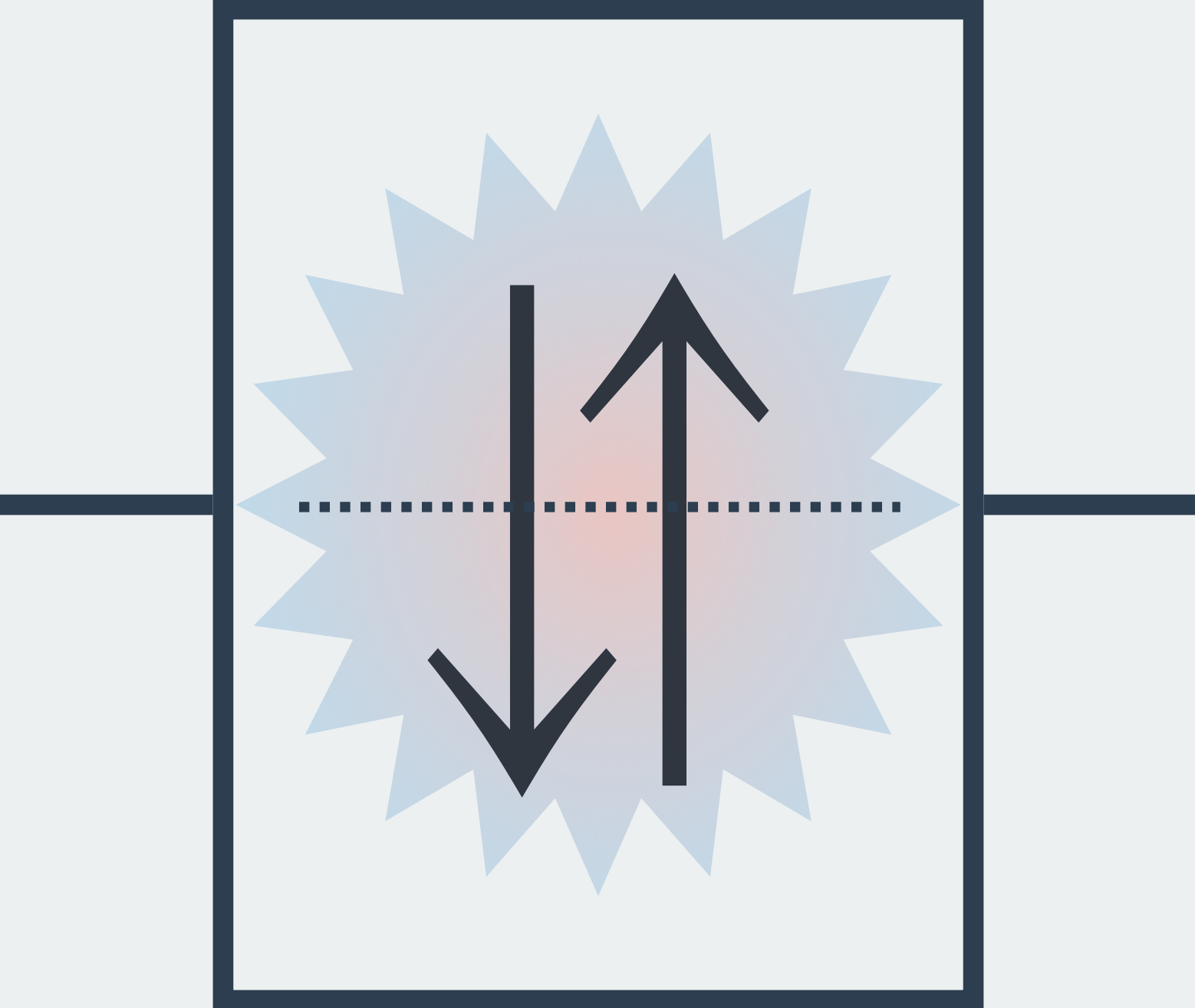


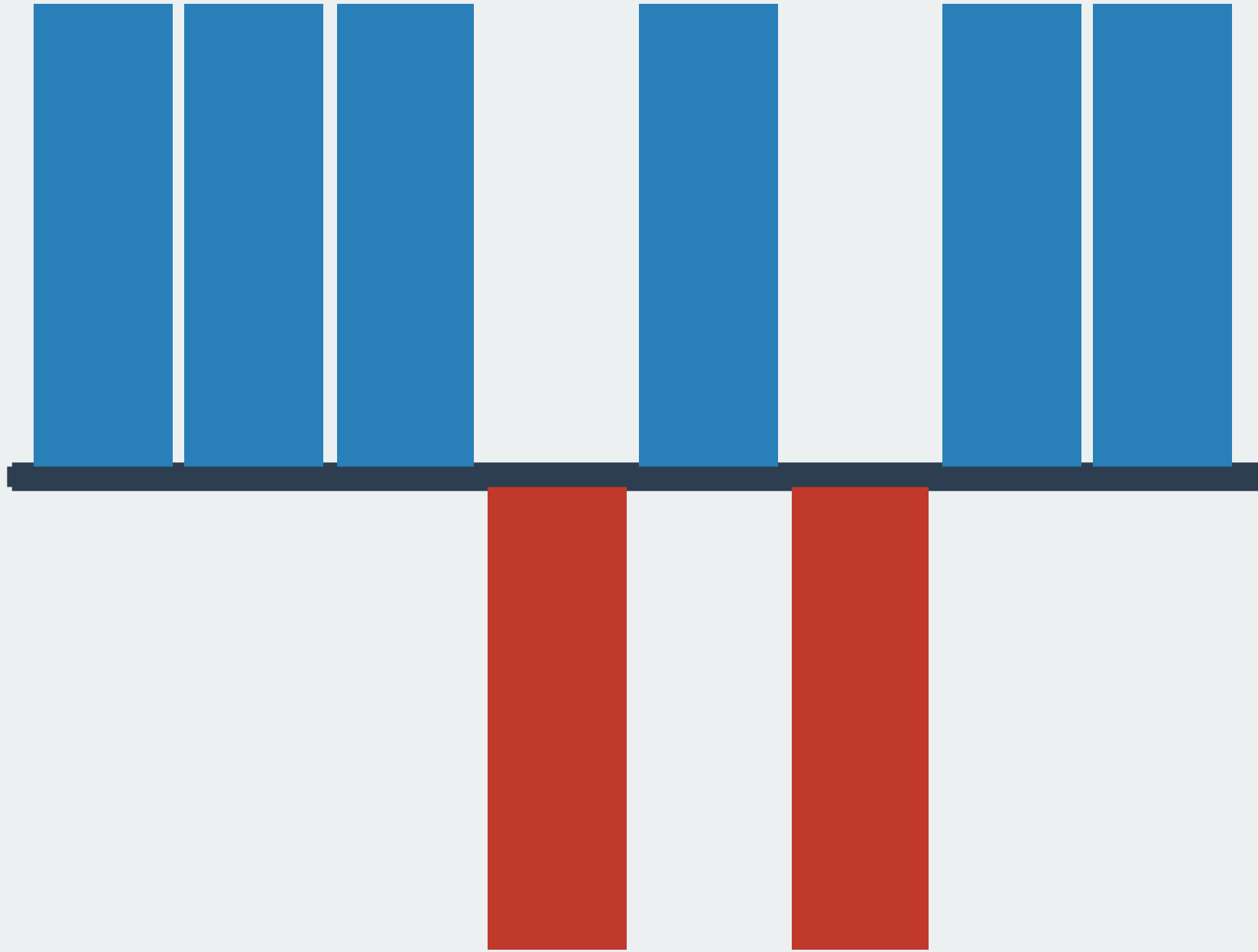
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24398732981279?

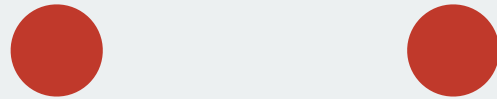
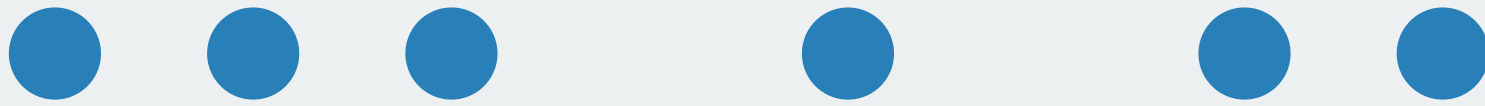


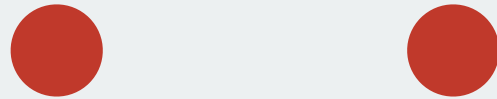
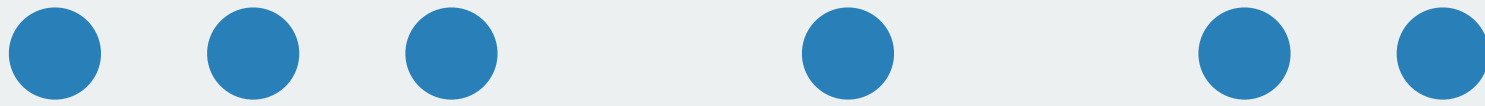


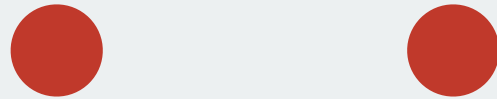
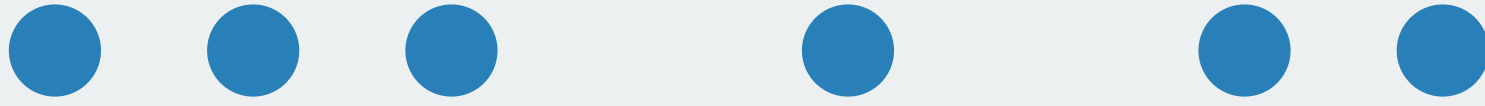


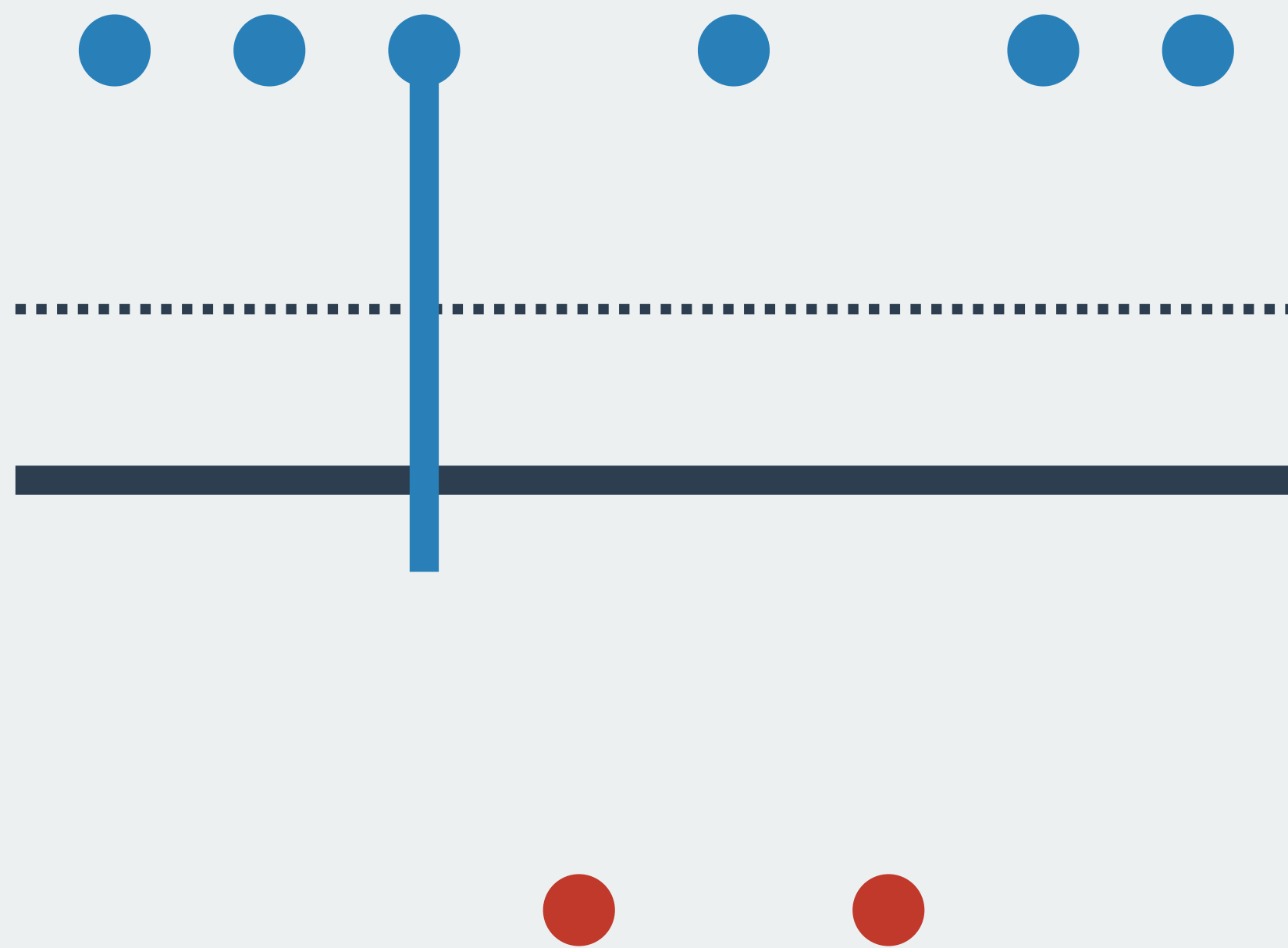


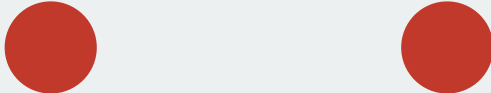
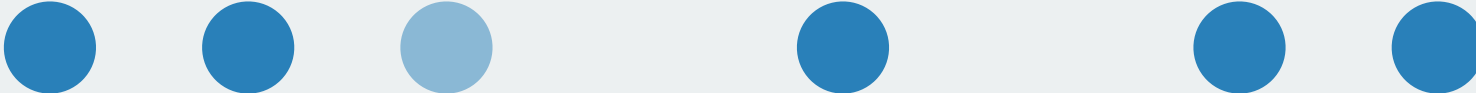


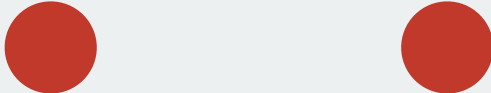
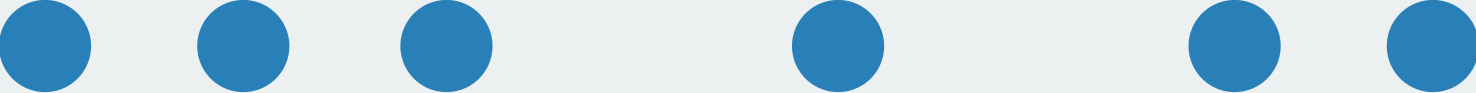
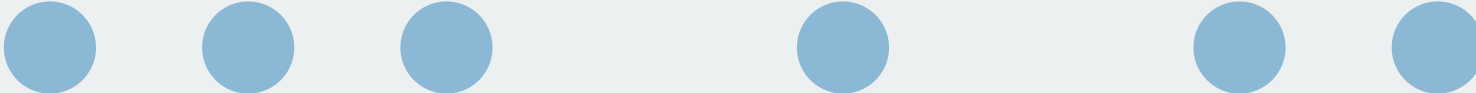


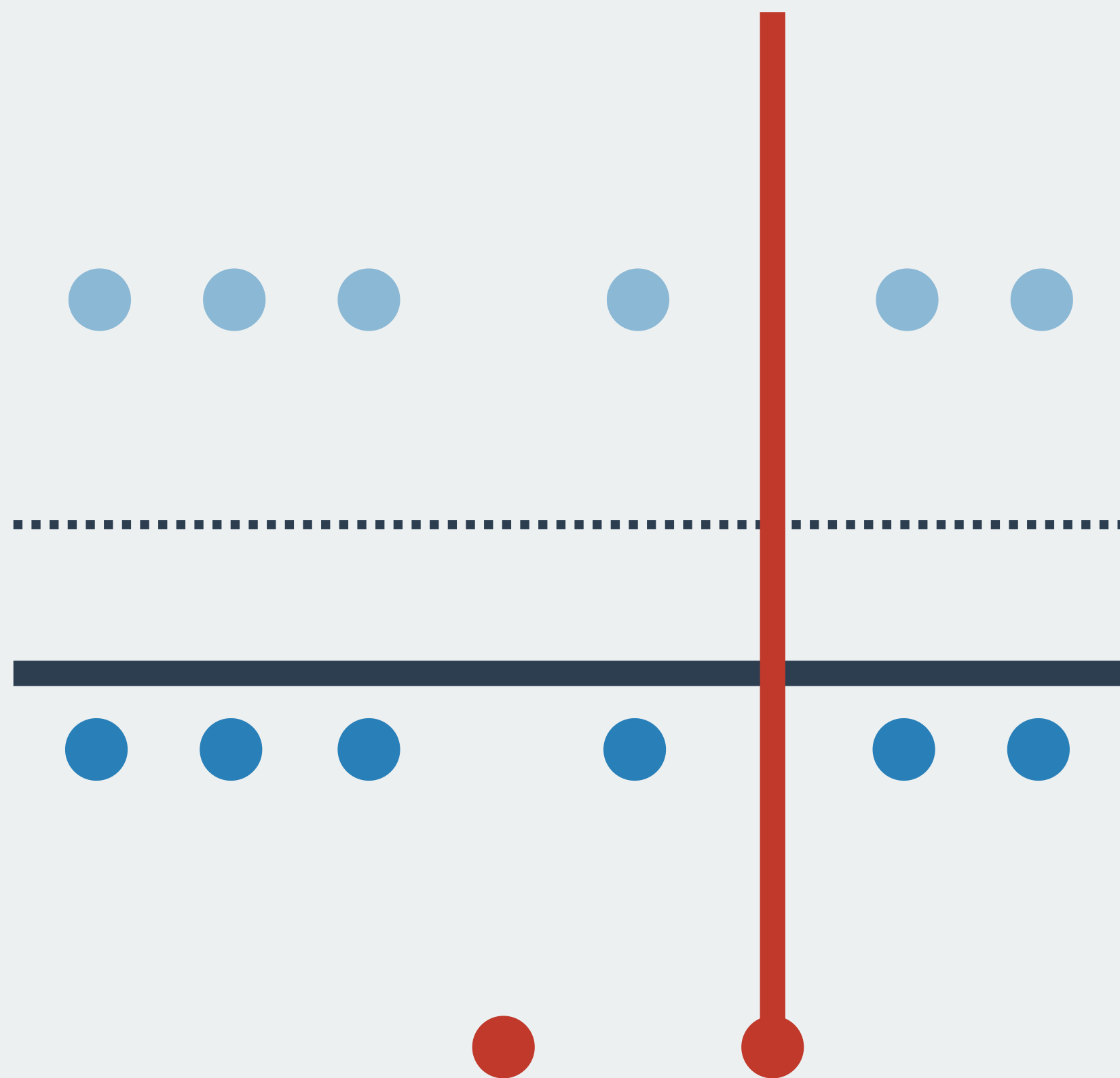


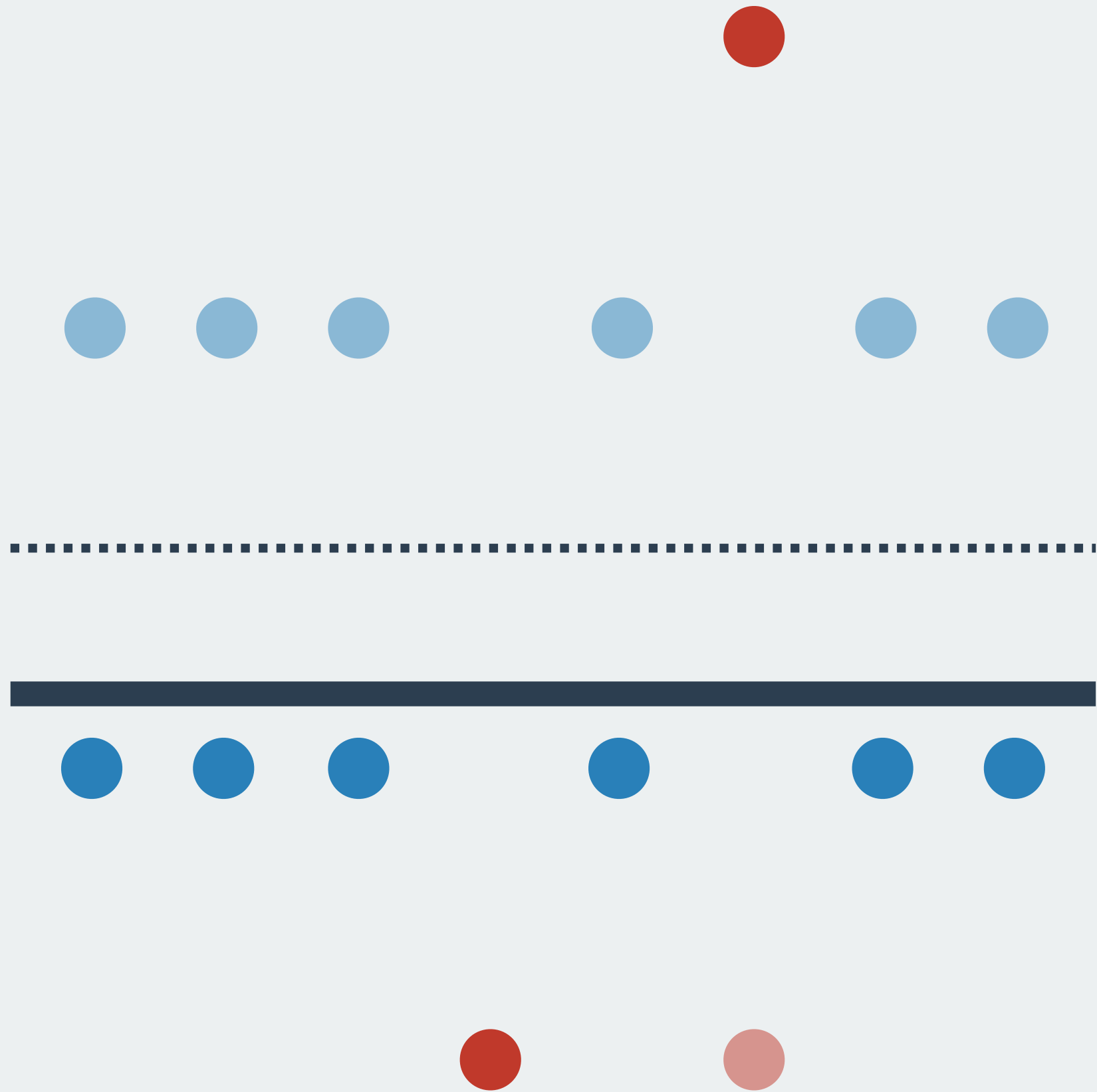


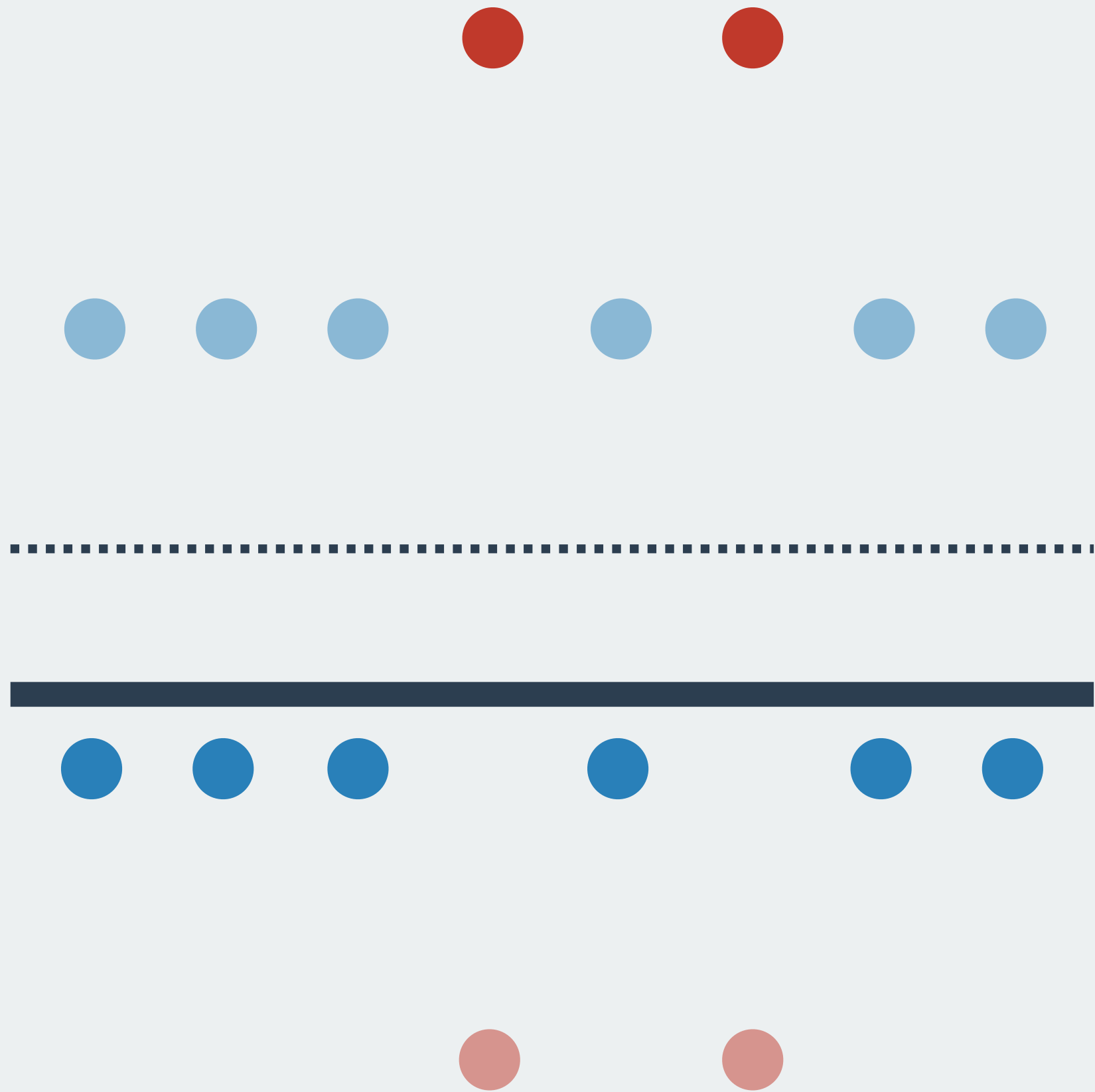


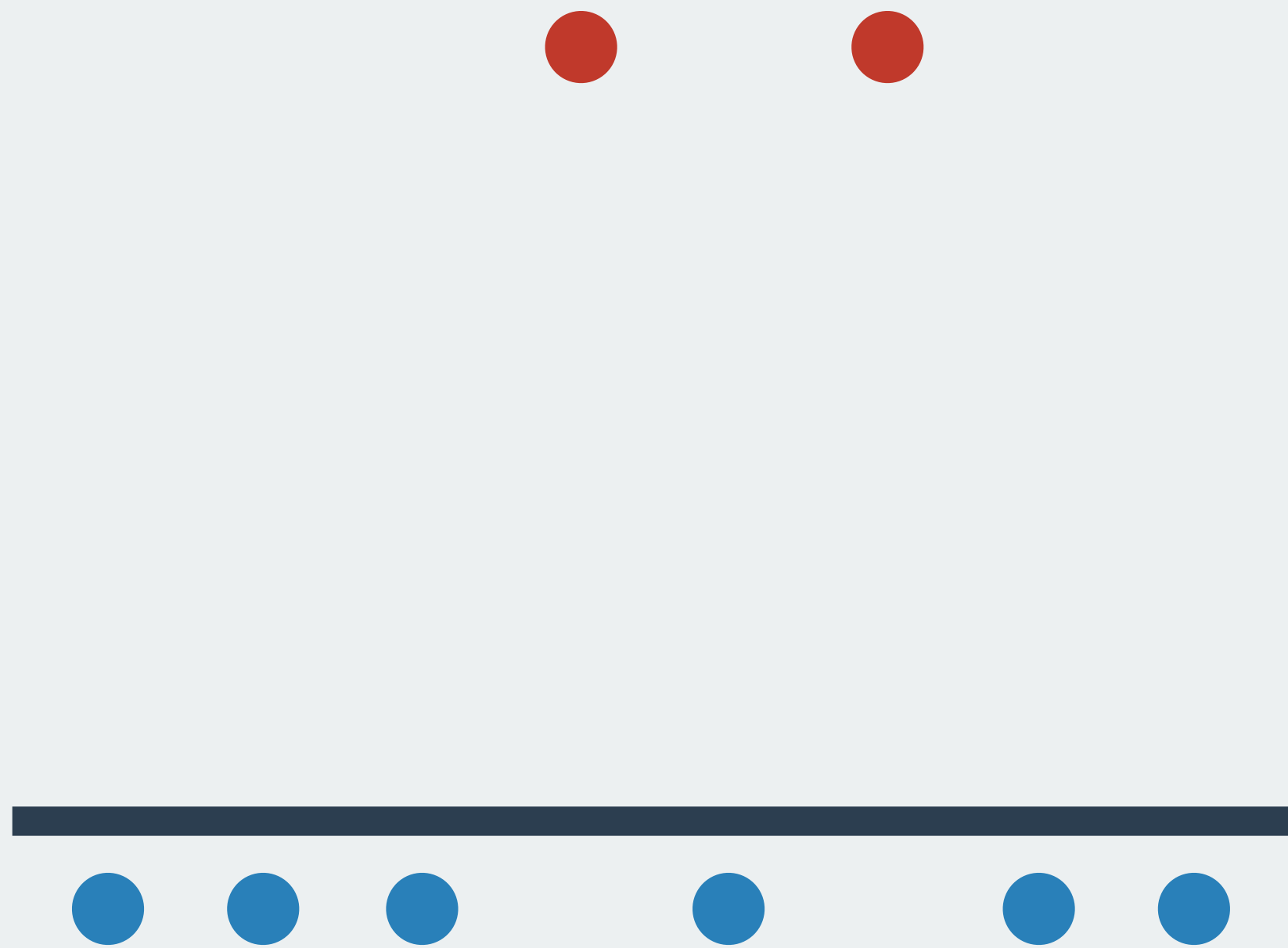


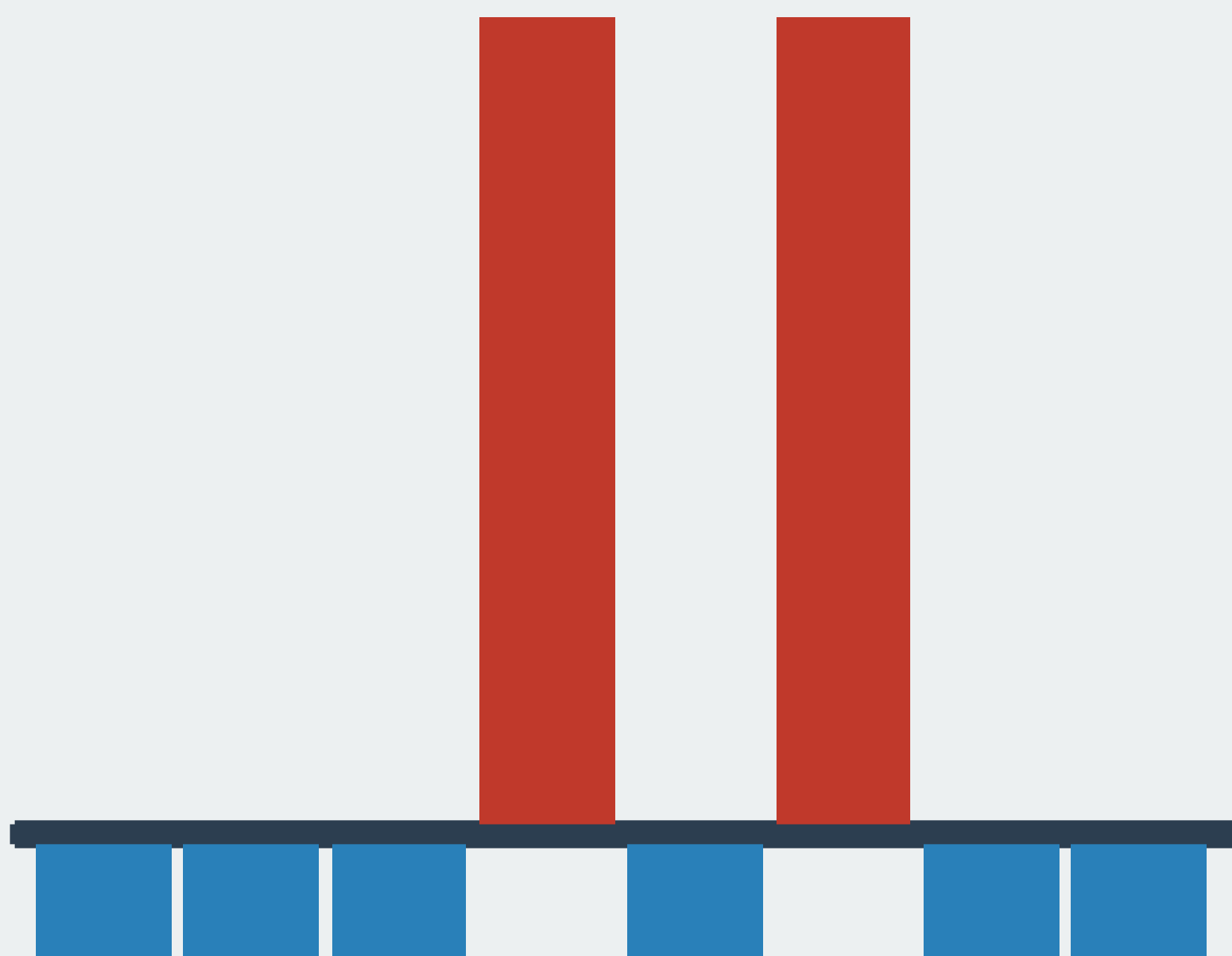


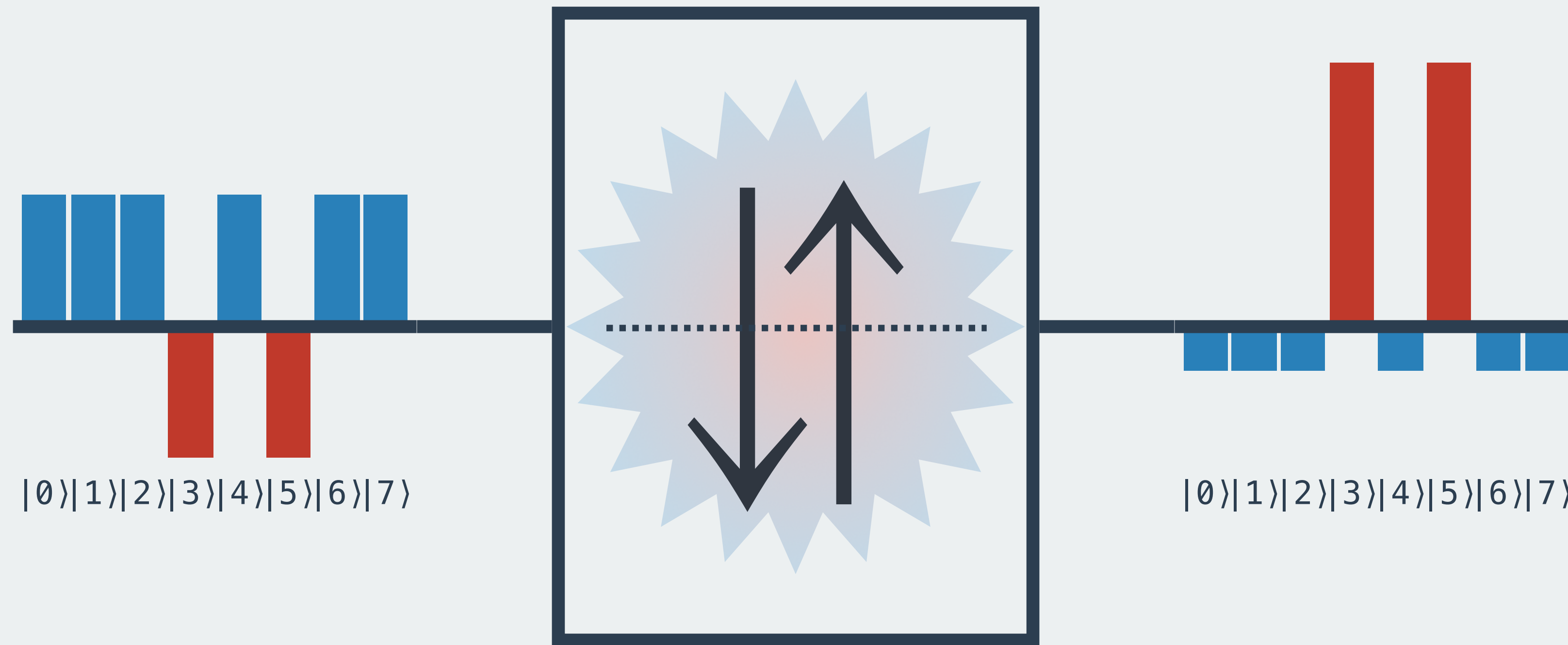


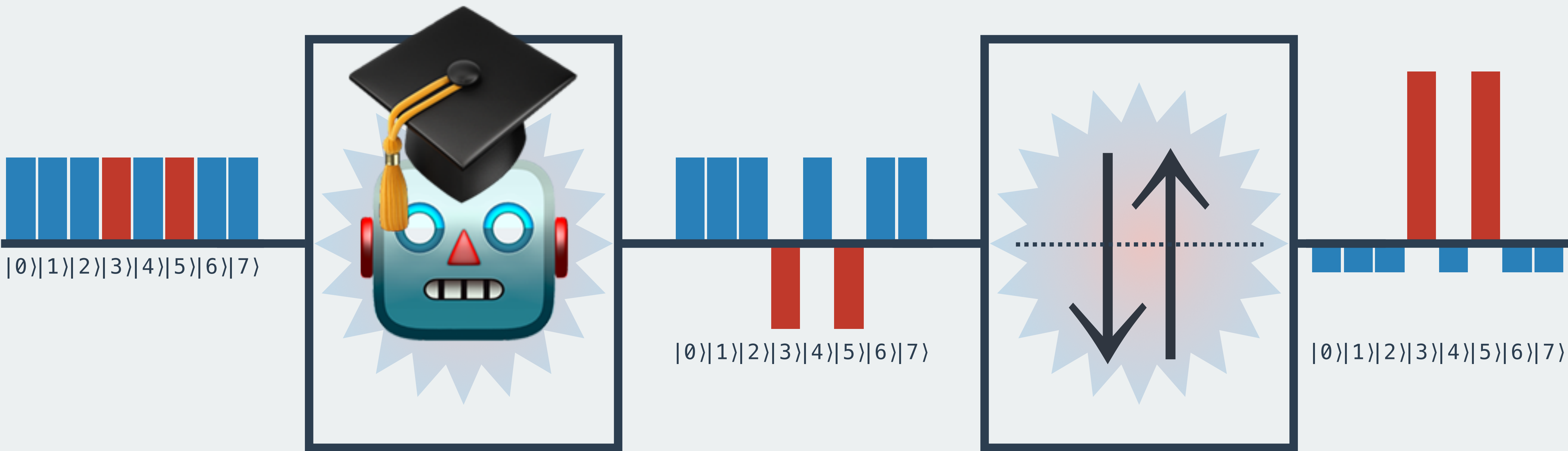


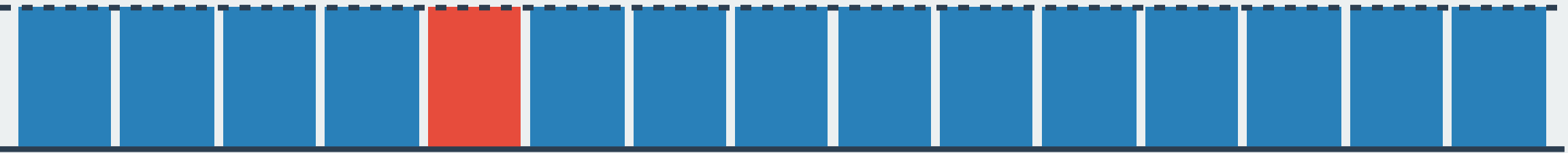


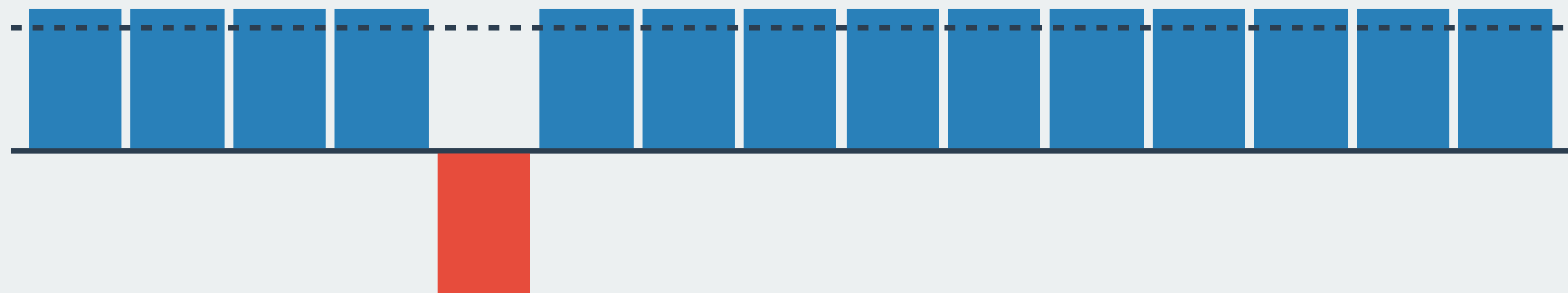


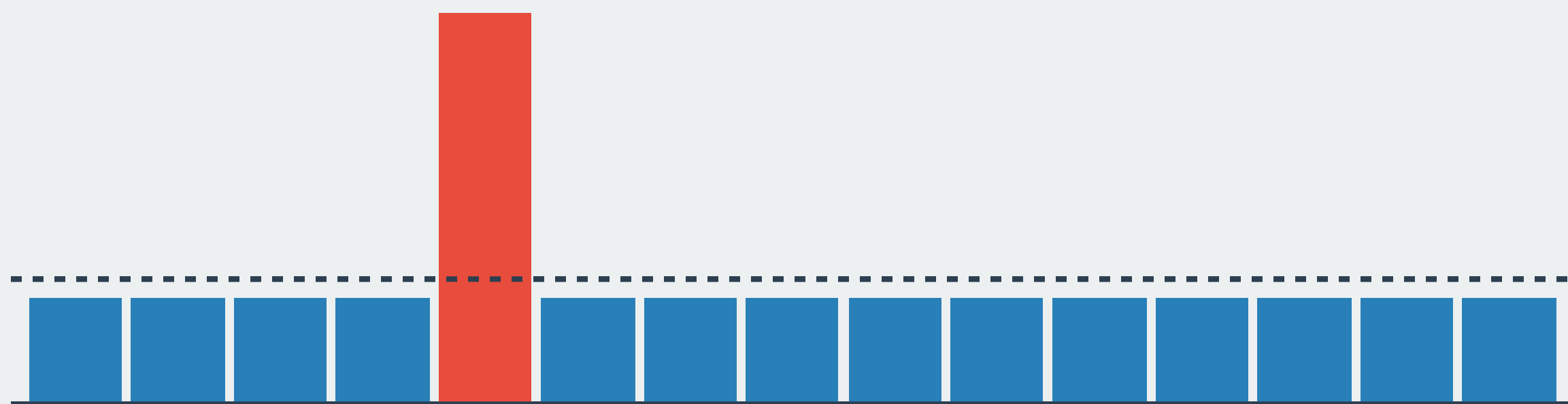


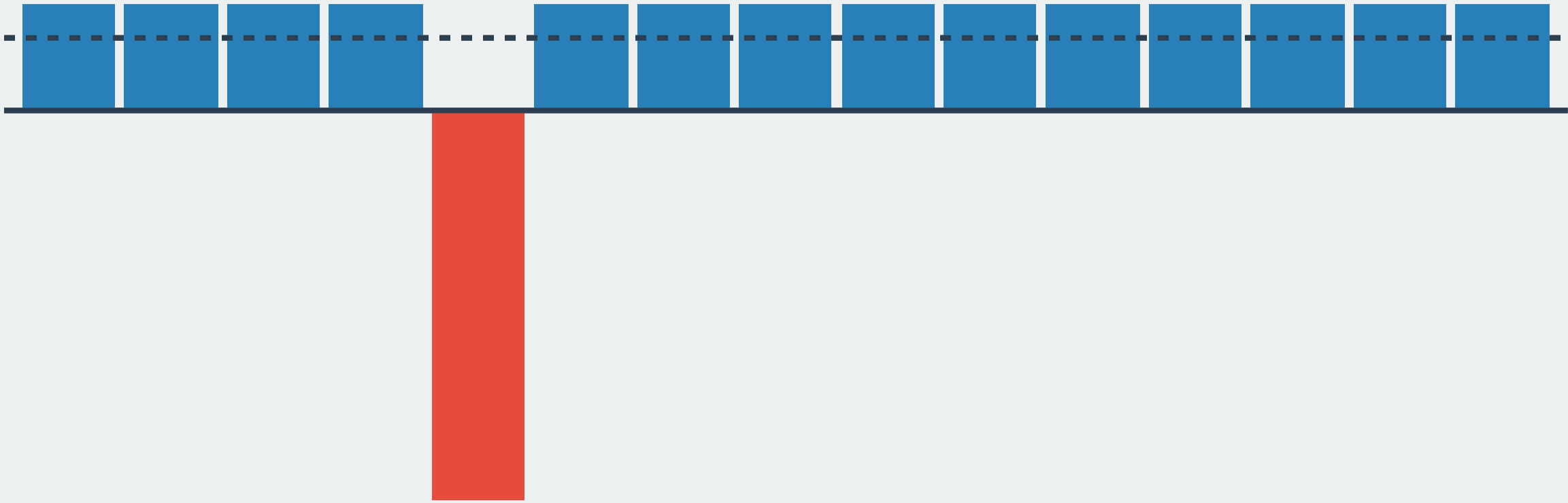




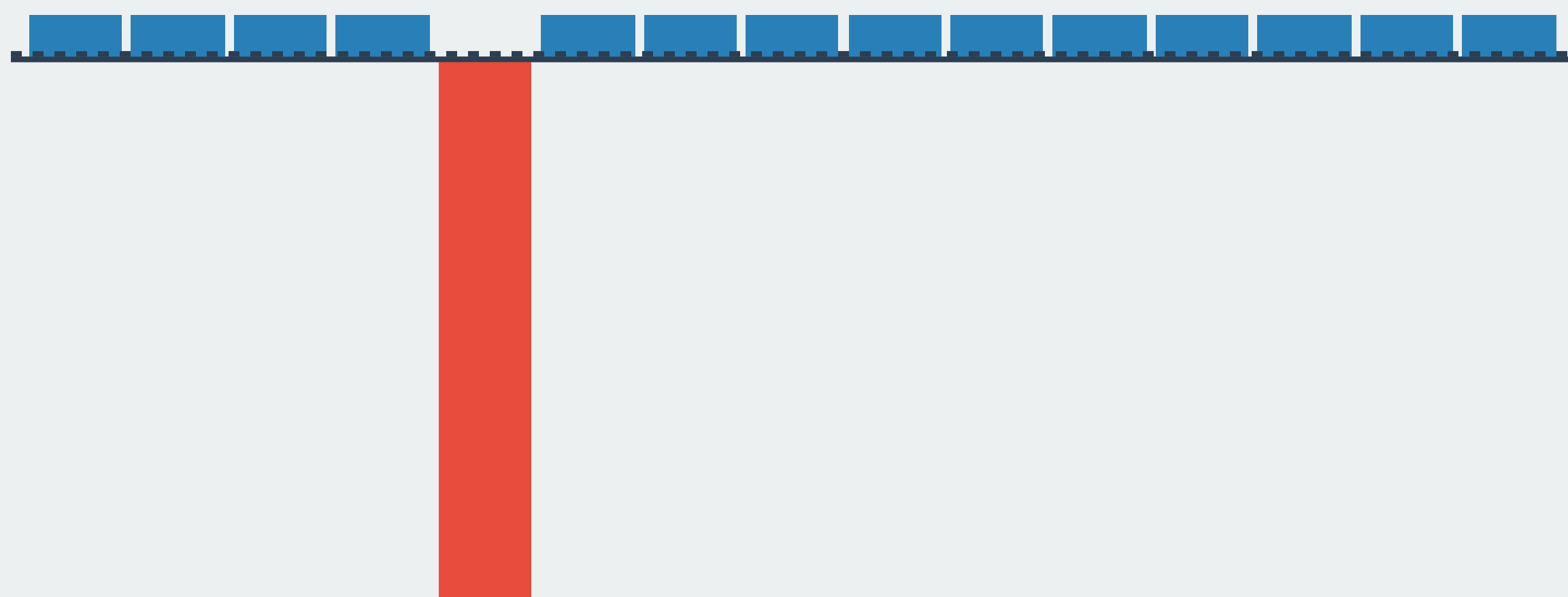


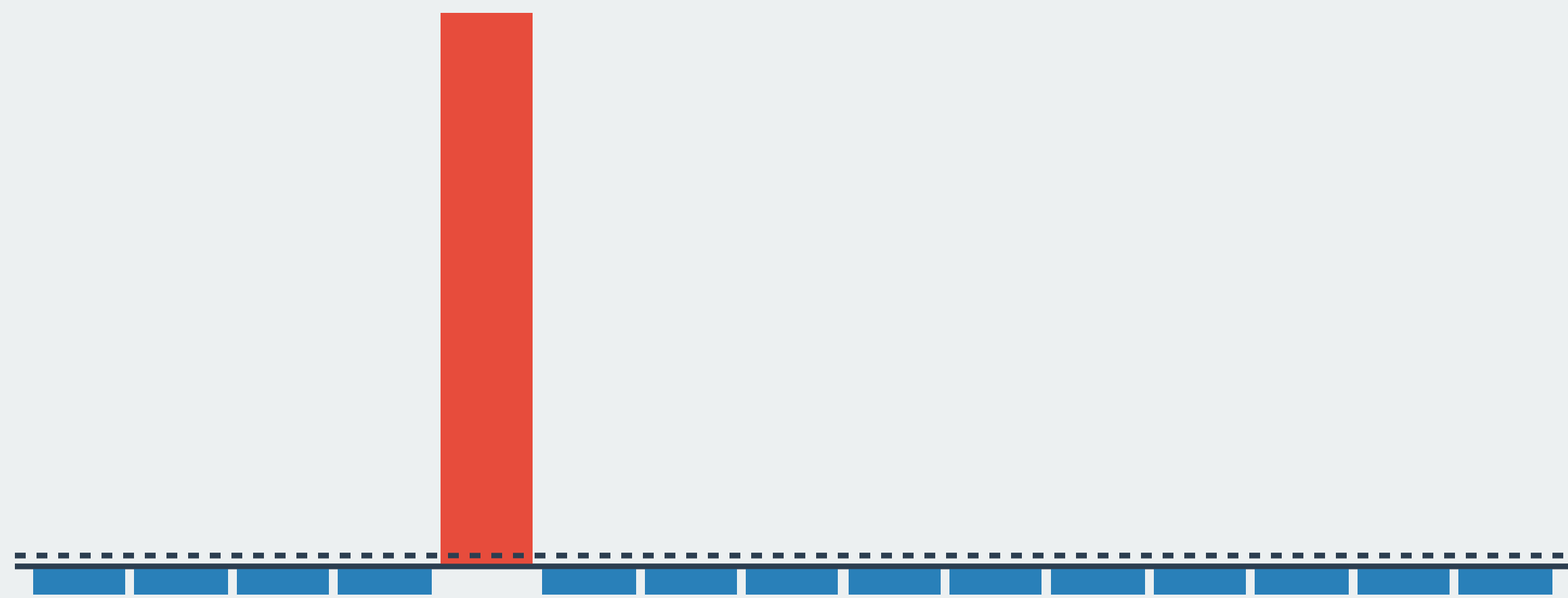


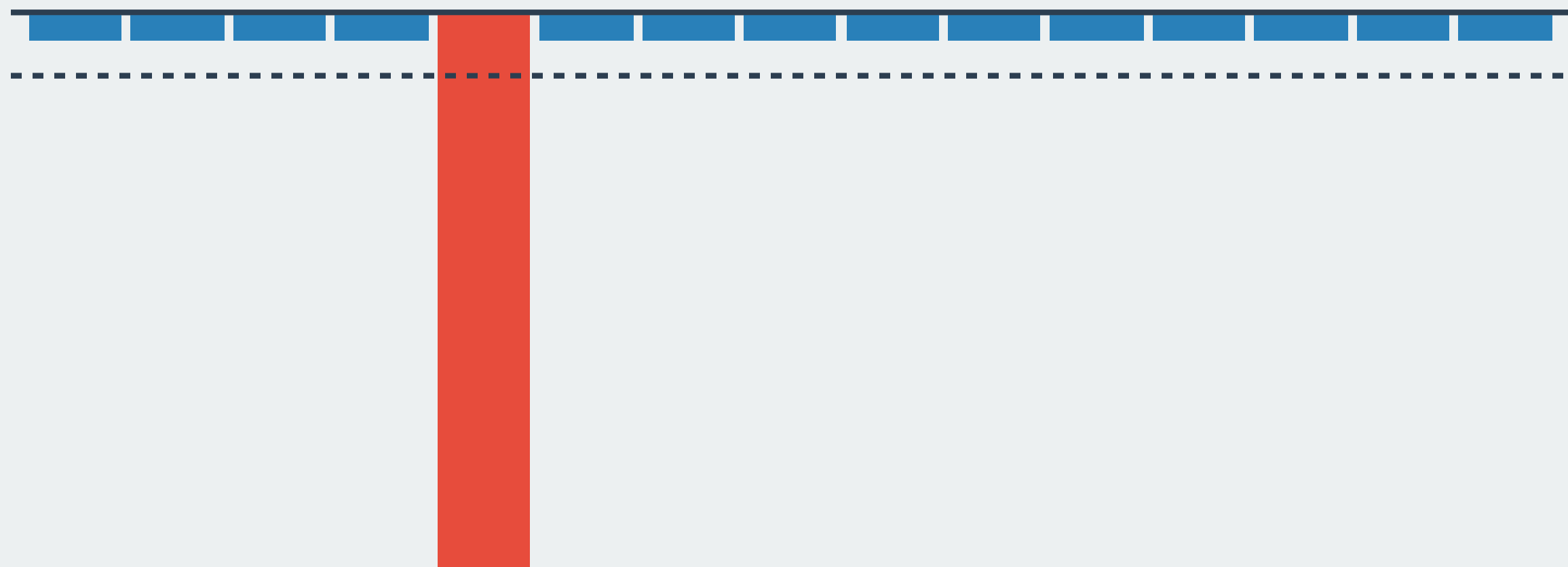


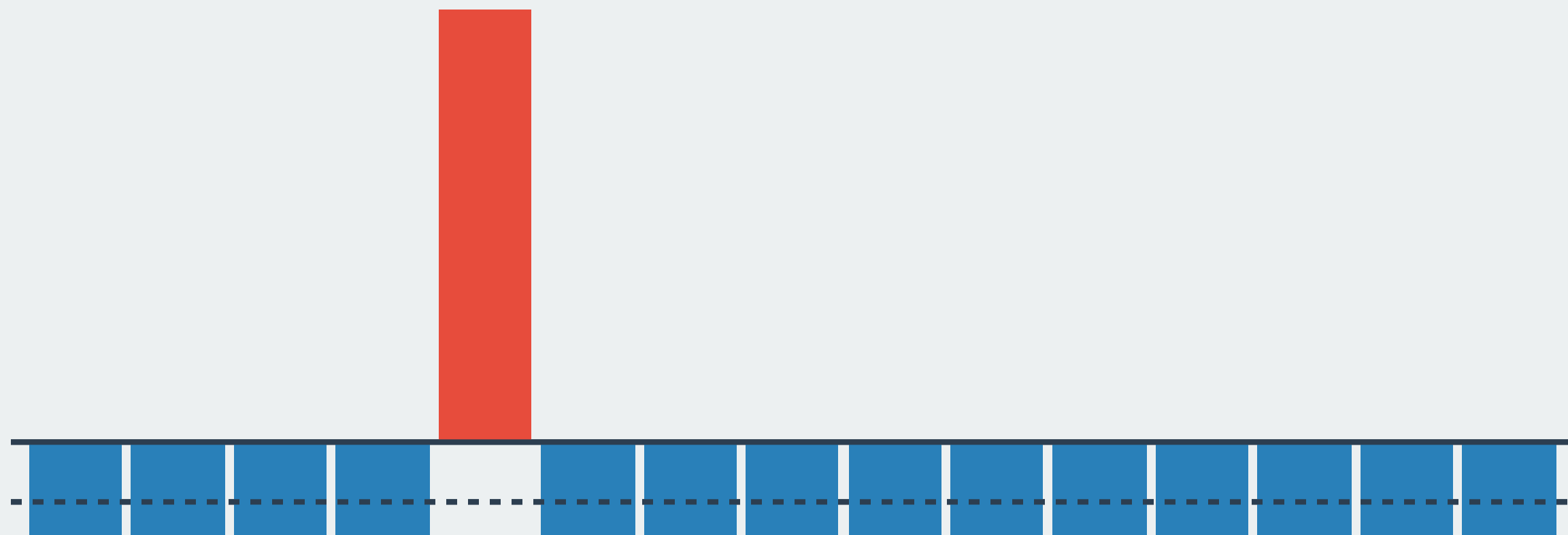


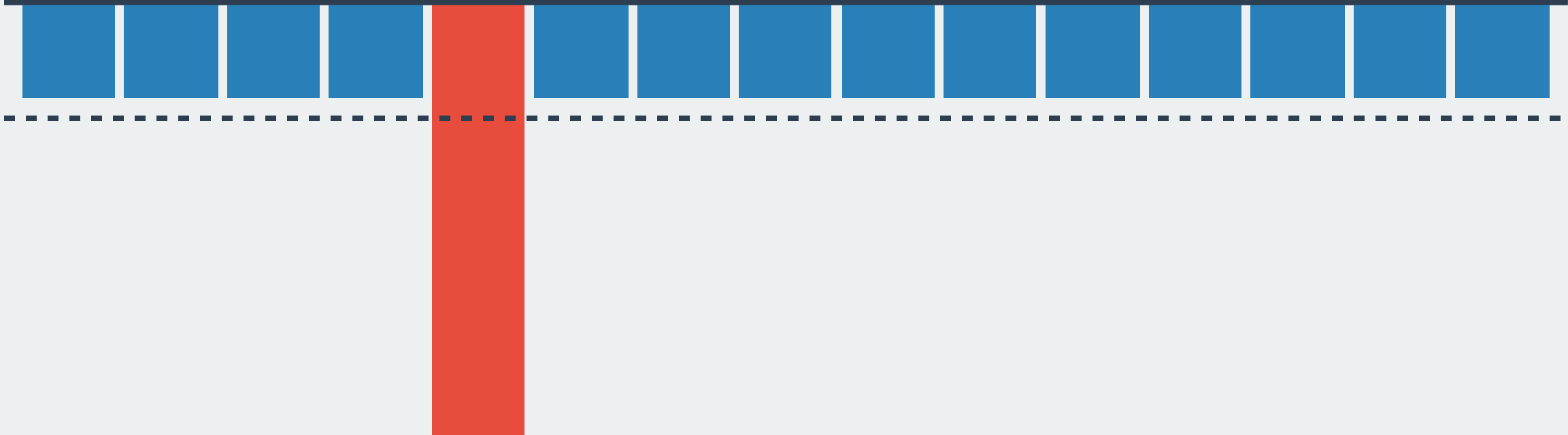


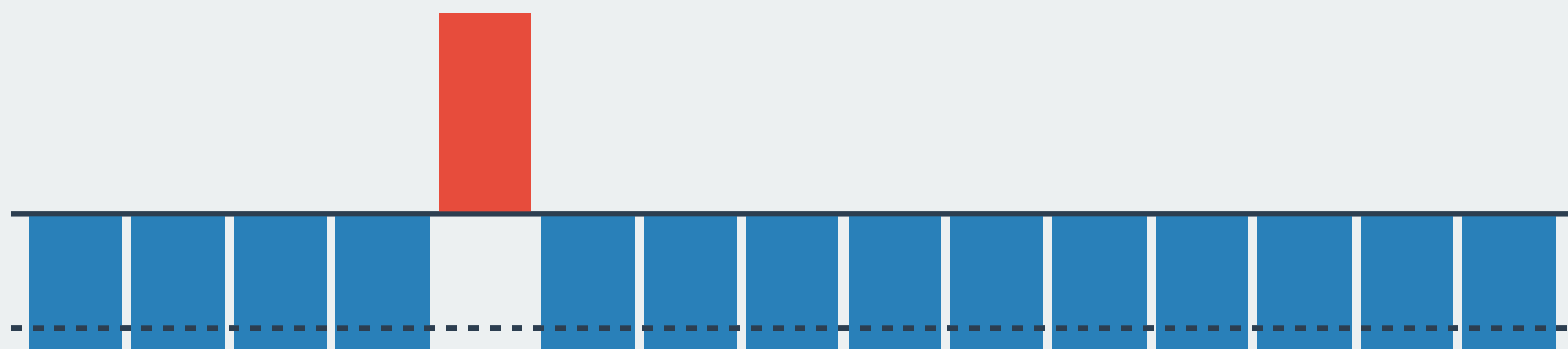


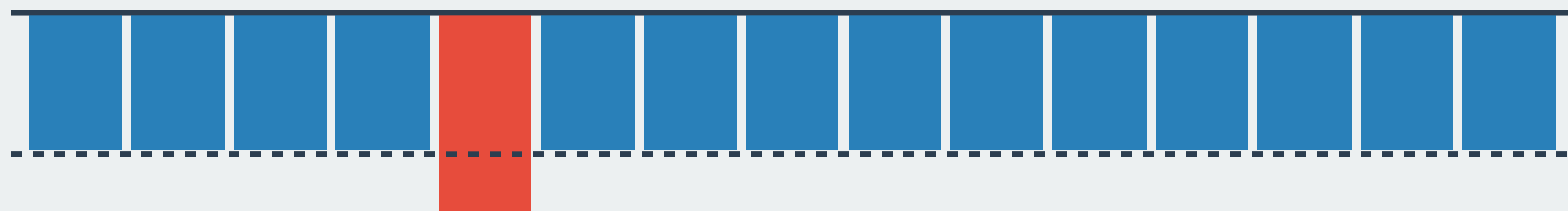


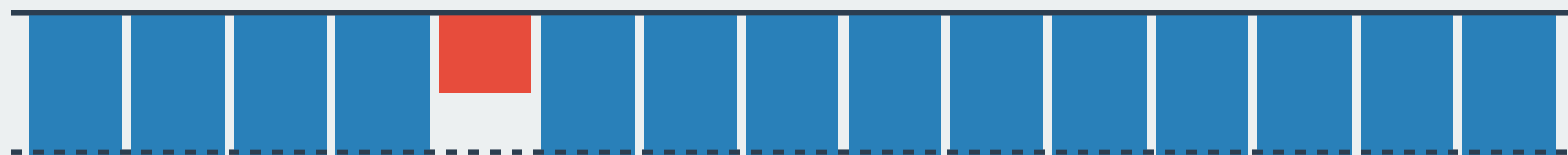




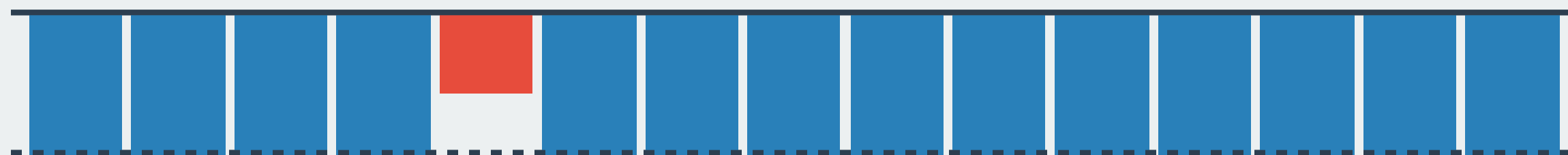








$$\approx \frac{\pi}{4} \sqrt{N/p}$$



naivno iskanje

Groverjev algoritem

naivno iskanje

Groverjev algoritem

$$N/p$$

naivno iskanje

Groverjev algoritem

$$N/p$$

$$\approx \frac{\pi}{4} \sqrt{N/p}$$

naivno iskanje	Groverjev algoritem
N/p	$\approx \frac{\pi}{4} \sqrt{N/p}$
1 000 000	

naivno iskanje	Groverjev algoritem
N/p	$\approx \frac{\pi}{4} \sqrt{N/p}$
1 000 000	785

naivno iskanje	Groverjev algoritem
N/p	$\approx \frac{\pi}{4} \sqrt{N/p}$
1 000 000	785
1 000 000 000 000	

naivno iskanje	Groverjev algoritem
N/p	$\approx \frac{\pi}{4} \sqrt{N/p}$
1 000 000	785
1 000 000 000 000	785 000

naivno iskanje	Groverjev algoritem
N/p	$\approx \frac{\pi}{4} \sqrt{N/p}$
1 000 000	785
1 000 000 000 000	785 000
12 dni	

naivno iskanje	Groverjev algoritem
N/p	$\approx \frac{\pi}{4} \sqrt{N/p}$
1 000 000	785
1 000 000 000 000	785 000
12 dni	1 sekunda

SHOROV

ALGORITEM

Peter Shor, 1959-



$$21 = ? \cdot ?$$

$$D(21,10) = 1$$

$$10^0 \bmod 21 = 1$$

$$10^0 \bmod 21 = 1$$

$$10^1 \bmod 21 = 10$$

$$10^0 \bmod 21 = 1$$

$$10^1 \bmod 21 = 10$$

$$10^2 \bmod 21 = 16$$

$10^0 \bmod 21 = 1$	$10^8 \bmod 21 = 16$	$10^{16} \bmod 21 = 4$
$10^1 \bmod 21 = 10$	$10^9 \bmod 21 = 13$	$10^{17} \bmod 21 = 19$
$10^2 \bmod 21 = 16$	$10^{10} \bmod 21 = 4$	$10^{18} \bmod 21 = 1$
$10^3 \bmod 21 = 13$	$10^{11} \bmod 21 = 19$	$10^{19} \bmod 21 = 10$
$10^4 \bmod 21 = 4$	$10^{12} \bmod 21 = 1$	$10^{20} \bmod 21 = 16$
$10^5 \bmod 21 = 19$	$10^{13} \bmod 21 = 10$	$10^{21} \bmod 21 = 13$
$10^6 \bmod 21 = 1$	$10^{14} \bmod 21 = 16$	$10^{22} \bmod 21 = 4$
$10^7 \bmod 21 = 10$	$10^{15} \bmod 21 = 13$	$10^{23} \bmod 21 = 19$

$$10^6 \bmod 21 = 1$$

$$10^6 \bmod 21 = 1$$

$$(10^6 - 1) \bmod 21 = 0$$

$$10^6 \bmod 21 = 1$$

$$(10^6 - 1) \bmod 21 = 0$$

$$(10^3 - 1)(10^3 + 1) \bmod 21 = 0$$



$$D(10^3 - 1,21) = 3$$

$$D(10^3 - 1,21) = 7$$



$10^2 \bmod 21$

...

$10^{64} \bmod 21$





$10^2 \bmod 21$... $10^{64} \bmod 21$

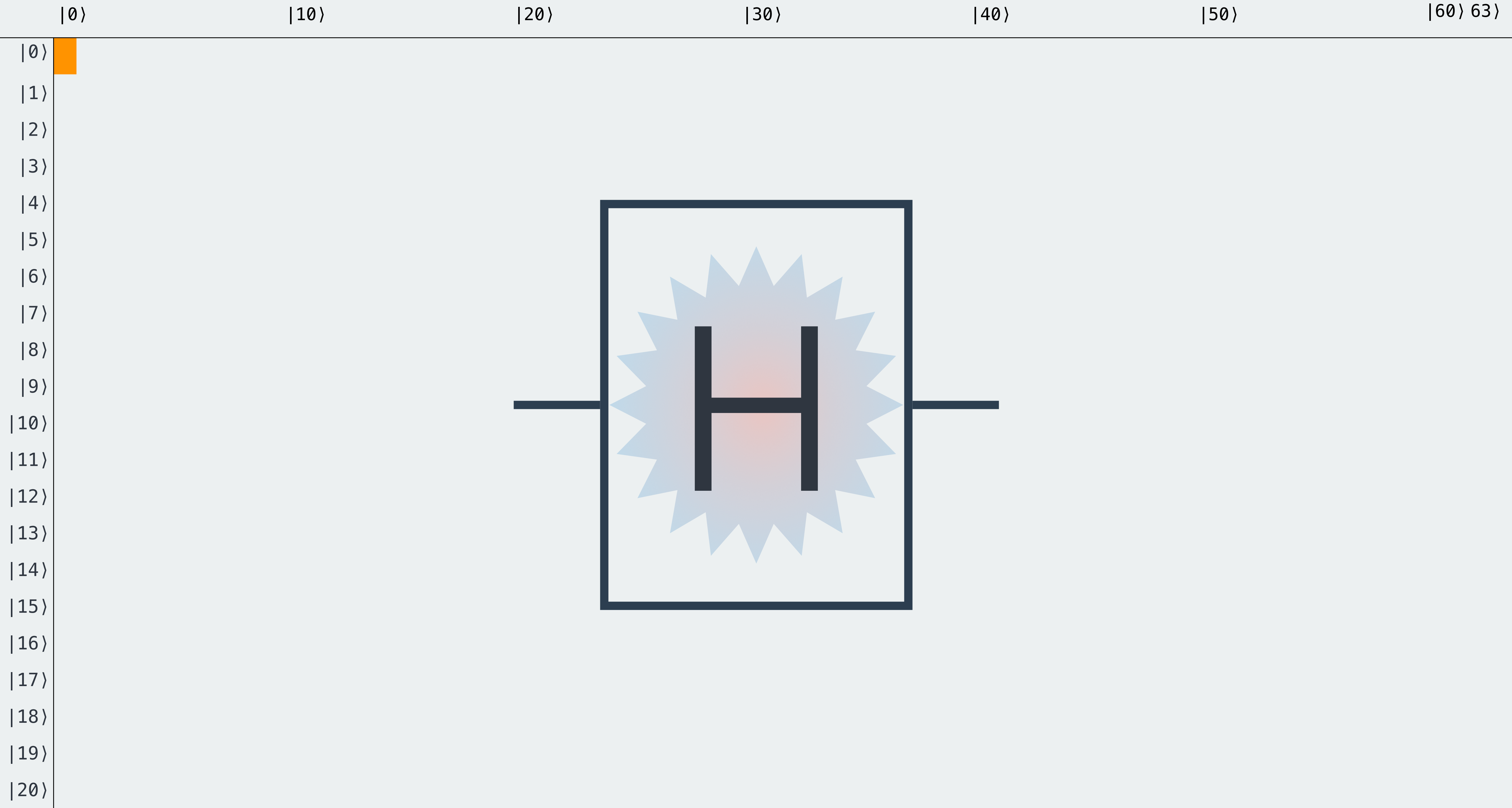
$21 \bmod 2$... $21 \bmod 20$



$10^2 \bmod 21$... $10^{64} \bmod 21$

$21 \bmod 2$... $21 \bmod 20$

	$ 0\rangle$	$ 10\rangle$	$ 20\rangle$	$ 30\rangle$	$ 40\rangle$	$ 50\rangle$	$ 60\rangle$ $ 63\rangle$
$ 0\rangle$							
$ 1\rangle$							
$ 2\rangle$							
$ 3\rangle$							
$ 4\rangle$							
$ 5\rangle$							
$ 6\rangle$							
$ 7\rangle$							
$ 8\rangle$							
$ 9\rangle$							
$ 10\rangle$							
$ 11\rangle$							
$ 12\rangle$							
$ 13\rangle$							
$ 14\rangle$							
$ 15\rangle$							
$ 16\rangle$							
$ 17\rangle$							
$ 18\rangle$							
$ 19\rangle$							
$ 20\rangle$							



| 0 \rangle

 $|10\rangle$

| 20 >

| 30 |

| 40

| 50

 $|60\rangle \quad |63\rangle$

10x

| 1)

27

13X

14

157

167

177

187

197

11

11

12

11

11

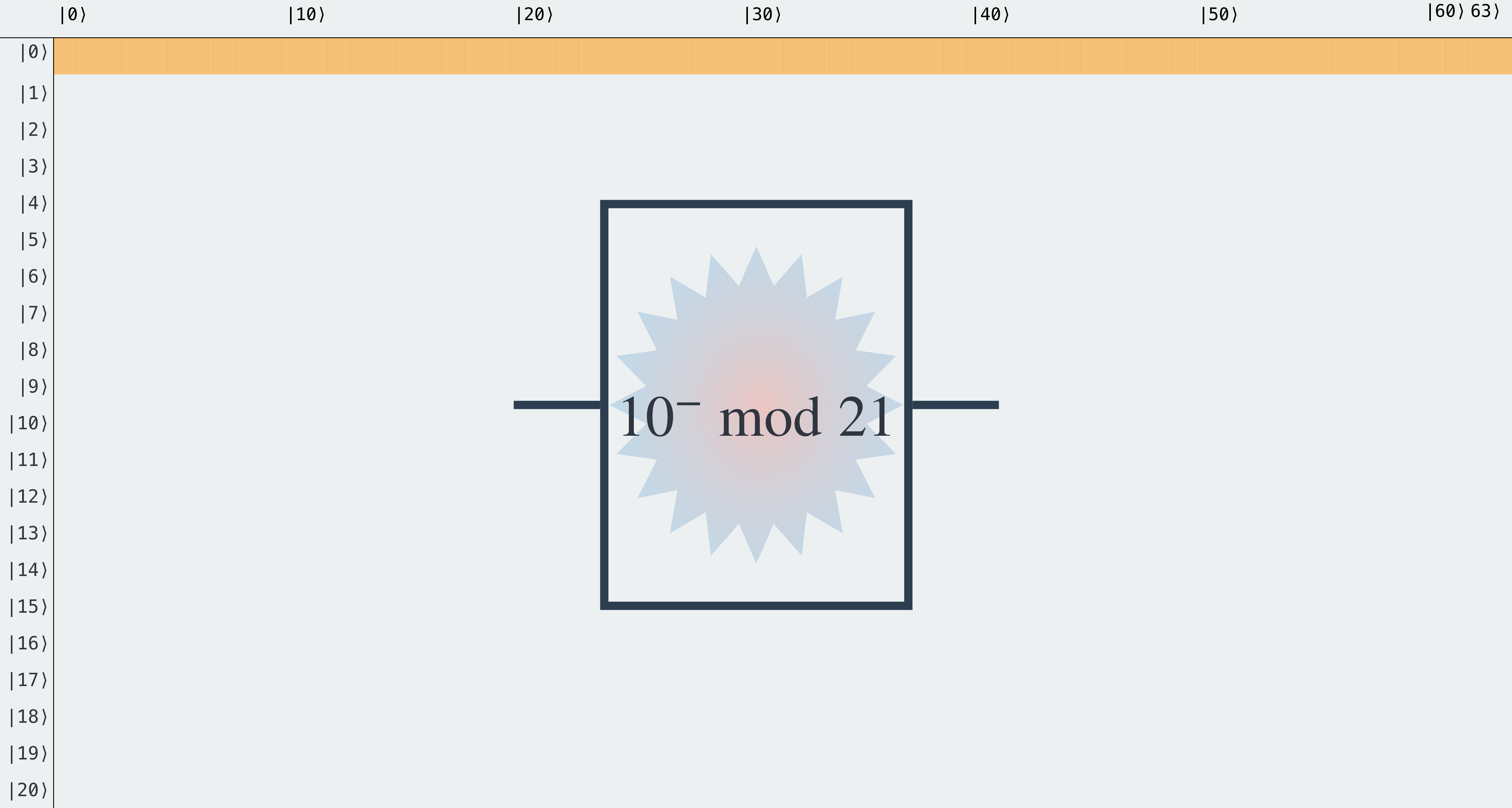
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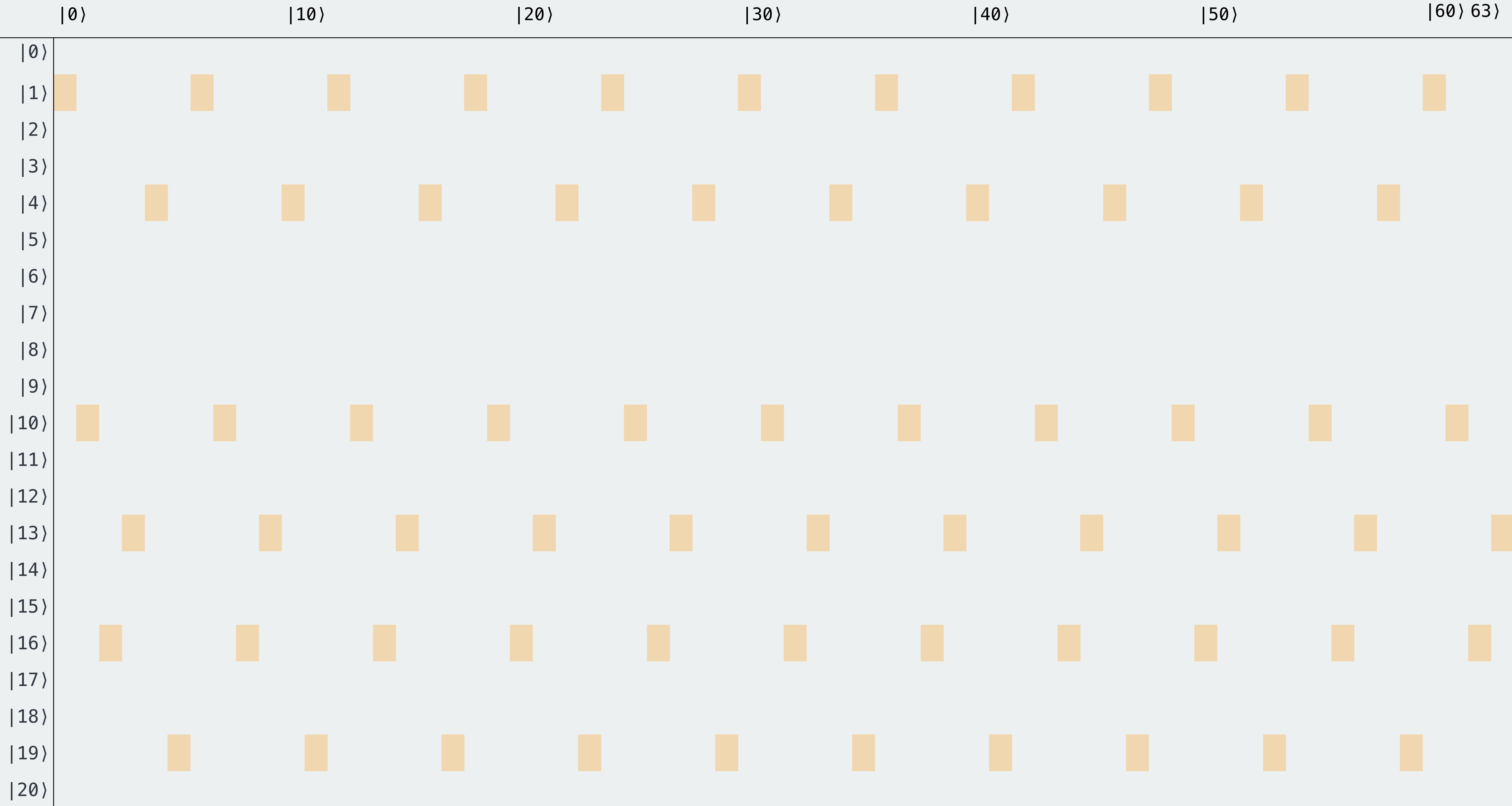
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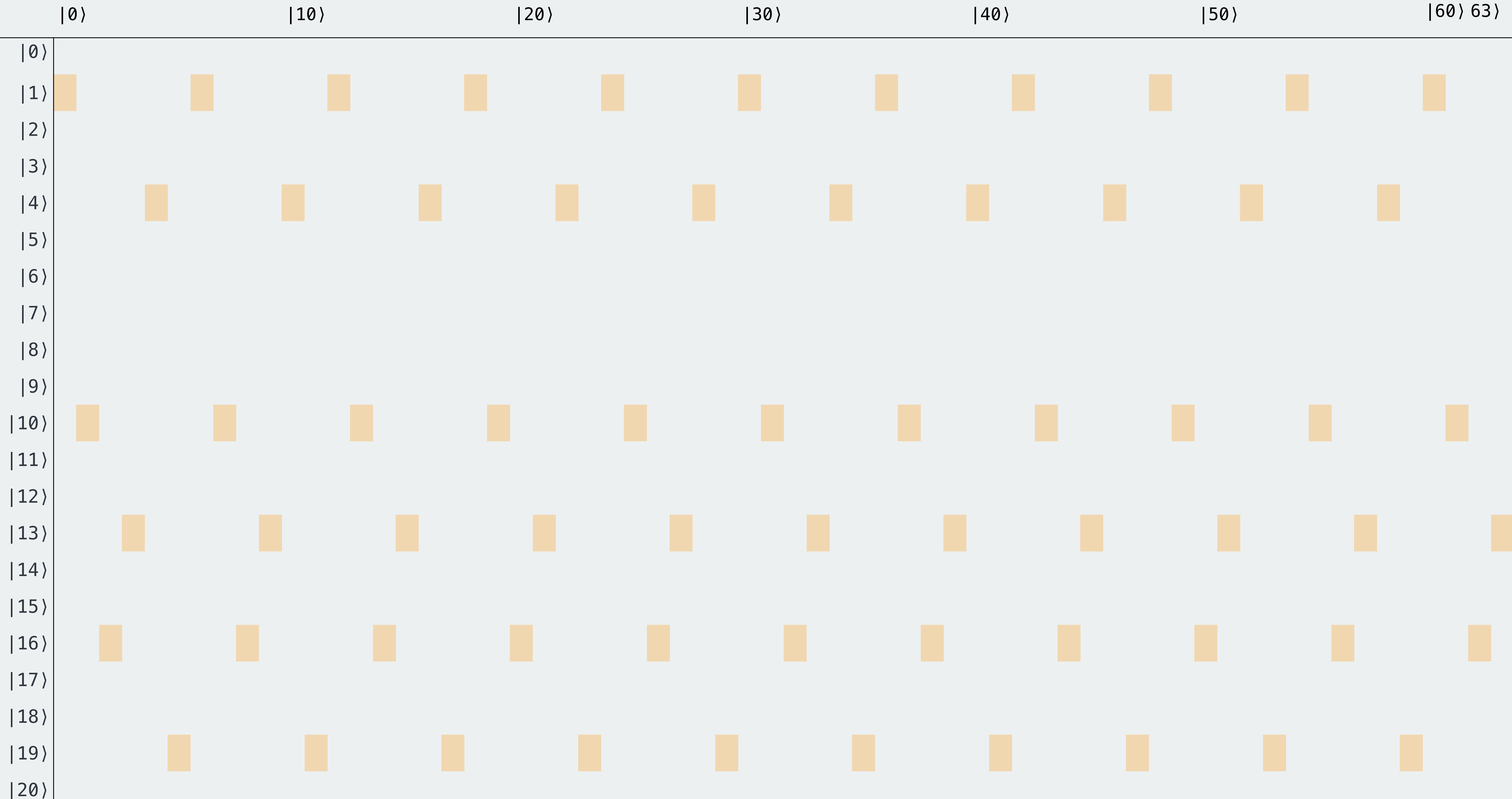
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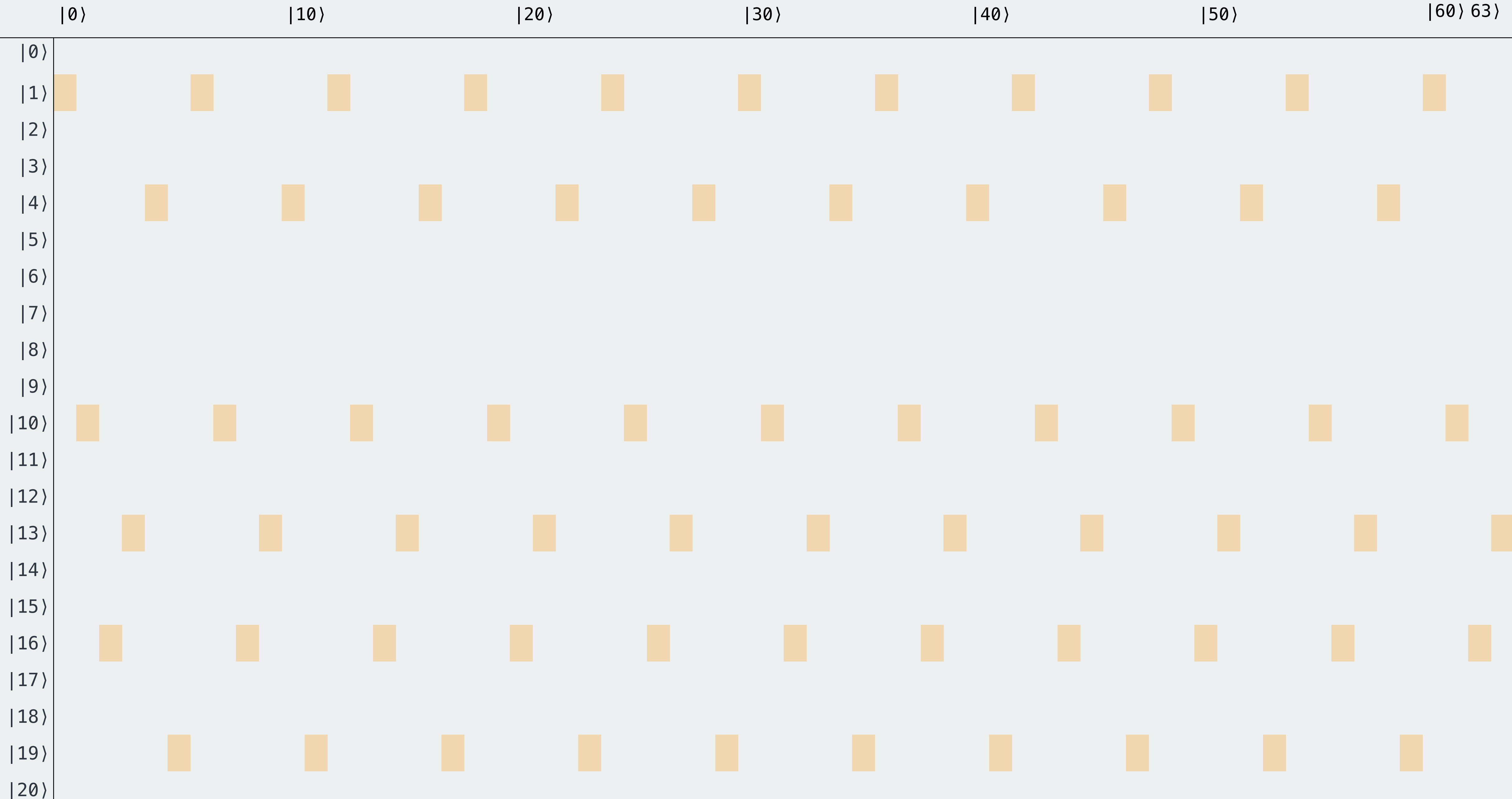
1

1









| 0 | x |

 $|10\rangle$

| 20 |

| 30 |

| 40

| 50

| 60 > 63

0

1

25

3

4

15

16

17

18

19

10

11

12

13

14

15

16

15

18

19

20

10x

 $|10\rangle$

| 20 |

130 Y

| 40 |

150

60%

63

 $|0\rangle$ $|1\rangle$ $|2\rangle$

3>

 $|4\rangle$

5)

| 67

| / >

187

197

| 10 |

11

| 127

| 137

| 14 ⟩

| 157

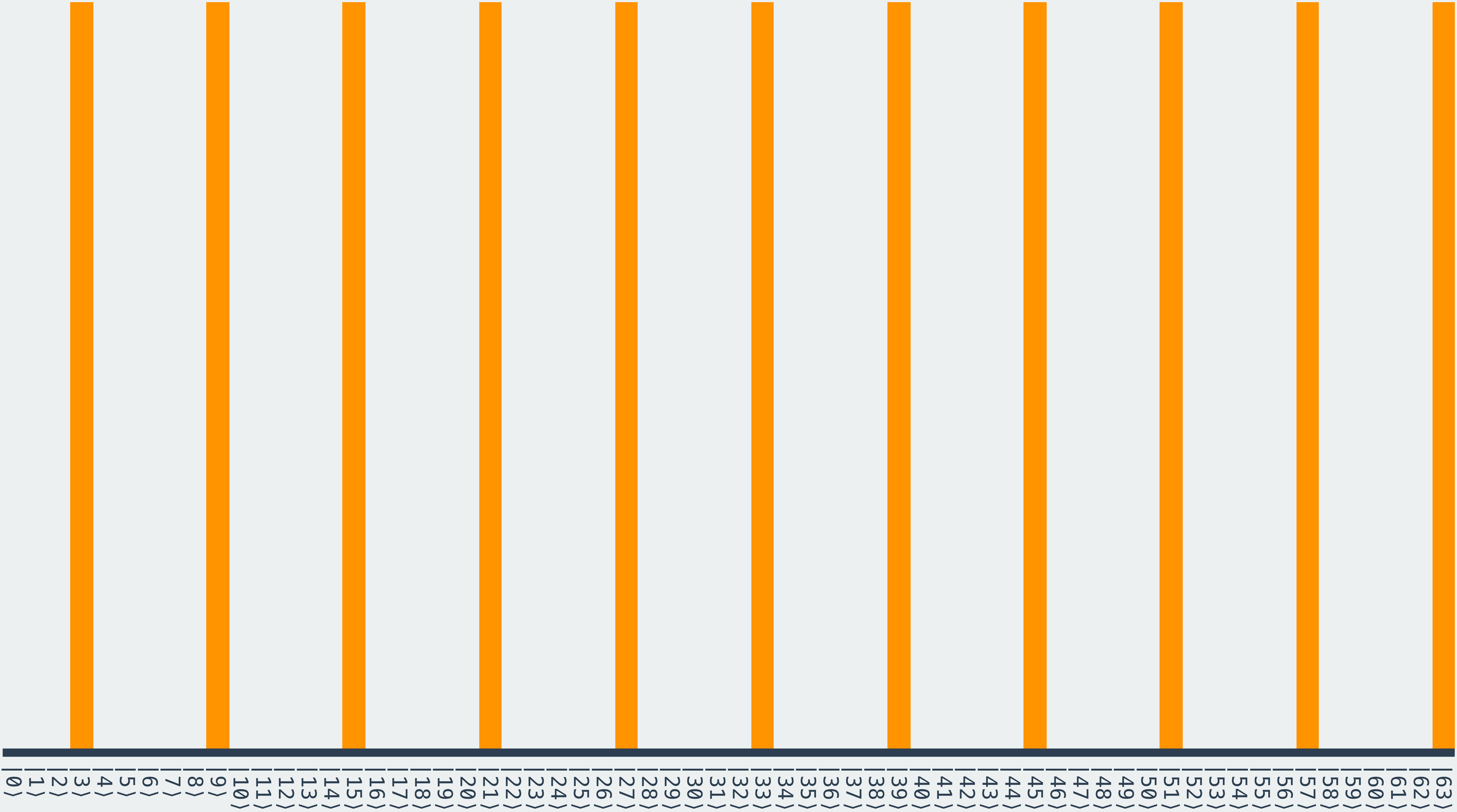
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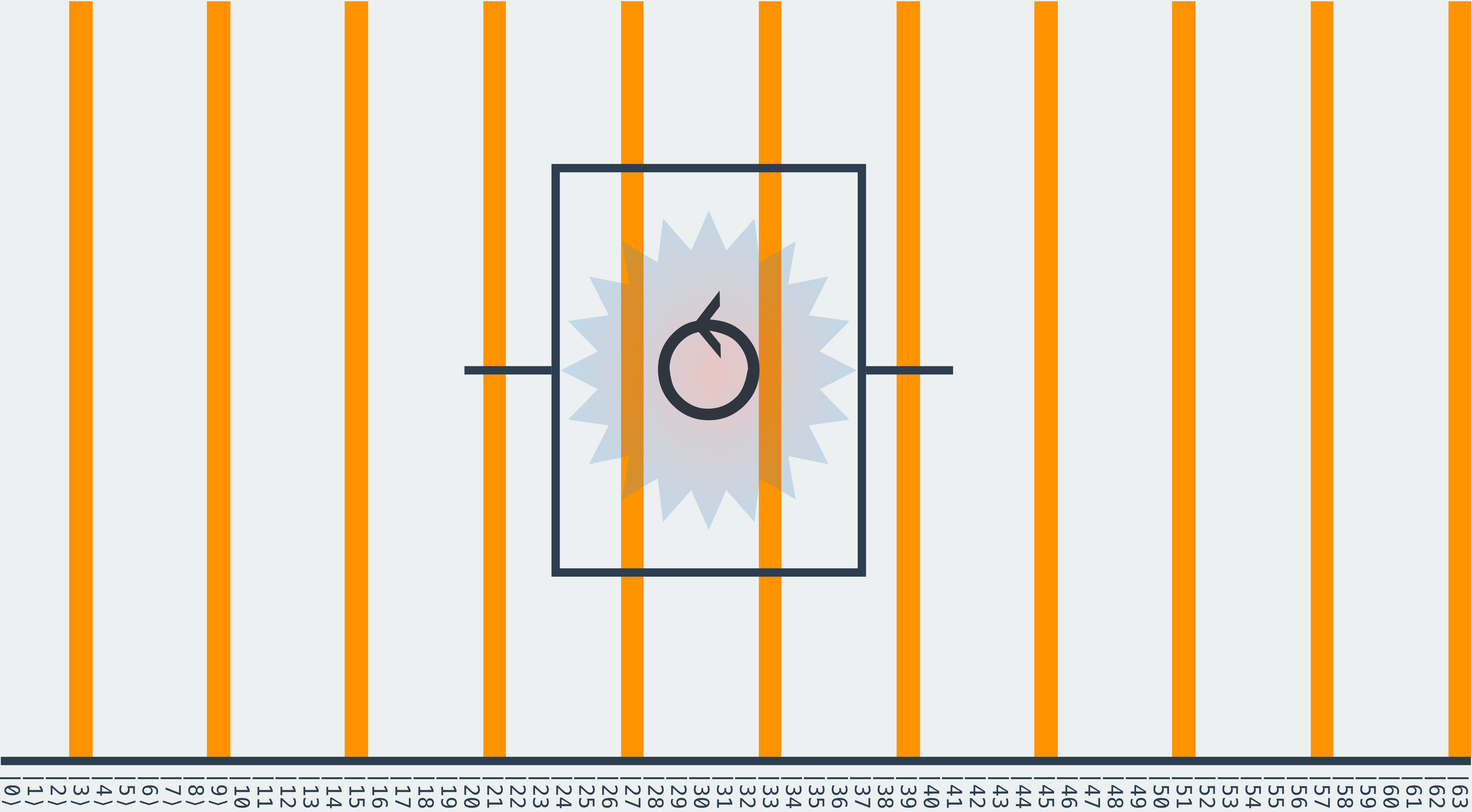
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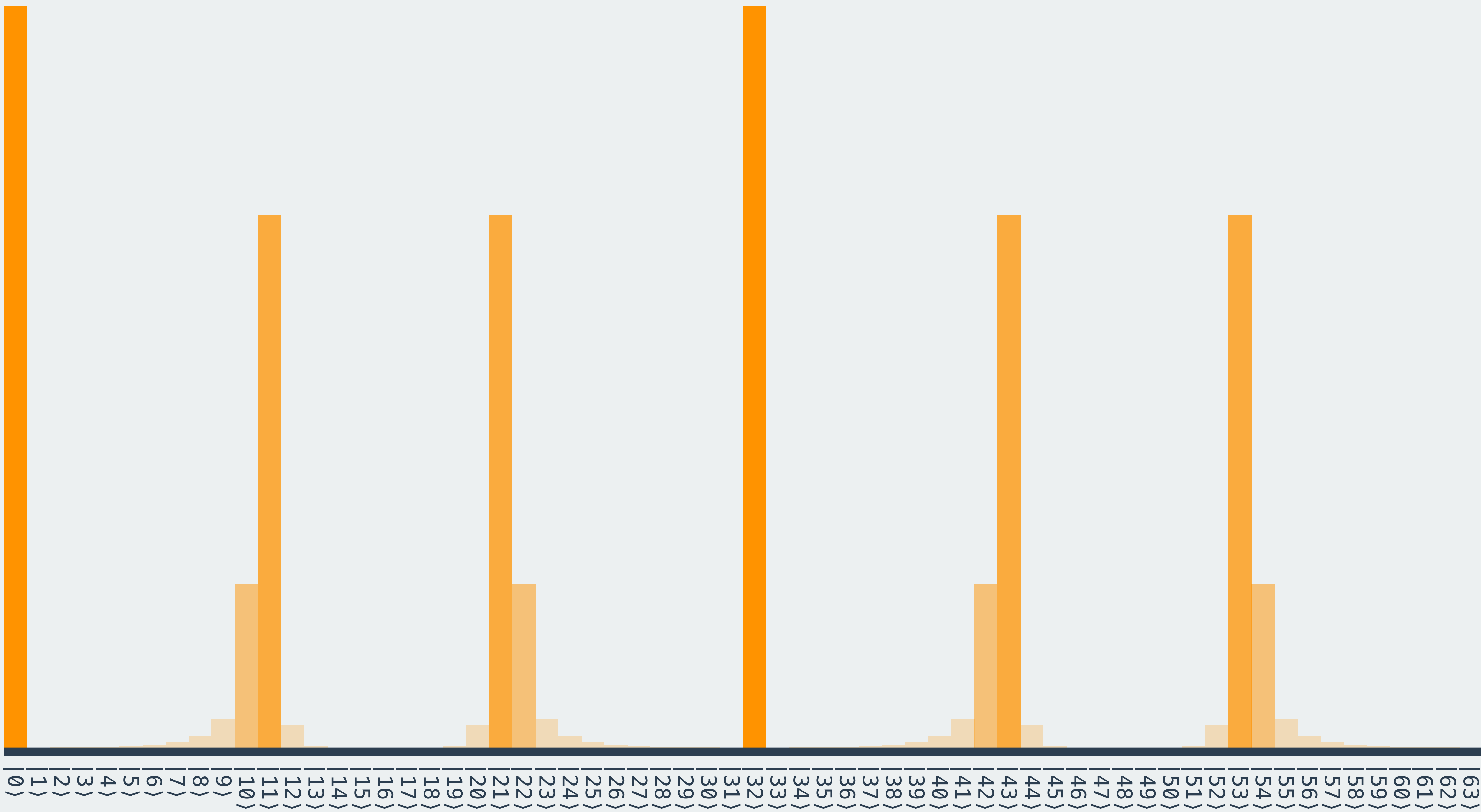
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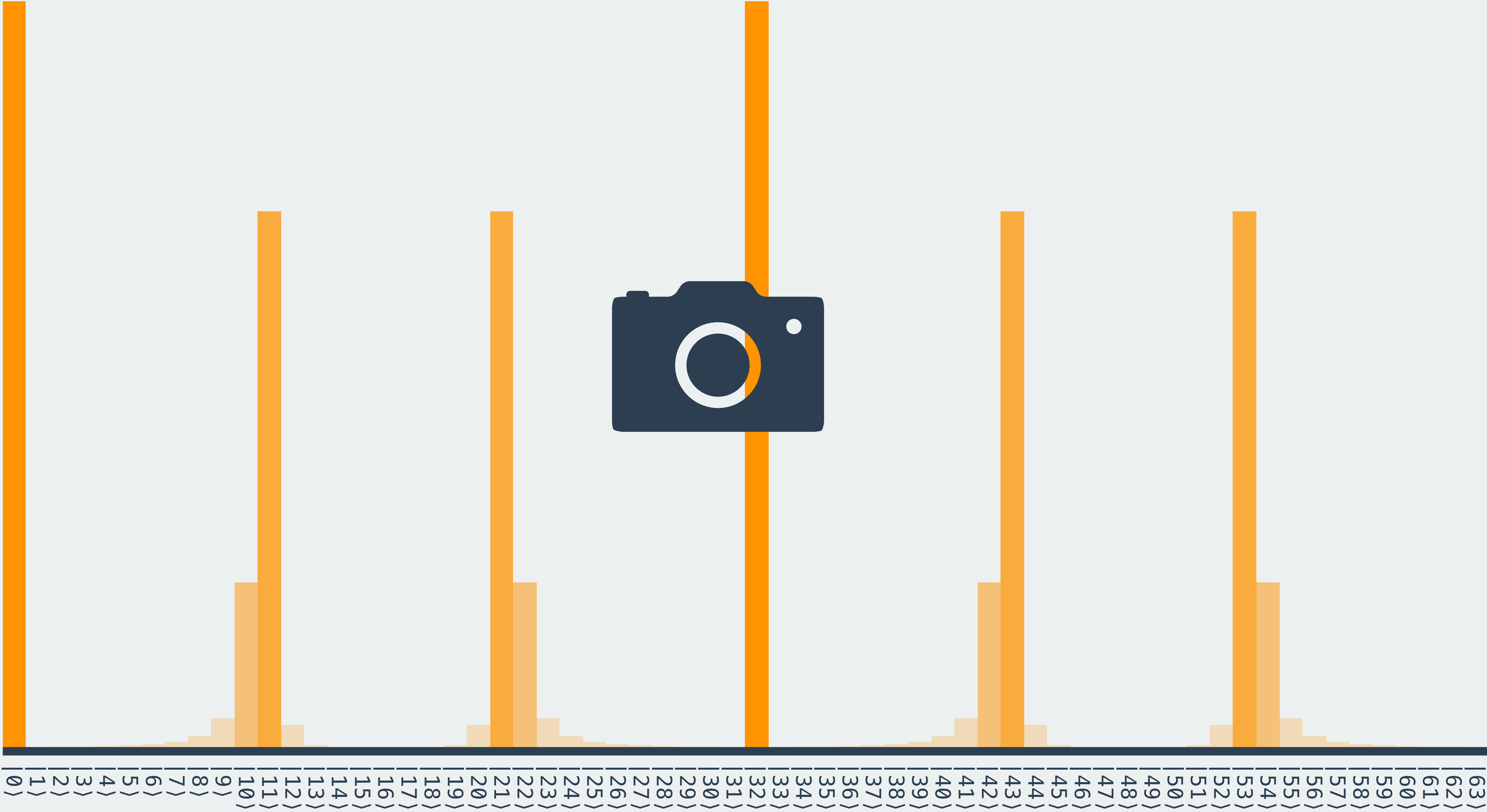
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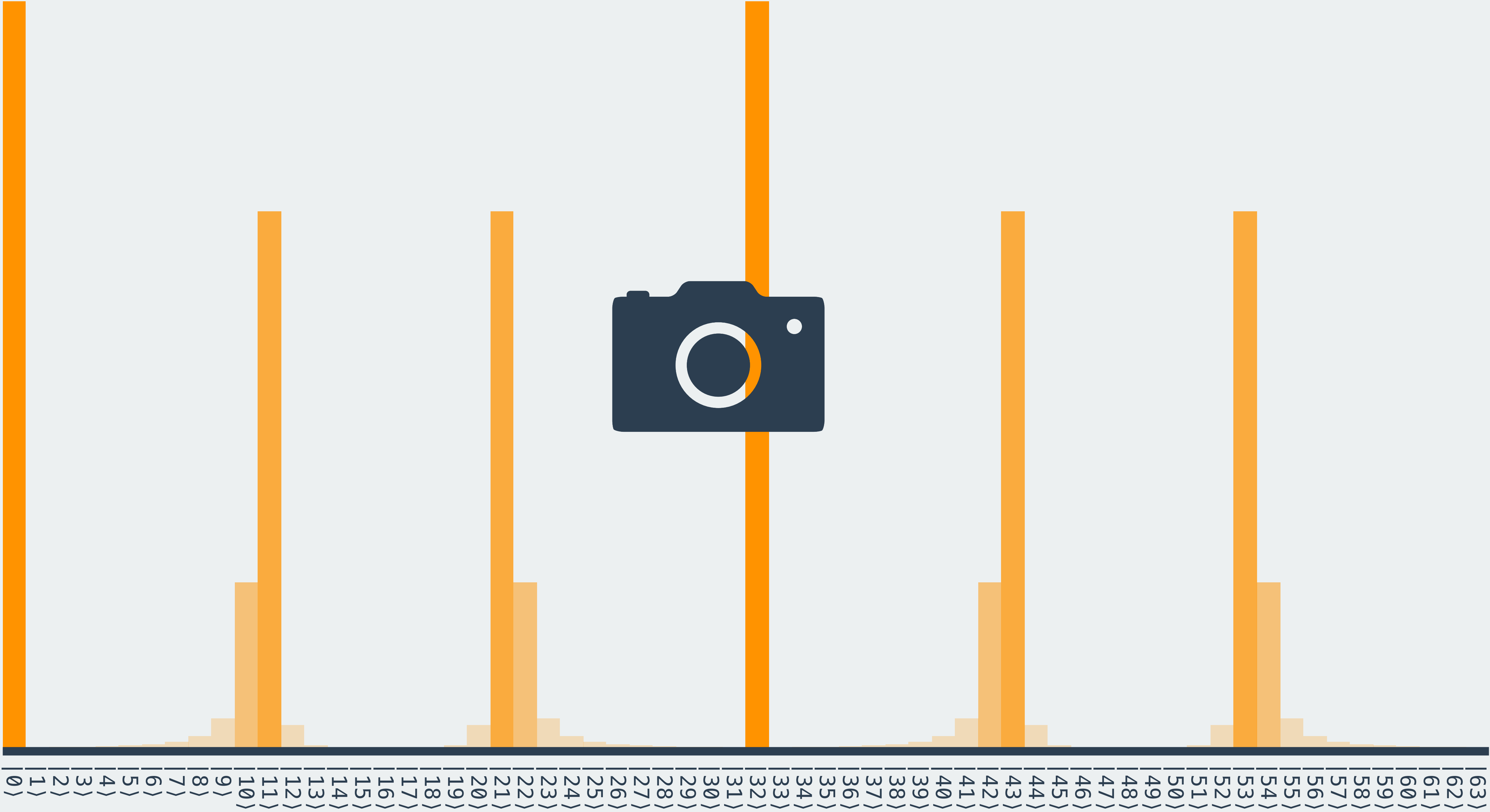
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|63>
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|4>
|3>
|2>
|1>
|0>

$$\frac{53}{64}$$

$$\frac{53}{64} = \frac{1}{1 + \frac{11}{53}}$$

$$\frac{53}{64} = \frac{1}{1 + \frac{11}{53}} \approx \frac{1}{1} = 1$$

$$\frac{53}{64} = \frac{1}{1 + \frac{11}{53}} \approx \frac{1}{1} = 1$$

$$= \frac{1}{1 + \frac{1}{4 + \frac{9}{11}}}$$

$$\frac{53}{64} = \frac{1}{1 + \frac{11}{53}} \approx \frac{1}{1} = 1$$

$$= \frac{1}{1 + \frac{1}{4 + \frac{9}{11}}} \approx \frac{1}{1 + \frac{1}{4}} = \frac{4}{5}$$

$$\frac{53}{64} = \frac{1}{1 + \frac{11}{53}} \approx \frac{1}{1} = 1$$

$$= \frac{1}{1 + \frac{1}{4 + \frac{9}{11}}} \approx \frac{1}{1 + \frac{1}{4}} = \frac{4}{5}$$

$$= \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{2}{9}}}}$$

$$\frac{53}{64} = \frac{1}{1 + \frac{11}{53}} \approx \frac{1}{1} = 1$$

$$= \frac{1}{1 + \frac{1}{4 + \frac{9}{11}}} \approx \frac{1}{1 + \frac{1}{4}} = \frac{4}{5}$$

$$= \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{2}{9}}}} \approx \frac{1}{1 + \frac{1}{4 + \frac{1}{1}}} = \frac{5}{6}$$

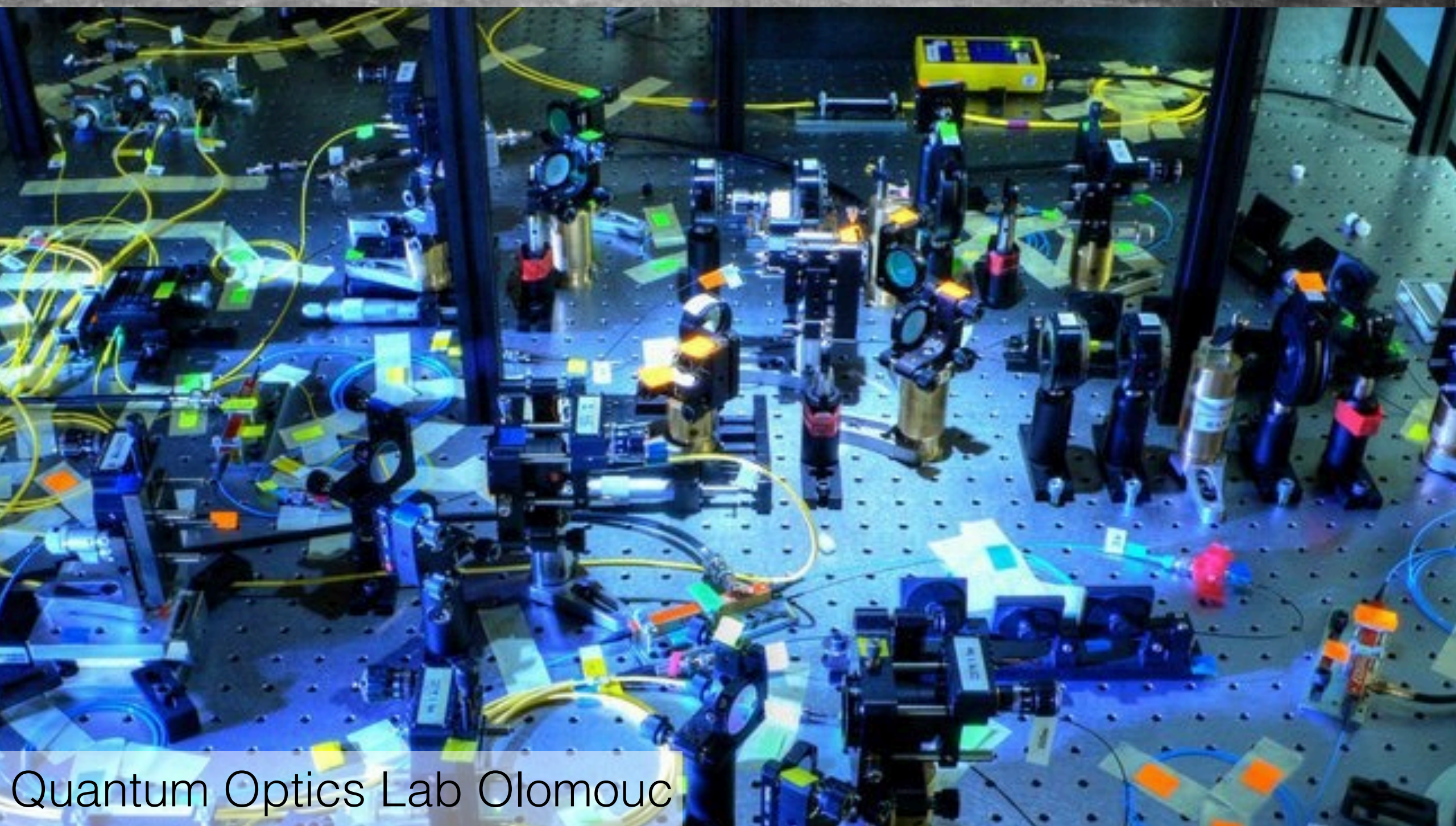
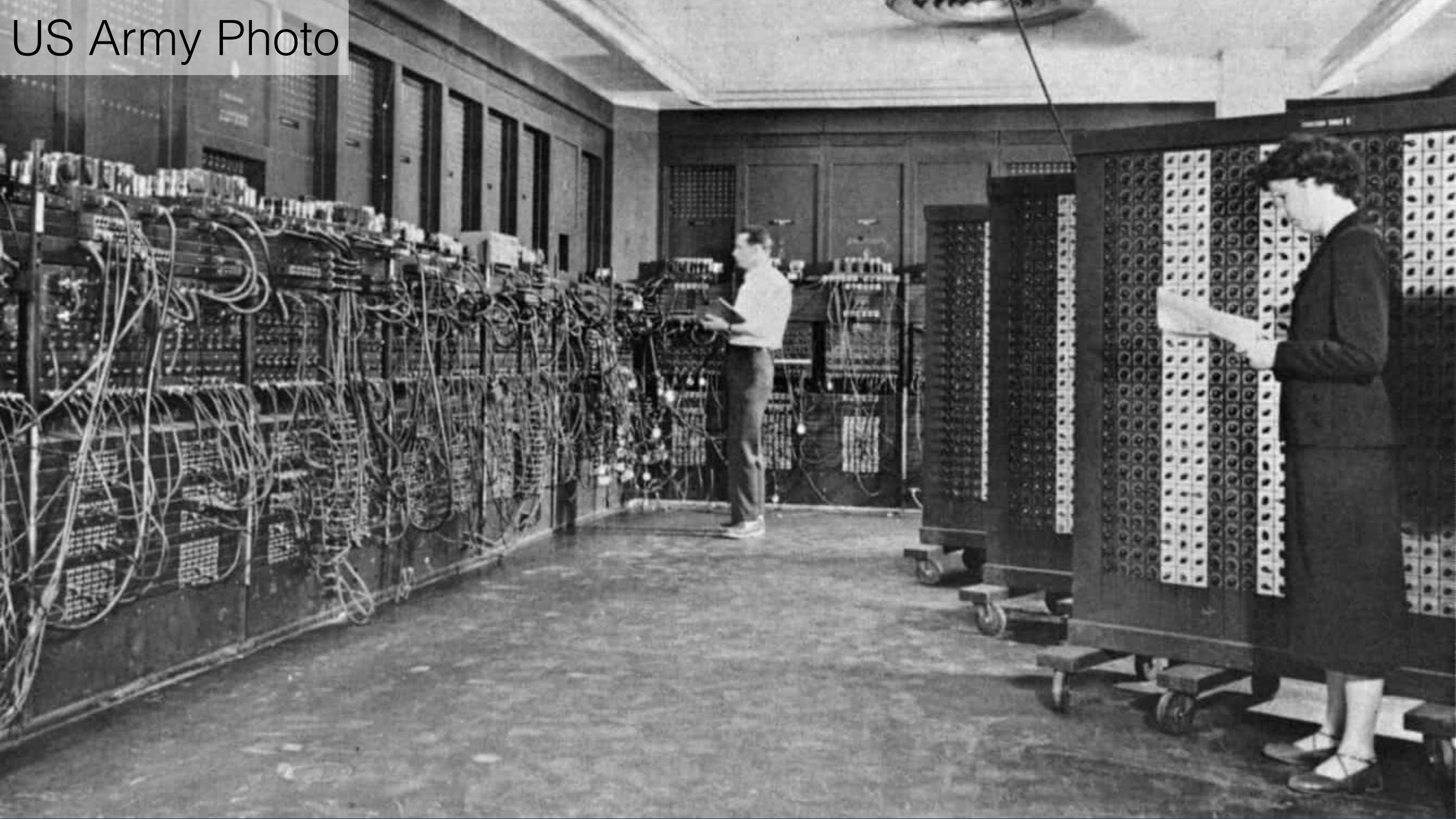


Shorov
algoritim



Quantum Optics Lab Olomouc

US Army Photo



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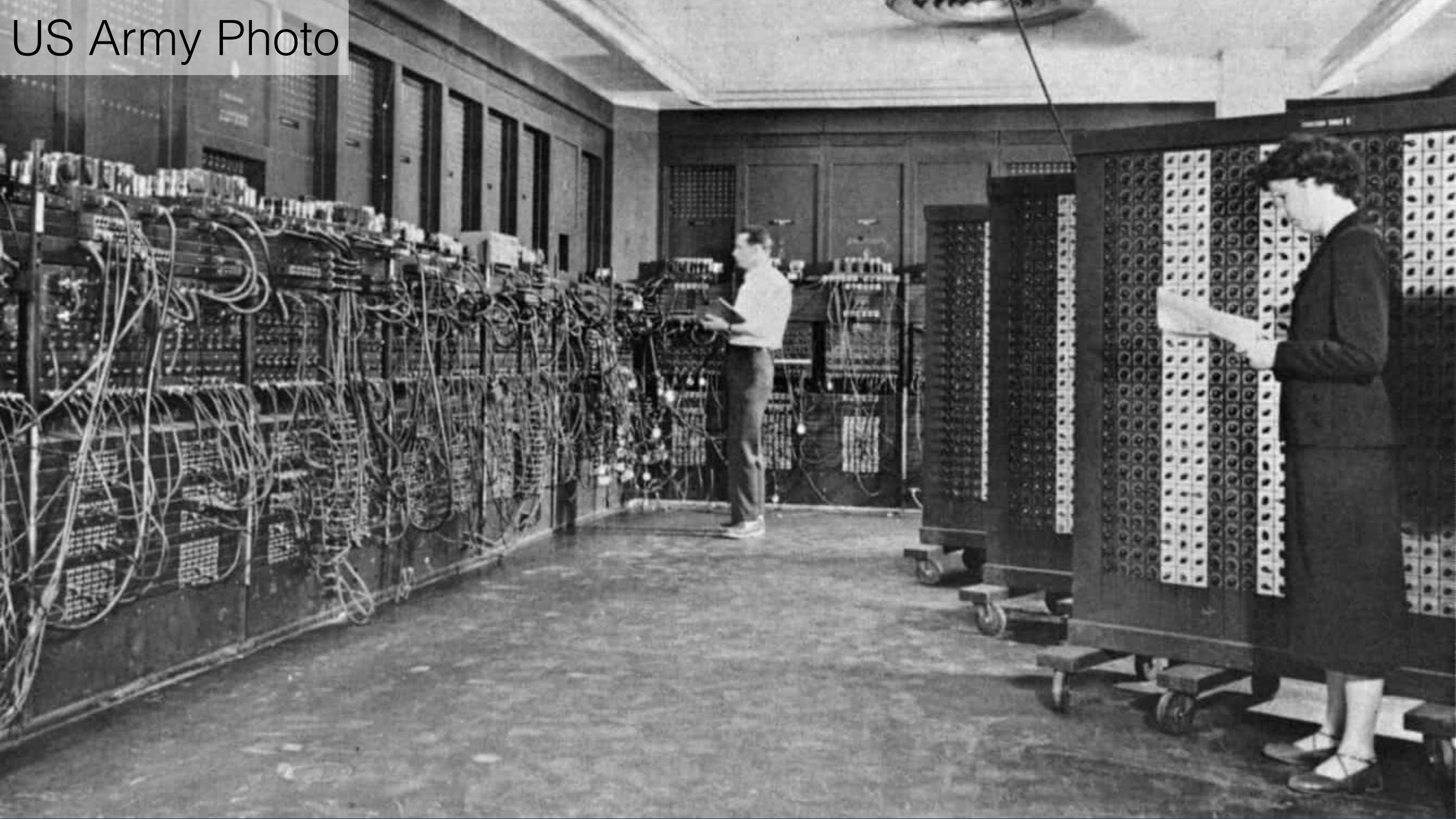
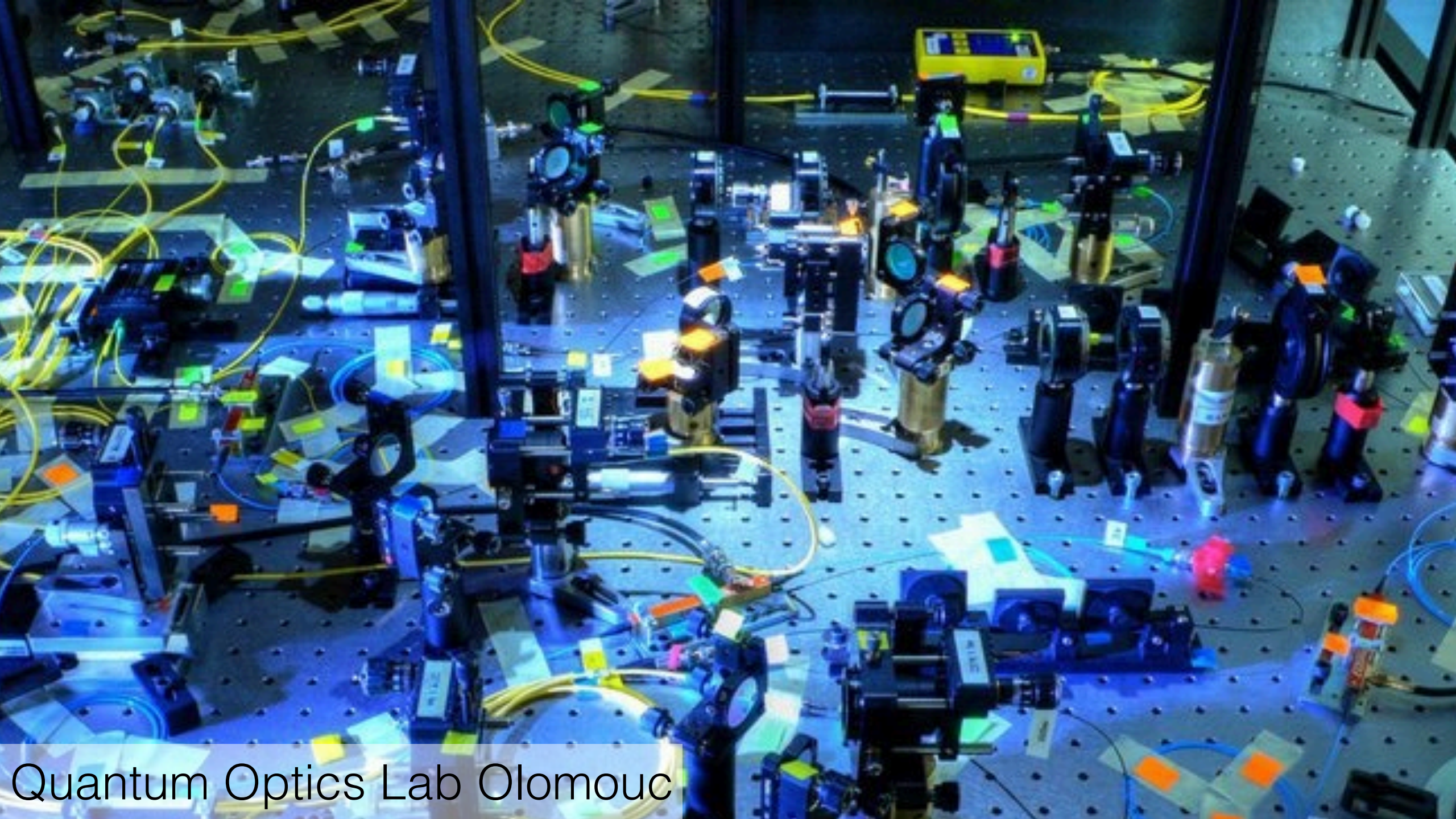
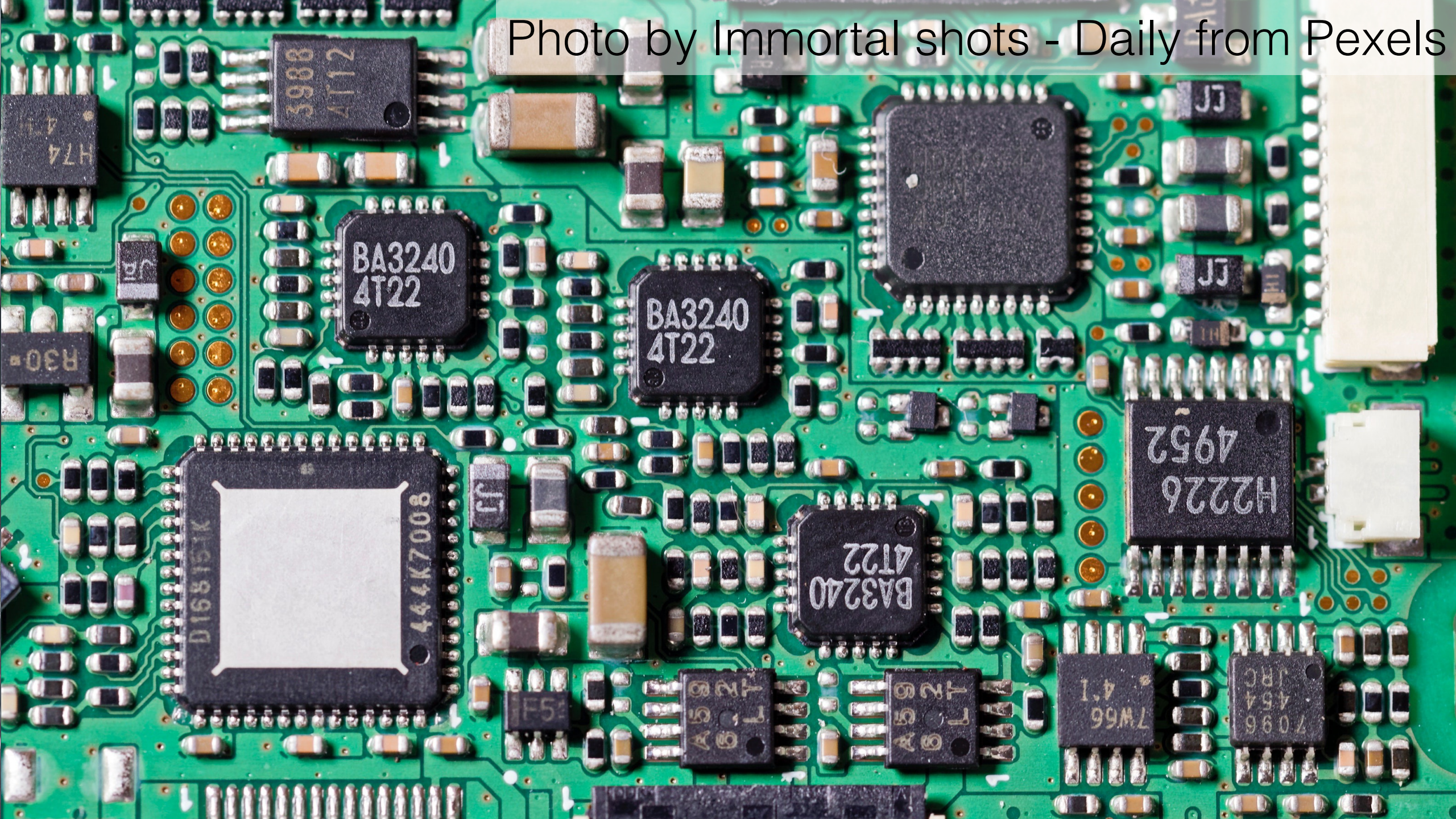


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US Army Photo

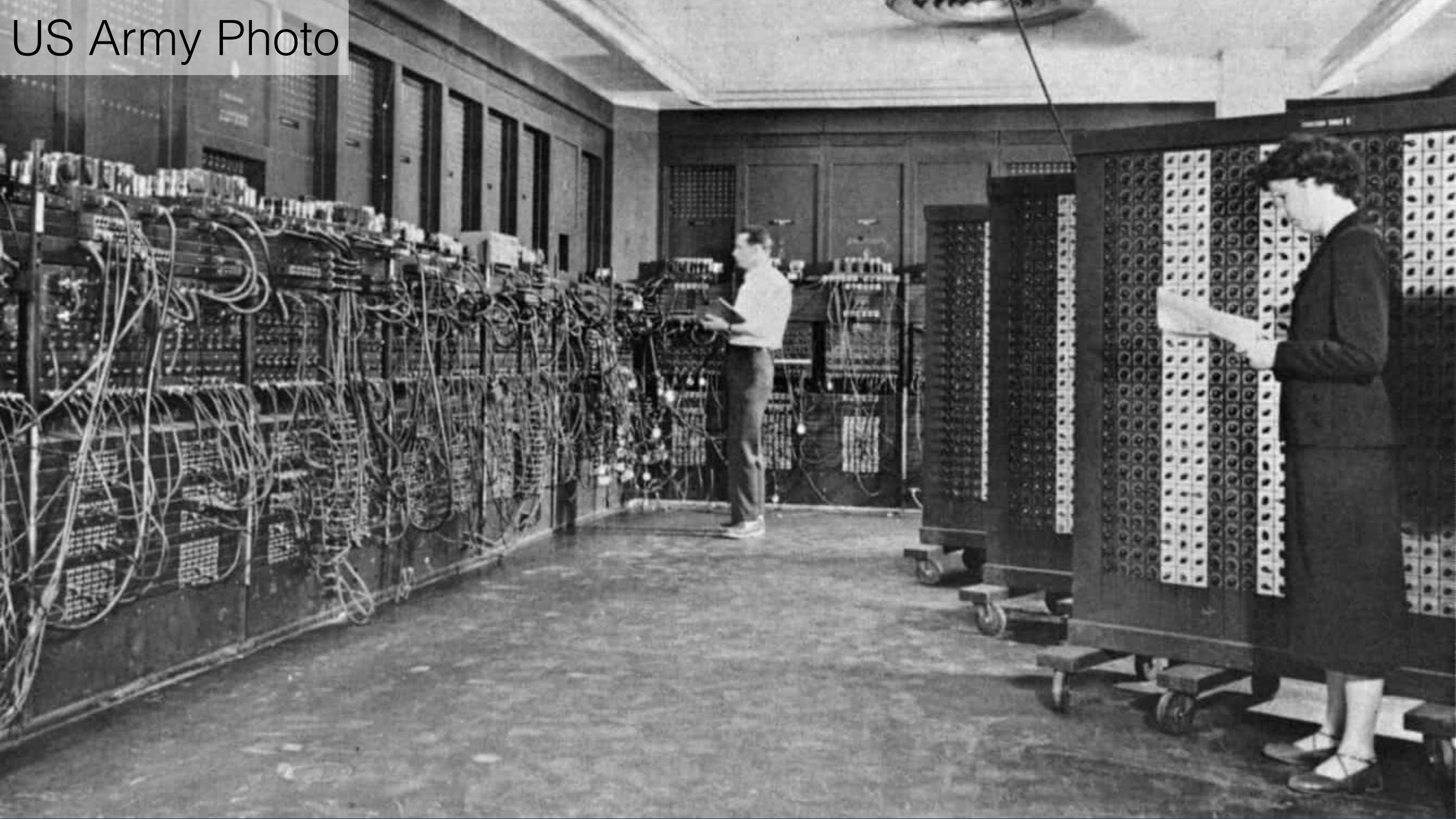
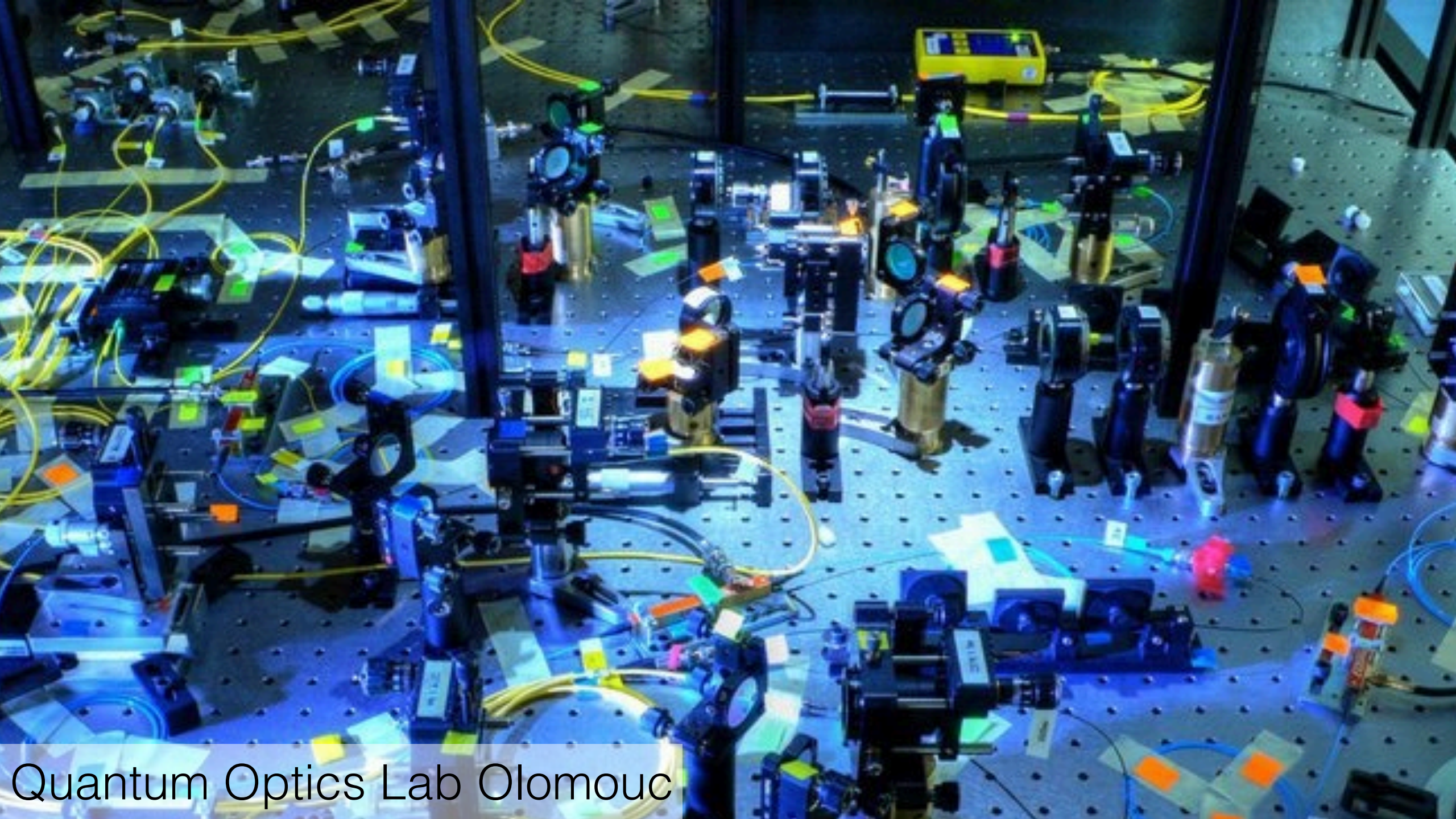
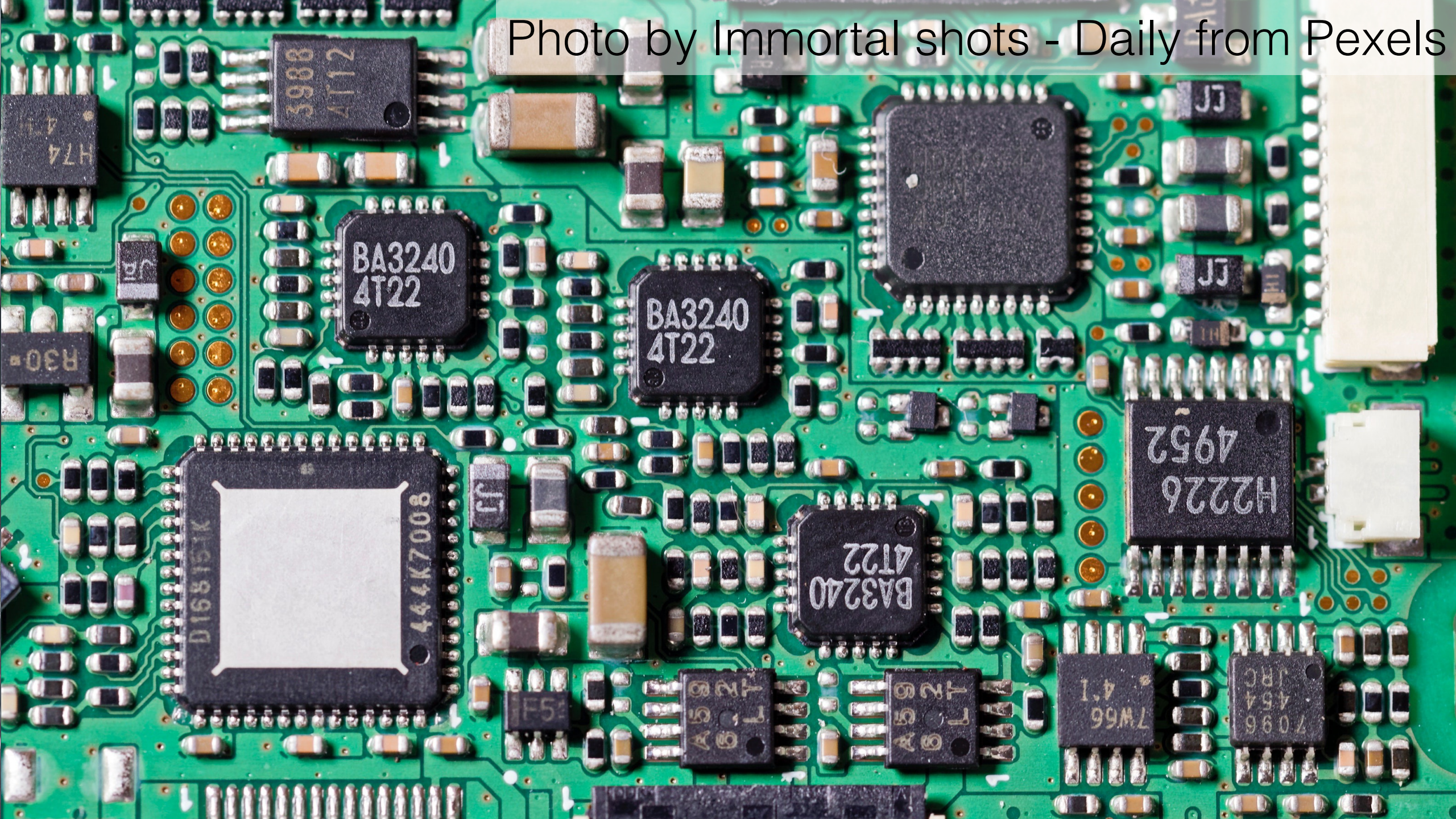


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OSNOVE KVANTNEGA RAČUNALNIŠTVA

MATIJA PRETNAR

Fakulteta za matematiko in fiziko
Univerza v Ljubljani

Math. Subj. Class. (2010): 68Q12, 81P68

Kvantno računalništvo skuša z izkoriščanjem kvantnih pojavov, kakršni sta superpozicija in prepletenost, učinkoviteje reševati računsko zahtevne probleme. V članku si ogledamo postulate kvantne mehanike, kvantno teleportacijo, Deutsch's algoritem, ki je bil prvi kvantni algoritem, ter najznamenitejša kvantna algoritma: Groverjev algoritem za iskanje v neurejeni tabeli in Shorov algoritem za razcep na praštevila.

THE BASICS OF QUANTUM COMPUTING

Quantum computing uses quantum phenomena such as superposition and entanglement in order to efficiently solve computationally hard problems. The paper looks at the quantum mechanics postulates, quantum teleportation, Deutsch's algorithm, which was the first quantum algorithm, and at two most famous quantum algorithms: Grover's search algorithm and Shor's factorization algorithm.

Uvod

V svetu majhnih delcev ne veljajo več pravila klasične Newtonove mehanike, kot smo jih navajeni, temveč nastopijo nenavadna pravila kvantne mehanike. Po teh pravilih se delci obnašajo tako, kot bi bili na več koncih hkrati, vendar le toliko časa, dokler jim ne izmerimo položaja. Ko pa ga enkrat izmerimo, se obnašajo le v skladu z rezultatom meritve. Primer takega vedenja vidimo v poskusu z dvojno režo (slika 1), v katerem streljamo fotone proti zaslonu z dvema ozkima režama, za zaslonom pa postavimo senzor, ki meri, kam so prileteli fotoni, ki jim je uspelo priti mimo.

Če eno od rež zapremo, največ zadetkov po pričakovanjih izmerimo za drugo režo. Če odpremo obe reži, bi tako pričakovali dva vrhova, za vsako režo po enega. V resnici pa dobimo interferenčni vzorec. Lahko bi si ga poskušali razložiti s tem, da fotoni na svoji poti motijo drug drugega, vendar dobimo tako sliko tudi, če poskrbimo, da naenkrat streljamo samo en foton. Torej obe možni poti, po katerih bi lahko potoval foton, motita druga drugo. Še več: če postavimo dodaten senzor, s katerim poskušamo ugotoviti, skozi katero režo je šel foton, interferenca izgine, za zaslonom pa izmerimo vrhova, ki smo ju pričakovali prej.

Kvantno računalništvo poskuša te neobičajne kvantne pojave izkoristiti za učinkovitejše računanje. S kvantnim računalništvom je prišel znani fizik

Obzornik mat. fiz.

HVALA ZA
POZORNOST!